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A SERIES OF TEXTBOOKS PREPARED FOR THE STUDENTS OF THE INTERNATIONAL CORRESPONDENCE SCHOOLS AND CONTAINING IN PERMANENT FORM THE INSTRUCTION PAPERS, EXAMINATION QUESTIONS, AND KEYS USED IN THEIR VARIOUS COURSES

ARITHMETIC
SPELLING
PENMANSHIP
VERTICAL PENMANSHIP

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11

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PREFACE

Formerly it was our practice to send to each student entitled to receive them a set of volumes printed and bound especially for the Course for which the student enrolled. In consequence of the vast increase in the enrolment, this plan became no longer practicable and we therefore concluded to issue a single set of volumes, comprising all our textbooks, under the general title of I. C. S. Reference Library. The students receive such volumes of this Library as contain the instruction to which they are entitled. Under this plan some volumes contain one or more Papers not included in the particular Course for which the student enrolled, but in no case are any subjects omitted that form a part of such Course. This plan is particularly advantageous to those students who enroll for more than one Course, since they no longer receive volumes that are, in some cases, practically duplicates of those they already have. This arrangement also renders it much easier to revise a volume and keep each subject up to date.

Each volume in the Library contains, in addition to the text proper, the Examination Questions and (for those subjects in which they are issued) the Answers to the Examination Questions.

In preparing these textbooks, it has been our constant endeavor to view the matter from the student's standpoint, and try to anticipate everything that would cause him trouble. The utmost pains have been taken to avoid and correct any and all ambiguous expressions—both those due to faulty rhetoric and those due to insufficiency of statement or explanation. As the best way to make a statement, explanation, or description clear is to give a picture or a

diagram in connection with it, illustrations have been used almost without limit. The illustrations have in all cases been adapted to the requirements of the text, and projections and sections or outline, partially shaded, or full-shaded perspectives have been used, according to which will best produce the desired results.

The method of numbering pages and articles is such that each part is complete in itself; hence, in order to make the indexes intelligible, it was necessary to give each part a number. This number is placed at the top of each page, on the headline, opposite the page number; and to distinguish it from the page number, it is preceded by a section mark (§). Consequently, a reference, such as § 3, page 10, can be readily found by looking along the inside edges of the headlines until § 3 is found, and then through § 3 until page 10 is found.

INTERNATIONAL CORRESPONDENCE SCHOOLS

CONTENTS

ARITHMETIC.

DEFINITIONS.

1. **Arithmetic** is the science of numbers and the art of computation.

2. A **unit** is *one*, or a single thing, as *one* boy, *one* horse, *one*, *one* dozen.

3. A **number** is a unit or a collection of units, as *three* apples, *five* boys, *seven.*

4. The **unit of a number** is one of the units included in the collection of units forming the number. Thus, the unit of *twelve* is *one*, of *twenty* dollars is *one* dollar.

5. A **concrete number** is a number applied to some particular kind of object or quantity, as three *horses*, five *dollars*, ten *pounds.*

6. An **abstract number** is a number not applied to any object or quantity, as *three*, *five*, *ten.*

7. **Like numbers** are numbers that express units of the *same kind*, as 6 *days* and 10 *days*, 2 *feet* and 5 *feet.*

8. **Unlike numbers** are numbers that express units of *different kinds*, as ten *months* and eight *miles*, seven *dollars* and five *feet.*

NOTATION AND NUMERATION.

9. Numbers are expressed in three ways: (1) By words. (2) By figures. (3) By letters.

10. **Notation** is the art of expressing numbers by figures or letters.

11. **Numeration** is the art of reading numbers expressed by figures or letters.

ARABIC NOTATION.

12. The **Arabic notation** is the method of expressing numbers by figures. This method employs ten characters, called **figures,** to represent numbers, viz.:

Figures	0	1	2	3	4	5	6	7	8	9
Names	*naught, cipher, or zero*	*one*	*two*	*three*	*four*	*five*	*six*	*seven*	*eight*	*nine*

The first figure (0) is called **naught, cipher,** or **zero,** and, when standing alone, has no value.

The other nine figures are called **digits,** and each one has a value of its own.

An **integer** is any whole number.

13. Since there are only ten *figures* used in expressing numbers, each *figure* must have different *values*, determined by the way in which it is used.

14. The value of a figure depends upon its *position* in relation to others.

15. Figures have **simple** values and **local, or place,** values.

16. The **simple** value of a figure is the value it expresses when standing alone.

17. The **local,** or **place,** value of a figure is its value as determined by its position in a number.

Thus, 6 standing alone means *six ones*..........	6
In the second place it denotes *six tens*..........	60
In the third place it denotes *six hundreds*........	600
In the fourth place it denotes *six thousands*......	6,000
In the fifth place it denotes *six ten-thousands*....	60,000
In the sixth place it denotes *six hundred-thousands*.	600,000
In the seventh place it denotes *six millions*......	6,000,000

18. The **value** of a figure increases tenfold with each remove to the left.

19. The **cipher** has no value in itself, but it is useful in fixing the place of other figures. To represent the number *four hundred five*, only two significant figures are necessary, one to denote *four hundred*, and the other to denote *five;* but if these two figures are placed together, as 45, the 4, being in the second place, will mean 4 *tens*. To denote 4 *hundreds* it should be in the third place. A cipher, therefore, must be inserted in the *tens* place to show that the number is composed of *hundreds* and *units* only, and that there are no *tens*. *Four hundred five* is, therefore, written 405. If the number were *four thousand five*, two ciphers would be inserted, thus, 4,005. If it were *four hundred fifty*, the cipher would be in units place to show that there are no *units*, but only *hundreds* and *tens*, thus, 450. *Four thousand fifty* is written 4,050, the ciphers indicating that there are no hundreds and no units.

20. In reading numbers, it is usual to divide them by commas into groups of three figures each, called **periods,** beginning at the right. The first figure is said to belong to the *first order*, the second to the *second order*, etc. Each **period** contains three orders, named as shown in the table below.

TABLE.

Name of period.	*Trillions.*			*Billions.*			*Millions.*			*Thousands.*			*Units.*		
Name of order.	hundred-trillions.	ten-trillions.	trillions.	hundred-billions.	ten-billions.	billions.	hundred-millions.	ten-millions.	millions.	hundred-thousands.	ten-thousands.	thousands.	hundreds.	tens.	units.
Number.	9	8	7,	4	3	2,	1	9	8,	7	6	5,	4	3	2

The first period, beginning at the right, contains *units*, *tens*, *hundreds;* the second, *thousands*, *ten-thousands*, *hundred-thousands;* the third, *millions*, *ten-millions*, *hundred-millions;* etc.

The number in the table is read, *nine hundred eighty-seven trillion*, *four hundred thirty-two billion*, *one hundred ninety-eight million*, *seven hundred sixty-five thousand*, *four hundred thirty-two.*

21. The writing of numbers is called **notation,** and the reading of numbers is called **numeration.** It will be noticed that in reading and writing numbers the *s* at the end of thousands, millions, etc., is omitted.

ROMAN NOTATION.

22. **Roman notation** is a method of expressing numbers by means of seven capital letters. These letters are I, V, X, L, C, D, and M. Their values, when standing alone, are as follows: I = 1, V = 5, X = 10, L = 50, C = 100, D = 500, M = 1,000. By combinations of these letters all other numbers are expressed. Their combinations are in accordance with the following

PRINCIPLES.

1. *Repeating a letter repeats its value.*

Thus, II = 2, XX = 20, XXX = 30, CC = 200, CCC = 300.

V, D, and L are never repeated, and only I, X, C, and M are ever used more than once in any combination.

2. *If a letter precedes one of greater value, their difference is denoted; if it follows, their sum is denoted.*

Thus, IV = 4, VI = 6, IX = 9, XI = 11, XL = 40, LX = 60.

3. *A bar placed over a letter multiplies its value by one thousand.*

Thus, $\overline{\mathrm{X}}$ = 10,000, $\overline{\mathrm{L}}$ = 50,000, $\overline{\mathrm{XC}}$DXVII = 90,517.

23. The following table illustrates more fully the foregoing principles:

VII	= 7	LX	= 60	CLXIX	= 169	$\overline{\mathrm{IX}}$LX	= 9060
XIII	= 13	LXIX	= 69	CLXXX	= 180	$\overline{\mathrm{L}}$CXC	= 50190
XIV	= 14	LXX	= 70	CCXL	= 240	$\overline{\mathrm{XIX}}$	= 19000
XV	= 15	LXXX	= 80	CCCLIX	= 359	$\overline{\mathrm{XL}}$DX	= 40510
XX	= 20	XC	= 90	CCCCL	= 450	$\overline{\mathrm{XCVII}}$	= 97000
XXV	= 25	XCIX	= 99	CCCCXC	= 490	$\overline{\mathrm{DCC}}$	= 700000
XXVIII	= 28	CIX	= 109	DCCXL	= 740	$\overline{\mathrm{XCIX}}$	= 99000
XXIX	= 29	CXI	= 111	DCCCXI	= 811	$\overline{\mathrm{M}}$	= 1000000
XXX	= 30	CXIX	= 119	DCCCC	= 900	$\overline{\mathrm{MM}}$	= 2000000
XL	= 40	CLVI	= 156	DCCCCXL	= 940	$\overline{\mathrm{MDCC}}$	= 1700000

24. The four fundamental processes of arithmetic are **addition, subtraction, multiplication,** and **division.** They are called fundamental processes because all operations in arithmetic are based upon them.

ADDITION.

25. Addition is the *process* of *finding* a number that is equal to two or more numbers taken together. The sign of addition is +. It is read *plus*, and means *more.* Thus, 5 + 6 is read 5 *plus* 6, and means that 5 and 6 are to be added.

26. The sign of equality is =. It is read *equals*, or *is equal to*. Thus, $5 + 6 = 11$ may be read, *5 plus 6 equals 11.*

27. Only like numbers can be added. Thus, 6 dollars can be added to 7 dollars, and the sum will be 13 dollars, but 6 *dollars* cannot be added to 7 *feet.*

28. The following table gives the sum of any two numbers from 1 to 12:

1 and 1 is 2	2 and 1 is 3	3 and 1 is 4	4 and 1 is 5
1 and 2 is 3	2 and 2 is 4	3 and 2 is 5	4 and 2 is 6
1 and 3 is 4	2 and 3 is 5	3 and 3 is 6	4 and 3 is 7
1 and 4 is 5	2 and 4 is 6	3 and 4 is 7	4 and 4 is 8
1 and 5 is 6	2 and 5 is 7	3 and 5 is 8	4 and 5 is 9
1 and 6 is 7	2 and 6 is 8	3 and 6 is 9	4 and 6 is 10
1 and 7 is 8	2 and 7 is 9	3 and 7 is 10	4 and 7 is 11
1 and 8 is 9	2 and 8 is 10	3 and 8 is 11	4 and 8 is 12
1 and 9 is 10	2 and 9 is 11	3 and 9 is 12	4 and 9 is 13
1 and 10 is 11	2 and 10 is 12	3 and 10 is 13	4 and 10 is 14
1 and 11 is 12	2 and 11 is 13	3 and 11 is 14	4 and 11 is 15
1 and 12 is 13	2 and 12 is 14	3 and 12 is 15	4 and 12 is 16
5 and 1 is 6	6 and 1 is 7	7 and 1 is 8	8 and 1 is 9
5 and 2 is 7	6 and 2 is 8	7 and 2 is 9	8 and 2 is 10
5 and 3 is 8	6 and 3 is 9	7 and 3 is 10	8 and 3 is 11
5 and 4 is 9	6 and 4 is 10	7 and 4 is 11	8 and 4 is 12
5 and 5 is 10	6 and 5 is 11	7 and 5 is 12	8 and 5 is 13
5 and 6 is 11	6 and 6 is 12	7 and 6 is 13	8 and 6 is 14
5 and 7 is 12	6 and 7 is 13	7 and 7 is 14	8 and 7 is 15
5 and 8 is 13	6 and 8 is 14	7 and 8 is 15	8 and 8 is 16
5 and 9 is 14	6 and 9 is 15	7 and 9 is 16	8 and 9 is 17
5 and 10 is 15	6 and 10 is 16	7 and 10 is 17	8 and 10 is 18
5 and 11 is 16	6 and 11 is 17	7 and 11 is 18	8 and 11 is 19
5 and 12 is 17	6 and 12 is 18	7 and 12 is 19	8 and 12 is 20
9 and 1 is 10	10 and 1 is 11	11 and 1 is 12	12 and 1 is 13
9 and 2 is 11	10 and 2 is 12	11 and 2 is 13	12 and 2 is 14
9 and 3 is 12	10 and 3 is 13	11 and 3 is 14	12 and 3 is 15
9 and 4 is 13	10 and 4 is 14	11 and 4 is 15	12 and 4 is 16
9 and 5 is 14	10 and 5 is 15	11 and 5 is 16	12 and 5 is 17
9 and 6 is 15	10 and 6 is 16	11 and 6 is 17	12 and 6 is 18
9 and 7 is 16	10 and 7 is 17	11 and 7 is 18	12 and 7 is 19
9 and 8 is 17	10 and 8 is 18	11 and 8 is 19	12 and 8 is 20
9 and 9 is 18	10 and 9 is 19	11 and 9 is 20	12 and 9 is 21
9 and 10 is 19	10 and 10 is 20	11 and 10 is 21	12 and 10 is 22
9 and 11 is 20	10 and 11 is 21	11 and 11 is 22	12 and 11 is 23
9 and 12 is 21	10 and 12 is 22	11 and 12 is 23	12 and 12 is 24

This table should be carefully committed to memory. Since 0 has no value, the sum of any number and 0 is the number itself; thus 17 and 0 is 17.

29. For *addition*, place the numbers to be added directly under each other, taking care to place *units* under *units*, *tens* under *tens*, *hundreds* under *hundreds*, and so on.

30. Example.—What is the sum of 131, 222, 21, 2, and 413?

Solution.—

$$\begin{array}{rr} & 131 \\ & 222 \\ & 21 \\ & 2 \\ & 413 \\ \hline \textit{sum} & 789 \text{ Ans.} \end{array}$$

Explanation.—After placing the numbers in proper order, begin at the bottom of the units column and add, mentally repeating the different sums. Thus, three, five, six, eight, nine, the sum of the numbers in the units column. Place the 9 directly beneath, as the units figure in the sum.

The sum of the numbers in the tens column is 8, which is the tens figure in the sum.

The sum of the numbers in the hundreds column is 7, which is the hundreds figure in the sum.

31. Example.—What is the sum of 61,803 + 43,429 + 47,712 + 62,138?

Solution.—

$$\begin{array}{rr} & 61803 \\ & 43429 \\ & 47712 \\ & 62138 \\ \hline & 22 \\ & 60 \\ & 2000 \\ & 13000 \\ & 200000 \\ \hline \textit{sum} & 215082 \text{ Ans.} \end{array}$$

Explanation.—The sum of the units column is 22; of the tens column it is 6 tens, or 60; of the hundreds column it is 20 hundreds, or 2,000; of the thousands column it is 13 thousands, or 13,000; and of the ten-thousands column the sum is 20 ten-thousands, or 200,000. The sum of these

numbers is 215,082. Ordinarily, the work would be performed as follows, the unnecessary cipher being omitted:

	61803
	43429
	47712
	62138
	22
	6
	20
	13
	20
sum	215082

This method is very convenient to use when adding a long column of figures.

32. In practice, addition is performed as follows:

	425	
	36	
	9215	
	4	
	907	
sum	10587	Ans.

EXPLANATION.—The sum of the numbers in the units column = 27 units, or 2 tens and 7 units. Write the 7 units as the first, or right-hand, figure in the sum. Reserve the 2 tens and add them to the tens column. The sum of the figures in the tens column plus the 2 tens reserved from the units column = 8, which is written as the second figure in the sum. There is nothing to carry to the next column. The sum of the numbers in the next column is 15, or 1 thousand and 5 hundreds. Write the 5 as the third, or hundreds, figure in the sum and carry the 1 to the next column: $1 + 9 = 10$, which is written at the left of the other figures.

The second method saves space and figures, but the first is to be preferred when adding a long column.

33. Example.—Add the following numbers:

Solution.—

$$\begin{array}{r} 890 \\ 82 \\ 90 \\ 393 \\ 281 \\ 80 \\ 770 \\ 83 \\ 492 \\ 80 \\ 383 \\ 84 \\ 191 \\ \hline \textit{sum}\ 3899 \end{array}\ \text{Ans.}$$

Explanation.—The sum of the first column is 19, or 1 ten and 9 units. Write 9 and carry 1 to the next column. The sum of the second column + 1 = 109 tens, or 10 hundreds and 9 tens. Write 9 and carry 10 to the next column. The sum of this column plus the 10 reserved is 38. Write 38. The total sum is 3,899. Ans.

34. Rule.—I. *Begin at the right, add each column separately, and write the sum, if it be only one figure, under the column added.*

II. *If the sum of any column consists of two or more figures, put the right-hand figure of the sum under that column and add the remaining figure or figures to the next column.*

35. Proof.—*To prove addition, add each column from top to bottom. If the same result is obtained as by adding from bottom to top, the work is probably correct.*

EXAMPLES FOR PRACTICE.

36. Find the sum of:

(*a*)	104 + 203 + 613 + 214.	Ans.	(*a*)	1,134.
(*b*)	1,875 + 3,143 + 5,826 + 10,832.		(*b*)	21,676.
(*c*)	4,865 + 2,145 + 8,173 + 40,084.		(*c*)	55,267.
(*d*)	14,204 + 8,173 + 1,065 + 10,042.		(*d*)	33,484.
(*e*)	10,832 + 4,145 + 3,133 + 5,872.		(*e*)	23,982.
(*f*)	214 + 1,231 + 141 + 5,000.		(*f*)	6,586.
(*g*)	123 + 104 + 425 + 126 + 327.		(*g*)	1,105.
(*h*)	6,354 + 2,145 + 2,042 + 1,111 + 3,333.		(*h*)	14,985.

RAPID ADDITION.

37. There is nothing more useful to the bookkeeper and business man than the ability to add rapidly and correctly; but this can be acquired only by persevering practice. Any time employed in practising addition will be well spent. If the student will practise addition ten or fifteen minutes daily for a month or so, he will be greatly benefited.

In order to become expert in adding, it is absolutely essential that when two figures are seen or heard pronounced, the student can instantly give their sum. Thus, 15 should suggest itself as soon as 6 and 9, or 8 and 7, are seen, or heard pronounced. It should not be necessary to say mentally, 6 and 9 are 15, but 6, 15, the 9 not being pronounced either mentally or orally. (In no case should the student contract the habit of adding aloud; it is not necessary, and it is a very difficult habit to break.)

In adding 5, 6, 1, 9, 7, 5, 2, 4, 8, 9, do not say 5 and 6 is 11, and 1 is 12, and 9 is 21, etc., but *think* 5, 11, 12, 21, 28, 33, 35, 39, 47, 56, repeating the sums about as fast as they can be pronounced.

While rapidity is of great importance, accuracy is more important, and accuracy is attained only by practice. Below will be found some examples on which the student can practise. He should construct similar ones for himself.

(1)	(2)	(3)	(4)	(5)
4175	9286	3582	48275	2681753
5698	5735	6896	27368	5176382
8527	8267	5275	59487	9628625
3489	3489	9182	61735	8763897
7678	9261	7364	98275	6374859
2537	7082	5385	63842	6283926

The answers to the above are: (1) 32,104; (2) 43,120; (3) 37,684; (4) 358,982; (5) 38,909,442.

38. To add rapidly, it is necessary for the student to become accustomed to group the figures of a column and

add the sums of the groups. It is most convenient to choose the groups, as far as possible, so that the sum of each group shall be either 10 or 20. An example will show how this is accomplished:

$$\begin{array}{r} 3 \\ 9 \\ 2 \\ 5 \\ 8 \\ 2 \\ 6 \\ 4 \\ 3 \\ 5 \\ 0 \\ 9 \\ 1 \\ 2 \\ \hline 59 \end{array}$$

Commencing at the bottom of the column, we see at once that 1 and 9 form a group whose sum is 10; hence we mentally say 2, 12 instead of 2, 3, 12. Now the next two figures, 5 and 3, we group together, and instead of adding 5 and 3 separately, we add the sum 8. The next two figures, 6 and 4, form a group whose sum is 10, and so do the next two figures, 8 and 2. Looking now at the four figures at the top of the column, we readily see that 5, 2, and 3 form a last group whose sum is 10, leaving only the 9 outside of a group. In adding the column, we would repeat mentally 2, 12, 20, 30, 40, 50, 59. Another grouping would readily appear to the skilful accountant; the 2 at the bottom and the 5 and 3 above the 0 form a group whose sum is 10. Recognizing this group, the mental addition would be: 10, 20, 30, 40, 50, 59.

This process of forming groups may be extended to include those whose sums are 15 or 20. By a judicious selection of

$$\begin{array}{l} 5 \\ \left.\begin{array}{r} 9 \\ 3 \\ 8 \end{array}\right\} 20 \\ \left.\begin{array}{r} 6 \\ 4 \end{array}\right\} 10 \\ 7 \\ \left.\begin{array}{r} 2 \\ 9 \\ 4 \end{array}\right\} 15 \\ \left.\begin{array}{r} 8 \\ 7 \end{array}\right\} 15 \\ \hline 72 \end{array}$$

the figures composing a group, its sum may usually be made either 10, 15, or 20, and these sums should always be sought in preference to others, since two 15's make 30, and the numbers 10, 20, and 30 are added with little mental labor. Thus, in the following example, the grouping is shown by the braces. The mental addition is 15, 30, 37, 47, 67, 72. After some practice, the student will be able to recognize a group whose sum is 10 or 20 almost instantly; he should persevere in the solution of examples until he is able to form the groups rapidly and with ease. It is not always possible to get consecutive numbers which will form groups whose sums are 10, 15, or 20. Such groups, however, can often be found by skipping one or more figures in the column. For instance, in the following example, the first two figures, 7 and 3, make a 10, and skipping the fourth

$$\begin{array}{r} 3 \\ 4 \\ 8 \\ 3 \\ 0 \\ 6 \\ 9 \\ 4 \\ 3 \\ \underline{7} \\ 47 \end{array}$$

figure 9, the third and fifth figures, 4 and 6, make another 10. Skipping the 8, the 3, 4, and 3 at the top of the column make another 10, thus making three groups of 10 and the figures 9 and 8. In adding, however, we should not leave the figures skipped to be added at the end of the operation, but should add them as they occur. In this example, we would say 10, 20, 29, 39, 47. The order of adding may be shown by the following arrangement:

$$\overbrace{7+3}^{10}+\overbrace{6+4}^{10}+9+\overbrace{3+4+3}^{10}+8.$$

The following examples may be used for practice:

(1)	(2)	(3)	(4)	(5)
3	13	317	1127	13103
5	27	226	2632	61706
2	92	232	1946	43285
6	35	518	3217	39134
3	64	396	1184	96286
4	22	435	1936	45173
0	30	707	2003	78135
9	97	992	5114	26232
3	18	311	976	19375
1	33	417	5634	8428

The answers are: (1) 36; (2) 431; (3) 4,551; (4) 25,769; (5) 430,857.

39. It is frequently advantageous to be able to add *horizontally*, as it is termed; by this is meant the adding of several numbers as they stand in a horizontal row, without arranging the numbers in vertical columns. Thus,

$$123+567+792+221+546=2{,}249. \quad \text{Ans.}$$

When adding in this manner, straight addition must be employed; i. e., the method of Art. **38** cannot be used. The process would be 6, 7, 9, 16, 19; 5, 7, 16, 22, 24; 7, 9, 16, 21, 22. As an example of this method, consider the following table, which is supposed to give, by days, the grain export, in bushels, of a certain city for one week. It is required to find

the amount of grain exported each day, the total amount of each kind of grain exported during the week, and, finally, the total amount of grain exported during the week.

GRAIN EXPORT OF A CITY FOR ONE WEEK (in bushels).

	Mon.	*Tues.*	*Wed.*	*Thur.*	*Fri.*	*Sat.*	*Totals.*
Corn........	28325	15236	35715	29128	75183	46217	*****
Wheat......	35719	41719	50108	32546	59275	81126	*****
Oats.........	12136	9237	18265	7268	6950	17230	*****
Barley......	18230	15738	21375	15928	19263	13637	*****
Rye.........	5275	6829	7201	11325	7825	13261	*****
Totals......	*****	*****	*****	*****	*****	*****	*****

The student should find the totals, and prove that the results are correct by adding the totals in the right-hand column, and then adding the totals in the bottom row; the two results should be the same, viz., 757,270 bushels. The other results are: corn, 229,804; wheat, 300,493; oats, 71,086; barley, 104,171; rye, 51,716; Mon., 99,685; Tues., 88,759; Wed., 132,664; Thurs., 96,195; Fri., 168,496; Sat., 171,471.

EXAMPLES FOR PRACTICE.

40. Find the sums of the following:

(1)	(2)	(3)	(4)	(5)	(6)
4568	15431	7386	49850	6542	62165
7391	29685	45371	17370	63834	16732
7854	73648	13764	68429	76343	85696
53469	34519	9887	23156	80931	71883
13470	78234	64348	21017	79883	50149
58143	7843	14627	67154	83578	31572
			64353	35647	76844

Explanation.—Having found the sums of the above, add the sums horizontally, and thus obtain the total of all the numbers. Ans.: (1) 144,895; (2) 239,360; (3) 155,383;

(4) 311,329; (5) 426,758; (6) 395,041; total, 1,672,766. Add all of the above groups horizontally, beginning with the numbers in the top row. Ans.: Top row, 145,942; second row, 180,383; third row, 325,734; fourth row, 273,845; fifth row, 307,101; sixth row, 262,917; seventh row, 176,844.

SUBTRACTION.

41. In arithmetic, **subtraction** is the process of finding how much greater one number is than another.

The greater of the two numbers is the **minuend.**

The lesser of the two numbers is the **subtrahend.**

The number left after subtracting the subtrahend from the minuend is the **difference,** or **remainder.**

42. The sign of subtraction is —. It is read **minus,** and means *less*. Thus, 12 — 7 is read 12 *minus* 7, and means that 7 is to be taken from 12.

43. Example.—From 7,568 take 3,425.

Solution.—	*minuend*	7 5 6 8	
	subtrahend	3 4 2 5	
	remainder	4 1 4 3	Ans.

Explanation.—Begin at the units column and subtract in succession each figure in the subtrahend from the one directly above it in the minuend, and write the remainders below the line. The result is the remainder.

44. When there are more figures in the minuend than in the subtrahend, and when some figures in the minuend are less than the figures directly under them in the subtrahend, proceed as in the following example:

First Method.—Example.—From 8,453 take 844.

Solution.—	*minuend*	8 4 5 3	
	subtrahend	8 4 4	
	remainder	7 6 0 9	Ans.

Explanation.—Begin at the units column to subtract. We cannot take 4 from 3, and must, therefore, borrow 1 *ten* from 5 in *tens* column, and add, or prefix, it to the 3 units, making 13 units. Then, 4 from 13 equals 9, which is written as the units figure of the remainder.

Since 1 *ten* was borrowed from 5 *tens*, only 4 *tens* remain; 4 from 4 equals 0, the *tens* figure of the remainder.

We cannot take 8 *hundreds* from 4 *hundreds*, and must borrow 1 *thousand* from 8 in the *thousands* column. This equals 10 *hundreds;* and 10 *hundreds* plus 4 *hundreds* equals 14 hundreds; 8 from 14 is 6, the *hundreds* figure of the remainder.

Since 1 *thousand* was borrowed from 8 *thousands*, only 7 thousands remain. There being nothing to subtract from this, the *thousands* figure of the remainder is 7.

45. Second Method.—Example.—From 84,532 take 8,447.

Solution.—

minuend	8 4 5 3 2	
subtrahend	8 4 4 7	
remainder	7 6 0 8 5	Ans.

Explanation.—As in the preceding example, 7 is taken from 12, leaving 5. Now, instead of subtracting 4 from 12, as explained above, 1 is added to 4, and the sum is subtracted from 13, leaving 8. That is, when 1 is borrowed from the minuend it is added to that figure of the subtrahend under the figure of the minuend from which it was borrowed. Continuing, 5 from 5 is 0, 8 from 14 is 6, and 1 from 8 is 7.

The method here described should be thoroughly learned, since it is easier than the first to apply in practice, and has other advantages that the student will doubtless notice.

46. It often happens that there are ciphers in the minuend, from which, of course, nothing can be borrowed. The following example will explain the difficulty:

Example.—From 20,000 take 8,763.

Solution.—

minuend	2 0 0 0 0	
subtrahend	8 7 6 3	
remainder	1 1 2 3 7	Ans.

Explanation.—Although 1 cannot be borrowed from tens column, it is customary to regard the ciphers as tens and increase each figure in the subtrahend by 1, explained in Art. **44.** Then, in performing the above subtraction, the process would be as follows: 3 from 10 is 7; 7 from 10 is 3; 8 from 10 is 2; 9 from 10 is 1; and 1 from 2 is 1.

47. Rule.—*Write the subtrahend under the minuend in the same manner as for addition, with the units of the same order standing in the same column.*

Commencing at the right, subtract each figure in the lower number from the one above it, and write the difference in the line below.

If any figure in the lower number is greater than the one above it, add 10 to the upper figure, perform the subtraction, and then add 1 to the next figure on the left, in the lower number. So proceed with the remaining figures.

48. Proof.—*Add the remainder to the subtrahend. The sum should equal the minuend. If not, the work is wrong.*

Proof of the last example:

subtrahend	8 7 6 3
remainder	1 1 2 3 7
minuend	2 0 0 0 0

49. There is still another method of subtraction better than either of the two previously described. It may be called the addition method. It is as follows:

Taking the example of Art. **44,**

minuend	8 4 5 3	
subtrahend	8 4 4	
remainder	7 6 0 9	Ans.

Instead of subtracting 4 from 13, we add to 4 a number that will make 13; this number is 9, since 4 and 9 is 13. Write the 9 and carry the 1, as in addition; then $4 + 1 = 5$.

Since $5+0=5$, for the next figure in the subtrahend we write 0, as shown. There is nothing to carry; hence, since 8 is greater than 4, we consider 4 to be 14, and find what number added to 8 will make 14; this is 6, which write below the line, as shown. We have 1 to carry, but, as we have no figure in the subtrahend to add it to, we say 1 and 7 is 8, and write the 7 below the line, as shown.

Again, consider the following example:

minuend	10000	
subtrahend	8763	
remainder	1237	Ans.

Here we say 3 and 7 is 10, and write the 7; then, 7 and 3 is 10 (carrying the 1 and adding it to the 6), and write the 3; next, 8 and 2 is 10, and write the 2; finally, 9 and 1 is 10, and write the 1. The student may use whichever method he prefers, but we strongly recommend the addition method as being less productive of errors.

EXAMPLES FOR PRACTICE.

50. From:

(*a*)	94,278 take 62,574.	Ans. (*a*)	31,704.
(*b*)	53,714 take 25,824.	(*b*)	27,890.
(*c*)	71,832 take 58,109.	(*c*)	13,723.
(*d*)	20,804 take 10,408.	(*d*)	10,396.
(*e*)	310,465 take 102,141.	(*e*)	208,324.
(*f*)	(81,043 + 1,041) take 14,831.	(*f*)	67,253.
(*g*)	(20,482 + 18,216) take 21,214.	(*g*)	17,484.
(*h*)	(2,040 + 1,213 + 542) take 3,791.	(*h*)	4.

51. In many cases it is inconvenient to write the subtrahend *under* the minuend. For example, it might be required to find the sum of a series of numbers, and to subtract this sum from some other number. In such cases, both time and space may be saved by writing the minuend under the subtrahend, drawing the line, and then subtracting downwards.

The rule given in Art. **47** may be applied to this case by changing the words *under* to read *above; above* to read *below; lower* to read *upper;* and *upper* to read *lower.*

EXAMPLE.—Writing the subtrahend above the minuend, subtract 5,267,148 from 10,342,927.

SOLUTION.—	*subtrahend*	5 2 6 7 1 4 8	
	minuend	1 0 3 4 2 9 2 7	
	remainder	5 0 7 5 7 7 9	Ans.

EXAMPLES FOR PRACTICE.

52. In the following examples, subtract the upper number from the lower:

(1)	(2)	(3)	(4)
23456789	37529510	9890978	19827
80706040	43184296	100010001	84362

(5)	(6)	(7)	(8)
7090508	9897960	81907	3161746789
8090403	123456789	94371	4213150000

Ans.—(1) 57,249,251; (2) 5,654,786; (3) 90,119,023; (4) 64,535; (5) 999,895; (6) 113,558,829; (7) 12,464; (8) 1,051,403,211.

GENERAL REMARKS ON SUBTRACTION.

53. Subtraction is a much simpler and easier operation than addition; but the student should, nevertheless, study the subject thoroughly. Its very simplicity is deceiving, and it is probable that more mistakes are made in subtraction than in addition. The student should practise subtraction until he feels that he has thoroughly mastered the subject. He should not try to subtract rapidly at first, but endeavor to make as few mistakes as possible. When he has attained such proficiency that he can solve, say, ten examples similar to those given in Art. **52,** a part of them having the

subtrahend above the minuend, without making a mistake, he may then strive for rapidity. A rapid calculator will subtract as fast as he can write the figures of the result.

It will be a great help to the student if he will reverse the addition table. At odd moments, when walking or working, let him say to himself, 6 from 11 is how much? 9 from 17 is what? etc., and in a short time the right answer will present itself without any mental effort whatever.

54. Addition and subtraction form an extremely important part of a bookkeeper's work. The books must *balance*, as it is termed; i. e., the sum of the columns on the debit side must *exactly* equal the sum of the columns on the credit side. If an error of even one cent is made it will manifest itself in the trial balance, and a day or more may be spent in finding the error. Hence, the importance of accuracy. It is also important that no time be wasted in adding the columns, and in subtracting the sums of the two columns to find the balance. To illustrate, we give part of a page of a ledger. The double vertical lines separate the debit side on the left from the credit side on the right.

DR.			PRICE & HOWARD.					CR.	
1888					1888				
June	4	Merchandise,	$1893	42	June	14	Merchandise,	$1538	38
"	20	"	149	37	"	30	Cash,	500	
July	3	Sundries,	115	26	July	17	Bills Receivable,	900	
Aug.	16	Merchandise,	1326	97	Aug.	1	Cash,	375	
"	30	Sundries,	490	63	"	24	Merchandise,	984	88
Sept.	19	Merchandise,	1085	75	Oct.	1	Balance,	763	14
			$5061	40				$5061	40
Oct.	1	Balance,	$ 763	14					

Now, what the bookkeeper has to do is to add the two columns and subtract the less sum from the greater; then he must write the difference in the column that contains the less sum, and also write it under the total of the column containing the greater sum. This difference is indicated by the word *balance*. In order not to mar the appearance of the ledger, no figures except those written above should be used. If the

student were obliged to write the two sums on a piece of waste paper in order to subtract them, considerable time would be lost, and confusion would result. There is a much shorter and easier method, when it can be readily seen which column contains the greater sum. It is as follows:

In the above account it is readily seen that the debit, or left-hand column, is the greater. Hence, add this column, and write the result, \$5,061.40, underneath (the period separates the dollars from the cents). Now add the extreme right-hand column—8, 16—and subtract 16 from the first figure (right-hand figure) of the total in the debit column. This figure is a cipher; hence, we prefix 2 to the cipher, making 20; whence, $20 - 16 = 4$, which write in the credit column, as shown. Now add the second column on the credit side, carrying the 2—thus, 2, 10, 13—obtaining 13 as the sum. Now subtract 13 from 4, the second figure of the total on the debit side. But 13 cannot be taken from 4; hence, we prefix 1 to the 4, and 13 from 14 leaves 1, which write in the second column on the credit side, as shown. Then add the third column of the credit side, first adding the 1 that was prefixed—thus, 1, 5, 10, 18—obtaining 18 as the sum; subtract 18 from 21 (prefixing 2) and get 3, which write in the third column on the credit side. The sum of the fourth column, with the 2 that was prefixed, is 20. Subtracting 20 from 6, prefixing 2 to the 6, getting 26, leaves 6, which write in the fourth column on the credit side. The sum of the fifth column, with the 2 that was prefixed, is 33. Subtracting 33 from 40, formed by prefixing 4 to 0, leaves 7, which write in the fifth column on the credit side. Carrying 4 to the sixth column and adding it to the 1 gives 5, and 5 from 5 equals 0. Hence, the balance is \$763 14. If the work has been done correctly, the debit and credit sides, when added, should give the same total. Adding the credit side, the total is \$5,061.40, the same as the debit side; hence, the work is correct.

55. The student will find the addition method of subtraction the best to use in this case. Thus, adding the first

column on the right, the sum is 16. The right figure of 5,061.40 is a cipher; hence, we write 2 before the cipher, getting 20, and say 16 and 4 is 20, and write the 4 on the credit side, as shown. Now, carrying the 2, the sum of the second column is 13, and 13 and 1 is 14, writing the 1 as before. Carrying the 1, the sum of the numbers in the third column is 18, and 18 and 3 is 21; write the 3 and carry the 2. The sum of the fourth column plus the 2 carried is 20, and 20 and 6 is 26; write the 6 and carry the 2. The sum of the fifth column plus the 2 carried is 33, and 33 and 7 is 40; write the 7 and carry the 4. The sum of the sixth column plus the 4 carried is 5, and 5 and 0 is 5.

The student should apply both methods in solving the following examples.

EXAMPLES FOR PRACTICE.

56. Let the student practise this method on the following examples, one of which is worked out. If it is not apparent which set of numbers is the greater, he should add up the two left-hand columns and be guided by the sums so obtained.

(1)

	12394		33604
	47826		9775
	52482		11628
	26103		84721
	40079		5109
	178884	*Balance*	34047
Balance	34047		178884

(2)

81720	81220
22222	21413
42730	37500
81075	41200
41630	71700
22500	
71946	

(3)

23725	11227
90000	21836
80000	71749
71824	64800
21875	11875
	53898
	20313

(4)

107500	37560
231842	21824
81210	71737
93840	24445
431200	94633
	22248
	10875

By the same method perform the following:

	(5)	(6)	(7)
minuend	$301.23	$4,214.60	9234621
subtrahend	24.99	875.95	897742
	63.75	469.98	3456987
	9.87	93.49	747678
	48.84	721.74	498765
	59.28	1803.42	1768496
remainder			

Ans.— (1) 34,047; (2) 110,790; (3) 31,726; (4) 662,270; (5) $94.50; (6) $250.02; (7) 1,864,953.

MULTIPLICATION.

57. To **multiply** a number is to *add* the number to itself a certain number of times.

58. **Multiplication** is the process of multiplying one number by another.

The *number* thus added to itself, or the number to be multiplied, is called the **multiplicand.**

The *number* that shows how many times the *multiplicand* is to be taken, or the *number* by which we *multiply*, is called the **multiplier.**

The result obtained by multiplying is called the **product.**

59. The sign of multiplication is $\times$. It is read *times*, or *multiplied by*. Thus, 9×6 is read 9 *times* 6, or 9 *multiplied by* 6.

60. It matters not in what order the numbers to be multiplied together are placed. Thus, 6×9 is the same as 9×6.

MULTIPLICATION TABLE.

61. In the following table, the product of any two numbers (neither of which exceeds twelve) may be found:

1 times 1 is 1	2 times 1 is 2	3 times 1 is 3
1 times 2 is 2	2 times 2 is 4	3 times 2 is 6
1 times 3 is 3	2 times 3 is 6	3 times 3 is 9
1 times 4 is 4	2 times 4 is 8	3 times 4 is 12
1 times 5 is 5	2 times 5 is 10	3 times 5 is 15
1 times 6 is 6	2 times 6 is 12	3 times 6 is 18
1 times 7 is 7	2 times 7 is 14	3 times 7 is 21
1 times 8 is 8	2 times 8 is 16	3 times 8 is 24
1 times 9 is 9	2 times 9 is 18	3 times 9 is 27
1 times 10 is 10	2 times 10 is 20	3 times 10 is 30
1 times 11 is 11	2 times 11 is 22	3 times 11 is 33
1 times 12 is 12	2 times 12 is 24	3 times 12 is 36
4 times 1 is 4	5 times 1 is 5	6 times 1 is 6
4 times 2 is 8	5 times 2 is 10	6 times 2 is 12
4 times 3 is 12	5 times 3 is 15	6 times 3 is 18
4 times 4 is 16	5 times 4 is 20	6 times 4 is 24
4 times 5 is 20	5 times 5 is 25	6 times 5 is 30
4 times 6 is 24	5 times 6 is 30	6 times 6 is 36
4 times 7 is 28	5 times 7 is 35	6 times 7 is 42
4 times 8 is 32	5 times 8 is 40	6 times 8 is 48
4 times 9 is 36	5 times 9 is 45	6 times 9 is 54
4 times 10 is 40	5 times 10 is 50	6 times 10 is 60
4 times 11 is 44	5 times 11 is 55	6 times 11 is 66
4 times 12 is 48	5 times 12 is 60	6 times 12 is 72
7 times 1 is 7	8 times 1 is 8	9 times 1 is 9
7 times 2 is 14	8 times 2 is 16	9 times 2 is 18
7 times 3 is 21	8 times 3 is 24	9 times 3 is 27
7 times 4 is 28	8 times 4 is 32	9 times 4 is 36
7 times 5 is 35	8 times 5 is 40	9 times 5 is 45
7 times 6 is 42	8 times 6 is 48	9 times 6 is 54
7 times 7 is 49	8 times 7 is 56	9 times 7 is 63
7 times 8 is 56	8 times 8 is 64	9 times 8 is 72
7 times 9 is 63	8 times 9 is 72	9 times 9 is 81
7 times 10 is 70	8 times 10 is 80	9 times 10 is 90
7 times 11 is 77	8 times 11 is 88	9 times 11 is 99
7 times 12 is 84	8 times 12 is 96	9 times 12 is 108
10 times 1 is 10	11 times 1 is 11	12 times 1 is 12
10 times 2 is 20	11 times 2 is 22	12 times 2 is 24
10 times 3 is 30	11 times 3 is 33	12 times 3 is 36
10 times 4 is 40	11 times 4 is 44	12 times 4 is 48
10 times 5 is 50	11 times 5 is 55	12 times 5 is 60
10 times 6 is 60	11 times 6 is 66	12 times 6 is 72
10 times 7 is 70	11 times 7 is 77	12 times 7 is 84
10 times 8 is 80	11 times 8 is 88	12 times 8 is 96
10 times 9 is 90	11 times 9 is 99	12 times 9 is 108
10 times 10 is 100	11 times 10 is 110	12 times 10 is 120
10 times 11 is 110	11 times 11 is 121	12 times 11 is 132
10 times 12 is 120	11 times 12 is 132	12 times 12 is 144

This table should be carefully committed to memory.

Since 0 has no value, the product of 0 and any number is 0.

62. Practise the following until you can give them all readily without referring to the table.

3×7	12×9	9×2	4×4	6×7
4×8	7×8	5×9	6×3	12×8
5×12	9×6	7×4	10×11	10×4
6×11	10×10	9×11	8×11	8×10
7×10	11×11	10×7	7×12	4×7
8×9	12×12	8×8	9×10	10×3
9×5	7×7	4×5	10×6	12×2
5×8	8×5	5×10	11×9	4×12
3×10	2×8	6×12	12×11	3×2
4×3	2×12	7×9	2×5	11×3
3×11	9×3	6×10	3×8	11×2
2×10	3×12	6×6	6×2	11×6
5×4	8×3	12×5	10×5	12×10
10×9	3×6	8×2	11×12	10×2
11×8	6×4	9×12	12×4	4×11
8×4	5×11	12×6	7×5	2×7
5×3	12×7	11×5	2×6	3×4
6×8	9×9	4×9	3×5	2×4
7×2	8×12	5×5	4×2	12×3
9×8	3×3	7×6	5×2	9×4
8×7	4×10	9×7	6×9	10×12
3×9	5×7	10×8	7×11	11×4
4×6	6×5	11×10	7×3	2×3
5×6	11×7	2×11	2×9	8×6

63. To multiply any number by a number of one figure.

EXAMPLE.—Multiply 425 by 5.

SOLUTION.—

multiplicand	4 2 5	
multiplier	5	
product	2 1 2 5	Ans.

EXPLANATION.—For convenience, the multiplier is written under the right-hand figure of the multiplicand. Multiplying the first figure at the right of the multiplicand, or 5, by the multiplier 5, the result is 5 times 5 units are 25 units, 2 tens and 5 units. Write the 5 units in units place in the product, and reserve the 2 tens to add to the product of tens. Multiplying the second figure of the multiplicand by the multiplier 5, the result is 10 tens, which plus the 2 tens reserved, is 12

tens, or 1 hundred plus 2 tens. Write the 2 tens in tens place, and reserve the 1 hundred to add to the product of hundreds. Multiplying the third, or last, figure of the multiplicand by the multiplier 5, the result is 20 hundreds, which plus the 1 hundred reserved, is 21 hundreds, or 2 thousands 1 hundred, which we write in the thousands and hundreds places, respectively.

Hence, the product is 2,125.

This result is the same as the sum of five 425's. Thus,

$$\begin{array}{rr} & 425 \\ & 425 \\ & 425 \\ & 425 \\ & \underline{425} \\ \textit{sum} & 2125 \text{ Ans.} \end{array}$$

EXAMPLES FOR PRACTICE.

64. Find the product of:

(*a*)	61,483 × 6.	(*a*)	368,898.
(*b*)	12,375 × 5.	(*b*)	61,875.
(*c*)	10,426 × 7.	(*c*)	72,982.
(*d*)	10,835 × 3.	(*d*)	32,505.
(*e*)	98,376 × 4.	Ans. (*e*)	393,504.
(*f*)	10,873 × 8.	(*f*)	86,984.
(*g*)	71,543 × 9.	(*g*)	643,887.
(*h*)	218,734 × 2.	(*h*)	437,468.

65. To multiply a number by a number of two or more figures.

EXAMPLE.—Multiply 475 by 234.

SOLUTION.—

$$\begin{array}{rr} \textit{multiplicand} & 475 \\ \textit{multiplier} & \underline{234} \\ & 1900 \\ & 1425 \\ & \underline{950} \\ \textit{product} & 111150 \text{ Ans.} \end{array}$$

EXPLANATION.—For convenience, the multiplier is generally written under the multiplicand, placing units under units, tens under tens, etc.

We cannot multiply by 234 at one operation; we must,

therefore, multiply by the parts and then add the **partial products.**

The parts by which we are to multiply are 4 units, 3 tens, and 2 hundreds. 4 times 475 = 1,900, the first partial product; 3 times 475 = 1,425, the second partial product, the right-hand figure of which is written directly under the figure multiplied by, or 3; 2 times 475 = 950, the third partial product, the right-hand figure of which is written directly under the figure multiplied by, or 2.

The sum of these three partial products is 111,150, which is the entire product.

66. Rule.—I. *Write the multiplier under the multiplicand, so that units are under units, tens under tens, etc.*

II. *Begin at the right, and multiply each figure of the multiplicand by each successive figure of the multiplier, placing the right-hand figure of each partial product directly under the figure used as a multiplier.*

III. *The sum of the partial products will be the required product.*

67. Proof.—*Review the work carefully, or multiply the multiplier by the multiplicand; if the results agree, the work is correct.*

68. The student will find the following test useful in determining whether the answer in multiplication is correct: *Find the sum of the digits in the multiplicand. If the sum consists of more than one figure, add the digits of the sum, and so continue until the sum is one figure. Do the same with the multiplier. Multiply together the final sums thus obtained, and if the result consists of more than one figure, add its digits until one figure is obtained. If this result is the same as is obtained by adding the digits of the product until one figure is obtained, the work is probably correct.*

To illustrate, multiply 837,295 by 4,631.

SOLUTION.—	*multiplicand*	8 3 7 2 9 5
	multiplier	4 6 3 1
	product	3 8 7 7 5 1 3 1 4 5

PROOF.— $8+3+7+2+9+5 = 34;\ 3+4 = 7$
$4+6+3+1 = 14;\ 1+4 = 5$
$35;\ 3+5 = 8.$
$3+8+7+7+5+1+3+1+4+5 = 44;\ 4+4 = 8.$

The proof given in this article is not absolute, because two or more errors might cause the product to fulfil the conditions of the test. But if, upon trial, the final sum of the digits of the product does not agree with that of the product of the final sums of the multiplicand and the multiplier, it is certain that the work is wrong.

69. There are many short methods of multiplication, and some of these will be given under the heading "Aliquot Parts." It is important that the student should notice the abbreviation that is possible when a cipher occurs in the multiplier, and when the multiplicand or the multiplier ends with one or more ciphers.

70. EXAMPLE.—Multiply 49,076 by 40,807.

SOLUTION.—

multiplicand	49076
multiplier	40807
partial products	343532
	392608
	196304
product	2002644332 Ans.

EXPLANATION.—The process is exactly the same as the preceding, except that when a cipher occurs in the multiplier it is not used to multiply by, the next digit of the multiplier being used instead. The first figure of the partial product is always written directly under the figure by which we multiply, as stated in the rule for multiplication, Art. **66.**

EXAMPLE.—Multiply 49,076 by 48,700.

SOLUTION.—

multiplicand	49076
multiplier	48700
partial products	343532
	392608
	196304
product	2390001200 Ans.

EXPLANATION.—Here the multiplier consists of three digits

and two ciphers, as in the preceding examples, but in this case the two ciphers occupy the units and tens places in the multiplier. Write the multiplier as shown, so that the ciphers lie to the right of the right-hand figure of the multiplicand, or, in other words, so that the right-hand *digit* of the multiplier lies under the right-hand digit of the multiplicand. Then, without paying attention to the ciphers on the right of the multiplier, multiply in the usual manner, annexing the two ciphers (the number of ciphers to the right of the right-hand digit of the multiplier) to the product, as shown. If the multiplicand ends in ciphers, the process is exactly the same; thus, multiplying 4,907,600 by 487:

multiplicand	4907600
multiplier	487
partial products	343532
	392608
	196304
product	2390001200 Ans.

If both multiplicand and multiplier end in ciphers, place the right-hand digits under each other, as above, and add to the product as many ciphers as are contained in both multiplicand and multiplier on the right of their right-hand digits.

EXAMPLE.—Multiply 590,000 by 420.

SOLUTION.—	*multiplicand*	590000
	multiplier	420
		118
		236
	product	247800000 Ans.

71. It would be well to apply the principle given in Art. **68** to all cases of multiplication, until the student has attained confidence. Thus, in the first example of Art. **70,** the number obtained by adding the digits of the multiplicand is $4+7+9+6 = 26$, and $2+6 = 8$; by adding the digits of the multiplier, $4+8+7 = 19$, $1+9 = 10$, and $1+0 = 1$. Whence, $8 \times 1 = 8$. Adding the digits of the product, $2+2+6+4+4+3+3+2 = 26$, or $2+6 = 8$; hence, the work is very probably correct.

72. To be able to multiply rapidly is almost as valuable to a bookkeeper or business man as to be able to add rapidly. The only way that any one can become expert in multiplying is by practice. To give the student a good idea of how a rapid multiplier would proceed, we will now give the entire process pursued, choosing two numbers whose digits consist of 7's, 8's, and 9's only, as these are the hardest to use.

Example.— $987789 \times 897 = ?$

Solution.—

987789	sum of digits = 48 = 12, or	3
897	sum of digits = 24, or	6
6914523		18, or 9
8890101		
7902312		
886046733	sum of digits = 45, or 9.	

Explanation.—Say *7 times 9 is 63;* write the 3 and carry 6. Say *7 times 8 is 56 and 6 is 62;* write the 2 and carry 6. Say *7 times 7 is 49 and 6 is 55;* write the 5 and carry 5. Say *7 times 7 is 49 and 5 is 54;* write the 4 and carry 5. Say *7 times 8 is 56 and 5 is 61;* write the 1 and carry 6. Say *7 times 9 is 63 and 6 is 69;* write the 69. We multiply by 9 and 8 in the same way, and then prove the work by the test given in Art. **68.**

73. After the student has attained considerable proficiency in multiplying as described in Art. **72,** he may shorten his work considerably by merely repeating the digit by which he multiplies, the product of that digit and the desired digit in the multiplicand, and the sum of this product and the number carried. Thus, instead of saying *7 times 8 is 56 and 6 is 62*, think *7, 56, 62.* In other words, in multiplying 987,789 by 9, think *9, 81* (write the 1); *9, 72, 80* (write the 0); *9, 63, 71; 9, 63, 70; 9, 72, 79;* and, finally, *9, 81, 88.* By practising this method for some time, he should be able to multiply nearly as fast as he can write the results.

74. Besides working the examples for practice which follow, the student should make up many others and work them out. He should continue to do this until he can

multiply rapidly, and with ease and certainty. This remark applies, also, to addition and subtraction, and to division, which follows. He should study the multiplication table until he can name the product of any two numbers between 1 and 12 instantly, without any hesitation whatever. He can learn to do this only by repeating the table over and over again. The result attained will be well worth the time and labor spent.

EXAMPLES FOR PRACTICE.

75. Find the product of:

		Ans.
(*a*)	3,842 × 26.	(*a*) 99,892.
(*b*)	3,716 × 45.	(*b*) 167,220.
(*c*)	1,817 × 124.	(*c*) 225,308.
(*d*)	675 × 38.	(*d*) 25,650.
(*e*)	1,875 × 33.	(*e*) 61,875.
(*f*)	4,836 × 47.	(*f*) 227,292.
(*g*)	5,682 × 543.	(*g*) 3,085,326.
(*h*)	3,257 × 246.	(*h*) 801,222.
(*i*)	2,875 × 302.	(*i*) 868,250.
(*j*)	17,819 × 1,004.	(*j*) 17,890,276.
(*k*)	38,674 × 205.	(*k*) 7,928,170.
(*l*)	18,304 × 100.	(*l*) 1,830,400.
(*m*)	7,834 × 10.	(*m*) 78,340.
(*n*)	87,543 × 1,000.	(*n*) 87,543,000.
(*o*)	48,763 × 100.	(*o*) 4,876,300.

DIVISION.

76. Division is the process of finding how many times one number is contained in another of the same kind, or it is the process of separating a number into a given number of equal parts. Thus, to separate 48 dollars into four equal amounts is division.

This form of division is generally called **partition.**

77. The **dividend** is the number to be divided, or to be separated into equal parts.

78. The **divisor** is the number by which the dividend is divided.

79. The **quotient** is the number showing how many times the dividend contains the divisor. In *partition* the quotient shows one of the equal parts of the dividend.

80. The **sign of division** is ÷. It is read *divided by*. Thus, 54 ÷ 9 denotes that 54 is to be divided by 9. In this case 54 is the *dividend* and 9 is the *divisor*.

81. To divide when the divisor consists of but one figure.

EXAMPLE.—What is the quotient of 875 ÷ 7?

SOLUTION.—

```
          divisor dividend quotient
                7 ) 8 7 5 ( 1 2 5  Ans.
                    7
                    ---
                    1 7
                    1 4
                    -----
                      3 5
                      3 5
                      ---
remainder               0
```

EXPLANATION.— 7 is contained in 8 hundreds 1 hundred times. Place the 1 as left-hand figure of the quotient. Multiply the divisor 7 by the 1 hundred of the quotient, and place the product, 7 hundreds, under the 8 hundreds in the dividend, and subtract. On the right of the remainder 1, bring down the next, or *tens* figure of the dividend, in this case 7, making 17 tens; 7 is contained in 17, 2 times. Write the 2 as the second figure of the quotient. Multiply the divisor 7 by the 2 in the quotient, and subtract the product from 17. To the remainder, 3, annex the next, or *units* figure of the dividend, in this case 5, making 35 units. 7 is contained in 35, 5 times, which is placed in the quotient. Multiplying the divisor by the last figure of the quotient, 5 times 7 = 35, which subtracted from 35, under which it is placed, leaves 0. Therefore, the quotient is 125. This method is called **long division.**

82. In **short division,** only the divisor, dividend, and quotient are written.

$$\begin{array}{rl} & \textit{dividend} \\ \textit{divisor}\ \ 7\,) & \underline{8^1\ 7^3\ 5} \\ \textit{quotient} & 1\ \ 2\ \ 5\ \ \text{Ans.} \end{array}$$

The operation is as follows: 7 is contained in 8 once and 1 remainder; 1 placed before 7 makes 17; 7 is contained in 17 2 times and 3 over; the 3 placed before 5 makes 35; 7 is contained in 35, 5 times. These partial quotients placed in order as they are found, make the entire quotient, 125.

83. If the divisor consists of *2 or more* figures, proceed as in the following example:

EXAMPLE.—Divide 2,702,826 by 63.

$$\begin{array}{llll} & \textit{divisor} & \textit{dividend} & \textit{quotient} \\ \text{SOLUTION.}— & 63\,) & 2702826 & (42902\ \ \text{Ans.} \\ & & \underline{252} & \\ & & \ \ 182 & \\ & & \ \ \underline{126} & \\ & & \ \ \ \ 568 & \\ & & \ \ \ \ \underline{567} & \\ & & \ \ \ \ \ \ \ 126 & \\ & & \ \ \ \ \ \ \ \underline{126} & \\ & \textit{remainder} & \ \ \ \ \ \ \ \ \ \ 0 & \end{array}$$

EXPLANATION.—As 63 is not contained in the first two figures, 27, we must use the first three figures, 270. Now, by trial we must find how many times 63 is contained in 270; 6 is contained in the first two figures of 270, 4 times. Place the 4 as the first figure in the quotient. Multiply the divisor, 63, by 4, and subtract the product 252 from 270. The remainder is 18, to which we annex the next figure of the dividend, 2, making 182. Now, 6 is contained in the first two figures of 182, 3 times, but on multiplying 63 by 3, we see that the product 189 is too great, so we try 2 as the second figure of the quotient. Multiplying the divisor 63 by 2, and subtracting the product 126 from 182, the remainder is 56, to which we annex the next figure of the dividend, making 568; 6 is contained in 56 about 9 times. Multiply the divisor 63 by 9, and subtract the product 567 from 568.

The remainder is 1, and bringing down the next figure of the dividend, 2, gives 12. As 12 is less than 63, we write 0 in the quotient and bring down the next figure, 6, making 126; 63 is contained in 126, 2 times, without a remainder. Therefore, 42,902 is the quotient.

84. Rule.—I. *Write the divisor at the left of the dividend with a curved line between them.*

II. *Find how many times the divisor is contained in the least number of the left-hand figures of the dividend that will contain it, and write the result at the right of the dividend, with a line between, as the first figure of the quotient.*

III. *Multiply the divisor by this quotient; write the product under the partial dividend used, and subtract, annexing to the remainder the next figure of the dividend. Divide as before, and continue thus until all the figures of the dividend have been used.*

IV. *If any partial dividend will not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend and proceed as before.*

V. *If there is at last a remainder, write it after the quotient, with the divisor underneath.*

85. Proof.—*Multiply the quotient by the divisor, and add the remainder, if any, to the product. The result will be the dividend.*

$$
\begin{array}{lrl}
 & \textit{divisor}\ \textit{dividend}\ \textit{quotient} & \\
\text{Thus,} & 63)4235(67\tfrac{14}{63} & \text{Ans.} \\
 & \underline{378} & \\
 & 455 & \\
 & \underline{441} & \\
 & \textit{remainder}\quad 14 & \\
\text{Proof.—} & \textit{quotient}\quad 67 & \\
 & \textit{divisor}\quad \underline{63} & \\
 & 201 & \\
 & \underline{402} & \\
 & 4221 & \\
 & \textit{remainder}\quad \underline{14} & \\
 & \textit{dividend}\quad 4235 &
\end{array}
$$

SHORT METHOD OF DIVISION.

86. The following method saves about half the figures required by the method just given, and we think that there will be fewer mistakes made when using it.

87. EXAMPLE.—Divide 39,913,910 by 5,494.

SOLUTION.—

```
       dividend       divisor
      39913910)5494
      -----------------
       14559   7265 quotient
        35711
         27470
          0000
```

EXPLANATION.—The addition method of subtraction (see Art. **49**) is used in this case; the divisor is written on the right of the dividend, and the quotient underneath the divisor. The different figures of the quotient are obtained in exactly the same manner as by the preceding method. Thus, the divisor is contained in the first five figures of the dividend 7 times, and 7 is written for the first figure of the quotient. Now, instead of multiplying the divisor by 7, writing the product under the first five figures of the dividend, and then subtracting, we multiply each figure of the divisor by 7, and by the addition method, subtract from the dividend, writing only the remainder. Thus, 7 times 4 is 28, and 5 is 33; write the 5 under the 3 in the dividend, and carry 3. Then, 7 times 9 is 63 and 3 is 66, and 66 and 5 is 71; write the 5 and carry 7. 7 times 4 is 28 and 7 is 35, 35 and 4 is 39; write the 4 and carry 3. 7 times 5 is 35 and 3 is 38, and 38 and 1 is 39; write the 1. Now bring down the next figure of the dividend, 9, and annex it to the remainder. $14,559 \div 5,494 = 2$; write 2 as the second figure of the quotient. Then, as above, $2 \times 4 = 8$, and $8 + 1 = 9$; write the 1 under the 9, as shown. $2 \times 9 = 18$, and $18 + 7 = 25$; write the 7 and carry the 2. $2 \times 4 = 8$, $8 + 2 = 10$, and $10 + 5 = 15$; write the 5 and carry the 1. $2 \times 5 = 10$, $10 + 1 = 11$, and $11 + 3 = 14$; write the 3. Bringing down 1, the next figure of the dividend, $35,711 \div 5,494 = 6$, the third figure of the quotient. Proceed as above with the remaining figures.

A fast computer would work as follows: In multiplying by 6, he would repeat to himself 6, 24, and 7 is 31 (writing the 7 and carrying the 3). 6, 54, 57, and 4 is 61. 6, 24, 30, and 7 is 37. 6, 30, 33, and 2 is 35.

The object of writing the divisor on the right is to make it easier to multiply by the figures of the quotient; it also saves space, as may readily be seen. The student is strongly advised to learn this method thoroughly, and always to use it. The best way to attain facility in division, is first to practise dividing by small numbers, from 2 to 12, and using the method of short division. After he has become proficient in this, he should practise long division by the method just described. Some special methods which may be used when dividing by certain numbers will be mentioned farther on.

88. The principle given in Art. **68** may be used to test the work of division, when the principle has been slightly modified. Add the digits of the divisor, the dividend, the quotient, and the remainder, if any, as described in Art. **68**, obtaining a single figure for the sum of each. Multiply the number thus obtained for the divisor by the number obtained for the quotient, and add to the product the number obtained for the remainder, if any. If the work has been done correctly, the result must equal the number obtained for the dividend. Thus, in the last example, the sum of the digits in the divisor (reduced to a single figure) is 4, of those in the quotient, 2, and in the remainder, 0. Hence, $4 \times 2 = 8$; $8 + 0 = 8$. Adding the digits in the dividend, the result (reduced to a single figure) is also 8; hence, the work is very probably correct.

Applying this method to the example in Art. **85**, we have, for the divisor 9, for the quotient 4, for the remainder 5. Hence, $9 \times 4 + 5 = 41$, and $4 + 1 = 5$. For the dividend, $4 + 2 + 3 + 5 = 14$, and $1 + 4 = 5$, also. The student will find this principle very useful.

Addition, subtraction, multiplication, and division, are the four corner stones of arithmetic; everything else in arithmetic depends upon them.

EXAMPLES FOR PRACTICE.

89. Divide the following:

			Ans.
(*a*)	126,498 by 58.	(*a*)	2,181.
(*b*)	3,207,594 by 767.	(*b*)	4,182.
(*c*)	11,408,202 by 234.	(*c*)	48,753.
(*d*)	2,100,315 by 581.	(*d*)	3,615.
(*e*)	969,936 by 4,008.	(*e*)	242.
(*f*)	7,481,888 by 1,021.	(*f*)	7,328.
(*g*)	1,525,915 by 5,003.	(*g*)	305.
(*h*)	1,646,301 by 381.	(*h*)	4,321.

CANCELATION.

90. **Cancelation** is the process of shortening operations in division by casting out equal factors from both dividend and divisor.

91. The **factors** of a number are those numbers which, when multiplied together, will equal that number. Thus, 5 and 3 are the factors of 15, since $5 \times 3 = 15$. Likewise, 8 and 7 are the factors of 56, since $8 \times 7 = 56$.

92. A **prime number** is a number that cannot be divided by any number except itself and 1; 1 is not considered a factor. Thus, 2, 3, 11, 29, etc. are prime numbers.

93. A **prime factor** of a number is any factor that is a prime number.

Any number that is not a prime is called a **composite** number, and may be produced by multiplying together its prime factors. Thus, 60 is a composite number, and is equal to the product of its prime factors, $2 \times 2 \times 3 \times 5$.

Two numbers are said to be **prime to each other** when they have no common factor, as, for example, 15 and 28; there is no number, except 1, that will divide *both* 15 and 28 without a remainder.

94. Canceling equal factors from both dividend and divisor does *not* change the quotient.

The canceling of a factor in both dividend and divisor is the *same as dividing them both by the same number*, which, by a principle of division, does not change the quotient.

Write the numbers forming the dividend above the line, and those forming the divisor below it.

95. Example.—Divide $4 \times 45 \times 60$ by 9×24.

Solution.—Placing the dividend over the divisor, and canceling

$$\frac{\cancel{4} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\cancel{9} \times \underset{\cancel{6}}{\cancel{24}}} = 50. \quad \text{Ans.}$$

Explanation.—The 4 in the dividend and 24 in the divisor are both divisible by 4, since 4 divided by 4 equals 1, and 24 divided by 4 equals 6. Cancel the 4 and the 24, and write the 6 under 24. Thus,

$$\frac{\cancel{4} \times 45 \times 60}{9 \times \underset{6}{\cancel{24}}} =$$

60 in the dividend and 6 in the divisor, are divisible by 6, since 60 divided by 6 equals 10, and 6 divided by 6 equals 1. Cancel the 60 and write 10 over it; also, cancel the 6. Thus,

$$\frac{\cancel{4} \times 45 \times \overset{10}{\cancel{60}}}{9 \times \underset{\cancel{6}}{\cancel{24}}} =$$

Again, 45 in the dividend and 9 in the divisor are each divisible by 9, since 45 divided by 9 equals 5, and 9 divided by 9 equals 1. Cancel the 45 and write the 5 over it; also, cancel the 9. Thus,

$$\frac{\cancel{4} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\cancel{9} \times \underset{\cancel{6}}{\cancel{24}}} =$$

Since there are no two remaining numbers (one in the dividend and one in the divisor) divisible by any number greater than 1 without a remainder, it is impossible to cancel further.

Multiply together all the uncanceled numbers in the dividend and divide their product by the product of all the uncanceled numbers in the divisor. The result will be the quotient. The product of all the uncanceled numbers in

the dividend is $5 \times 10 = 50$, and there are no uncanceled numbers in the divisor.

$$\text{Hence, } \frac{\cancel{4} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\cancel{9} \times \underset{6}{\cancel{24}}} = 5 \times 10 = 50. \quad \text{Ans.}$$

96. Rule.—I. *Cancel the common factors from both the dividend and divisor.*

II. *Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

EXAMPLES FOR PRACTICE.

97. Divide:

		Ans.
(*a*)	$14 \times 18 \times 16 \times 40$ by $7 \times 8 \times 6 \times 5 \times 3$.	(*a*) 32.
(*b*)	$3 \times 65 \times 50 \times 100 \times 60$ by $30 \times 60 \times 13 \times 10$.	(*b*) 250.
(*c*)	$8 \times 4 \times 3 \times 9 \times 11$ by $11 \times 9 \times 4 \times 3 \times 8$.	(*c*) 1.
(*d*)	$164 \times 321 \times 6 \times 7 \times 4$ by $82 \times 321 \times 7$.	(*d*) 48.
(*e*)	$50 \times 100 \times 200 \times 72$ by $1,000 \times 144 \times 100$.	(*e*) 5.
(*f*)	$48 \times 63 \times 55 \times 49$ by $7 \times 21 \times 11 \times 48$.	(*f*) 105.
(*g*)	$110 \times 150 \times 84 \times 32$ by $11 \times 15 \times 100 \times 64$.	(*g*) 42.
(*h*)	$115 \times 120 \times 400 \times 1,000$ by $23 \times 1,000 \times 60 \times 800$.	(*h*) 5.

ARITHMETIC.

FRACTIONS.

1. A **fraction** is one or more of the equal parts of a unit.

2. Two numbers are required to express a fraction, one called the **numerator**, and the other, the **denominator**.

3. The numerator is placed above the denominator, with a *line* between them, as $\frac{2}{3}$. Here 3 is the *denominator*, and shows into how many equal parts the unit is divided. The *numerator* 2 shows how many of these equal parts are taken or considered. The denominator also indicates the name of the parts.

$\frac{1}{2}$ is read one-half.
$\frac{3}{4}$ is read three-fourths.
$\frac{3}{8}$ is read three-eighths.
$\frac{5}{16}$ is read five-sixteenths.
$\frac{29}{47}$ is read twenty-nine forty-sevenths.

4. In the expression "$\frac{3}{4}$ of an apple," the denominator 4 shows that the apple is divided into four *equal* parts, and the numerator 3 shows that three of these parts, or fourths, are taken or considered.

If each of the parts, or fourths, of the apple were cut into two equal pieces, there would then be twice as many pieces as before, or $4 \times 2 = 8$ pieces in all; one of these pieces would be called one-eighth, and would be expressed in figures as $\frac{1}{8}$. Three of these pieces would be called three-eighths, written $\frac{3}{8}$. The words three-fourths, three-eighths, five-sixteenths, etc. are abbreviations of three one-fourths, three

one-eighths, five one-sixteenths, etc. It is evident that, the greater the denominator, the greater is the number of parts into which the unit is divided; consequently, the parts themselves are smaller, and the value of the fraction is less for the same number of parts taken. In other words, $\frac{7}{9}$, for example, is less than $\frac{7}{8}$, because, if a unit is divided into 9 parts, the parts are less than if the same unit had been divided into 8 parts; and, since $\frac{1}{9}$ is less than $\frac{1}{8}$, it is clear that 7 one-ninths is less than 7 one-eighths. Hence, also, $\frac{3}{8}$ is less than $\frac{3}{4}$.

5. The **value** of a fraction is the *numerator* divided by the *denominator;* as, $\frac{4}{2} = 2$, $\frac{6}{2} = 3$.

6. The line between the *numerator* and the *denominator* means *divided by*, or $\div$.

$\frac{3}{4}$ is equivalent to $3 \div 4$.
$\frac{5}{8}$ is equivalent to $5 \div 8$.

7. The *numerator* and *denominator* of a fraction are called the **terms** of a fraction.

8. The *value* of a fraction when its terms are equal is 1.

$\frac{4}{4}$, or four-fourths $= 1$.
$\frac{8}{8}$, or eight-eighths $= 1$.
$\frac{64}{64}$, or sixty-four sixty-fourths $= 1$.

9. A **proper fraction** is a fraction whose numerator is *less* than its denominator. Its value is *less* than 1, as $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{16}$.

10. An **improper fraction** is a fraction whose numerator *equals or is greater than the denominator*. Its value is 1 or *more* than 1, as $\frac{4}{4}$, $\frac{9}{8}$, $\frac{42}{32}$.

11. A **mixed number** is a *whole number and a fraction united.* $4\frac{2}{3}$ is a mixed number, and is equivalent to $4 + \frac{2}{3}$. It is read *four and two-thirds.*

REDUCTION OF FRACTIONS.

12. **Reduction of fractions** is the process of changing the form of fractions without changing their *value.*

13. A fraction is reduced to *higher terms* by *multiplying both terms of the fraction by the same number.* Thus,

$\frac{3}{4}$ is reduced to $\frac{6}{8}$ by multiplying both terms of the fraction by 2.

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}.$$

The *value* is not changed. For, suppose that a unit, say an apple, is divided into 8 equal parts. If these parts be arranged in 4 piles, each containing 2 parts, it is evident that each pile will be composed of the same part of the apple as would have been the case had the apple been originally cut into 4 equal parts. Now, if one of these piles (containing 2 parts) be removed, there will be 3 piles left, each containing 2 equal parts, or 6 equal parts in all, i. e., six-eighths. But, since one pile, or one-quarter, was removed, there are three-quarters left. Hence, $\frac{3}{4} = \frac{6}{8}$. The same reasoning may be applied to any similar case. Therefore, multiplying both terms of a fraction by the same number does not alter its value.

14. To reduce a fraction to an equivalent fraction having a given denominator.

EXAMPLE.—Reduce $\frac{7}{8}$ to an equivalent fraction having 96 for a denominator.

SOLUTION.—Both the numerator and the denominator must be multiplied by the same number in order not to change the value of the fraction. The denominator must be multiplied by some number which will make the product 96; this number is evidently $96 \div 8 = 12$, since $8 \times 12 = 96$. Hence, $\frac{7 \times 12}{8 \times 12} = \frac{84}{96}$. Ans.

15. Rule.—*Divide the given denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

EXAMPLE.—Reduce $\frac{3}{4}$ to 100ths.

SOLUTION.— $100 \div 4 = 25$; hence, $\frac{3 \times 25}{4 \times 25} = \frac{75}{100}$. Ans.

16. *A fraction is reduced to lower terms by dividing both terms by the same number.* Thus, $\frac{8}{10}$ is reduced to $\frac{4}{5}$ by dividing both terms by 2.

$$\frac{8 \div 2}{10 \div 2} = \frac{4}{5}.$$

That $\frac{8}{10} = \frac{4}{5}$ is readily seen from the explanation given in

Art. **13**; for, multiplying both terms of the fraction $\frac{4}{5}$ by 2, $\frac{4\times 2}{5\times 2} = \frac{8}{10}$, and, if $\frac{4}{5} = \frac{8}{10}$, $\frac{8}{10}$ must equal $\frac{4}{5}$. Hence, dividing both terms of a fraction by the same number does not alter its value.

17. *A fraction is reduced to lowest terms, or simplest form, when its numerator and denominator cannot both be divided by the same number without a remainder.* As, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{11}{24}$, $\frac{8}{15}$.

EXAMPLES FOR PRACTICE.

18. Reduce the following:

(*a*) $\frac{7}{16}$ to 128ths.	Ans.	(*a*) $\frac{56}{128}$.
(*b*) $\frac{24}{132}$ to its lowest terms.		(*b*) $\frac{2}{11}$.
(*c*) $\frac{64}{1000}$ to its lowest terms.		(*c*) $\frac{8}{125}$.
(*d*) $\frac{5}{7}$ to 49ths.		(*d*) $\frac{35}{49}$.
(*e*) $\frac{13}{16}$ to 10,000ths.		(*e*) $\frac{8125}{10000}$.

---

19. To reduce a whole number or a mixed number to an improper fraction.

Example 1.—How many *fourths* in 5?

Solution.—Since there are 4 *fourths* in 1, in 5 there will be 5×4 fourths, or 20 fourths; i.e., $5\times\frac{4}{4} = \frac{20}{4}$. Ans.

Example 2.—Reduce $8\frac{3}{4}$ to an improper fraction.

Solution.— $8\times\frac{4}{4} = \frac{32}{4}$. $\frac{32}{4}+\frac{3}{4} = \frac{35}{4}$. Ans.

20. Rule.—*Multiply the whole number by the denominator of the fraction, add the numerator to the product and place the denominator under the result. If it is desired to reduce a whole number to a fraction, multiply the whole number by the given denominator and the result will be the numerator of the required fraction.*

EXAMPLES FOR PRACTICE.

21. Reduce to improper fractions:

(*a*) $4\frac{1}{8}$.	Ans.	(*a*) $\frac{33}{8}$.
(*b*) $5\frac{1}{9}$.		(*b*) $\frac{46}{9}$.
(*c*) $10\frac{2}{10}$.		(*c*) $\frac{102}{10}$.
(*d*) $37\frac{3}{4}$.		(*d*) $\frac{151}{4}$.
(*e*) $50\frac{4}{5}$.		(*e*) $\frac{254}{5}$.
(*f*) Reduce 7 to a fraction whose denominator is 16.		(*f*) $\frac{112}{16}$.

22. To reduce an improper fraction to a whole or a mixed number.

Example.—Reduce $\frac{21}{4}$ to a mixed number.

Solution.— 4 is contained in 21, 5 times and 1 remaining (see Art. 5); as this remainder is also divided by 4, its value is $\frac{1}{4}$. Therefore, $5+\frac{1}{4}$, or $5\frac{1}{4}$, is the number.

23. Rule.—*Divide the numerator by the denominator, and write the result as in ordinary division.* (See part V of Rule, Art. **84,** § 1.)

EXAMPLES FOR PRACTICE.

24. Reduce to whole or mixed numbers:

(*a*) $\frac{145}{6}$.	Ans.	(*a*) $24\frac{1}{6}$.
(*b*) $\frac{185}{3}$.		(*b*) $61\frac{2}{3}$.
(*c*) $\frac{701}{6}$.		(*c*) $116\frac{5}{6}$
(*d*) $\frac{149}{3}$.		(*d*) $49\frac{2}{3}$.
(*e*) $\frac{76}{19}$.		(*e*) 4.
(*f*) $\frac{125}{25}$.		(*f*) 5.

---

25. **A common denominator** of *two or more fractions* is a number that will contain (i. e., which may be divided by) all of the *denominators* of the fractions without a remainder. The **least common denominator** is the least number that will contain all the denominators of the fractions without a remainder.

26. To find the least common denominator.

Example.—Find the least common denominator of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{16}$.

Solution.—We first place the denominators in a row, separated by commas.

$$\begin{array}{r|rrrr} 2 & 4, & 3, & 9, & 16 \\ \hline 2 & 2, & 3, & 9, & 8 \\ \hline 3 & & 3, & 9, & 4 \\ \hline & & & 3, & 4 \end{array}$$

$2 \times 2 \times 3 \times 3 \times 4 = 144$, the least common denominator. Ans.

Explanation.—Divide each of the denominators by some prime number that will divide at least two of them without a remainder if (possible), bringing down to the row below those

denominators which will not contain the divisor without a remainder. Dividing each of the numbers by 2, the second row becomes 2, 3, 9, 8, since 2 will not divide 3 and 9 without a remainder. Dividing again by 2, the result is 3, 9, 4. Dividing the third row by 3, the result is 3, 4. The numbers in the fourth row are now prime to each other (see Art. **93,** § 1), and the product of these numbers multiplied by the divisors will be the least common denominator. Thus, $2 \times 2 \times 3 \times 3 \times 4 = 144$, the least common denominator.

27. Example.—Find the least common denominator of $\frac{4}{9}$, $\frac{5}{12}$, and $\frac{7}{18}$.

Solution.—

$$\begin{array}{r|rrr} 3 & 9, & 12, & 18 \\ \hline 3 & 3, & 4, & 6 \\ \hline 2 & & 4, & 2 \\ \hline & & 2 & \end{array}$$

$3 \times 3 \times 2 \times 2 = 36$. Ans.

28. If one (or more) of the denominators is a factor of some other denominator, it need not be considered in the process of finding the least common denominator, for if the least common denominator will contain the larger denominator it will also contain any factor of it. Thus, in the last example, since 9 is a factor of 18, it need not be considered, and all that is necessary is to find the least common denominator of 12 and 18. Also, if *all* of the denominators have a common factor, whether prime or composite, that factor may be used as a divisor. For example, since 12 and 18 have the common factor 6, 6 may be used as a divisor instead of its prime factors 2 and 3. Hence, the entire operation of finding the least common denominator of $\frac{4}{9}$, $\frac{5}{12}$, $\frac{7}{18}$ reduces to $\begin{array}{r|rr} 6 & 12, & 18, \\ \hline & 2, & 3, \end{array}$ or least common denominator $= 6 \times 2 \times 3 = 36$, the same result as before.

29. Any number that will exactly contain another number is called a **multiple** of that number. Thus, 48 is a multiple of 6; also, of 8, of 12, etc. Any number that will exactly contain two or more numbers is called a **common multiple** of those numbers; and the least number that will

exactly contain two or more numbers is called the **least common multiple** of those numbers. Hence, the least common denominator of two or more fractions is the least common multiple of the denominators of the fractions.

30. The least common multiple of several numbers may often be determined by inspection. For example, if it is desired to find the least common multiple of 3, 5, and 10, simple inspection will show that the least common multiple is 30, the mental process being as follows: Since 5 is a factor of 10, 5 need not be considered; and since 3 and 10 are prime to each other, their least common multiple is their product, i. e., 3×10, or 30. Therefore, 30 is the least common multiple of 3, 5, and 10.

Again, consider the example of Art. **27.** Here, it is required to find the least common multiple of 9, 12, and 18. Since 9 is a factor of 18, it need not be considered. The least common multiple of 12 and 18 must, of course, contain 18 a certain number of times. To ascertain how many times 18 the least common multiple must be, divide 12 (the other number) by the greatest factor common to both 12 and 18; the quotient multiplied by 18 will be the required least common multiple. In the present case, the greatest factor is 6, and $12 \div 6 = 2$; $2 \times 18 = 36$, the least common multiple.

31. To reduce two or more fractions to equivalent fractions having the least common denominator.

EXAMPLE.—Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$ to equivalent fractions having the least common denominator.

SOLUTION.—The least common denominator is the least number that can be exactly divided by 3, 4, and 8. This number is readily seen by inspection to be 24. Now, reducing each fraction to a fraction having a denominator of 24 (Art. **14**), we obtain

$$\frac{2 \times 8}{3 \times 8} = \frac{16}{24}, \quad \frac{3 \times 6}{4 \times 6} = \frac{18}{24}, \quad \frac{7 \times 3}{8 \times 3} = \frac{21}{24}. \quad \text{Ans.}$$

32. Rule.—*Divide the least common denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

EXAMPLES FOR PRACTICE.

33. Reduce to fractions having a common denominator:

(*a*)	$\frac{3}{4}, \frac{5}{8}, \frac{7}{8}$.	(*a*)	$\frac{6}{8}, \frac{5}{8}, \frac{7}{8}$.
(*b*)	$\frac{3}{16}, \frac{3}{4}, \frac{7}{32}$.	(*b*)	$\frac{6}{32}, \frac{24}{32}, \frac{7}{32}$.
(*c*)	$\frac{7}{8}, \frac{7}{88}, \frac{10}{11}$.	Ans. (*c*)	$\frac{77}{88}, \frac{7}{88}, \frac{80}{88}$.
(*d*)	$\frac{3}{5}, \frac{5}{8}, \frac{11}{40}$.	(*d*)	$\frac{24}{40}, \frac{25}{40}, \frac{11}{40}$.
(*e*)	$\frac{4}{10}, \frac{6}{40}, \frac{9}{20}$.	(*e*)	$\frac{16}{40}, \frac{6}{40}, \frac{18}{40}$.
(*f*)	$\frac{7}{15}, \frac{17}{30}, \frac{21}{30}$.	(*f*)	$\frac{14}{30}, \frac{17}{30}, \frac{21}{30}$.

ADDITION OF FRACTIONS.

34. *Fractions cannot be added unless they have a common denominator.* We cannot add $\frac{3}{4}$ to $\frac{7}{8}$ as they now stand, since the denominators represent different parts of a unit. Fourths can be added to fourths, but not to eighths.

Suppose we divide an apple into 4 equal parts, and then divide 2 of these parts into 2 equal parts. It is evident that we shall have 2 one-fourths and 4 one-eighths. Now if we add these parts the result is $2 + 4 = 6$ something. But what is this something? It is not fourths, for six fourths are $1\frac{1}{2}$, and we had only one apple to begin with; neither is it eighths, for six eighths are $\frac{3}{4}$, which is less than 1 apple. By reducing the fourths to eighths, we have $\frac{2}{4} = \frac{4}{8}$; and, adding the other 4 eighths, $4 + 4 = 8$ eighths. The result is correct, since $\frac{8}{8} = 1$. Or we can, in this case, reduce the eighths to fourths. Thus, $\frac{4}{8} = \frac{2}{4}$; whence, adding $2 + 2 = 4$ quarters or fourths, a correct result, since $\frac{4}{4} = 1$.

Before adding, fractions should be reduced to a common denominator, preferably the *least* common denominator.

35. Example.—Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$.

Solution.—The *least common denominator*, or the *least number* that will exactly contain all the *denominators*, is 8.

$$\tfrac{1}{2} = \tfrac{4}{8},\ \tfrac{3}{4} = \tfrac{6}{8},\ \text{and } \tfrac{4}{8} + \tfrac{6}{8} + \tfrac{5}{8} = \frac{4+6+5}{8} = \tfrac{15}{8} = 1\tfrac{7}{8}.\quad \text{Ans.}$$

Explanation.—As the *denominator* indicates the names of the *parts*, only the *numerators* are added to obtain the total number of *parts* indicated by the *denominator*. Thus, 4 one-eighths plus 6 one-eighths plus 5 one-eighths = 15 one-eighths.

36. Example 1.—What is the sum of $12\frac{3}{4}$, $14\frac{5}{8}$, and $7\frac{5}{16}$?

Solution.—The least common denominator in this case is 16.

$$\begin{array}{rcl} 12\frac{3}{4} & = & 12\frac{12}{16} \\ 14\frac{5}{8} & = & 14\frac{10}{16} \\ 7\frac{5}{16} & = & 7\frac{5}{16} \\ \hline \end{array}$$

$$\textit{sum} \quad 33 + \frac{27}{16} = 33 + 1\frac{11}{16} = 34\frac{11}{16}. \quad \text{Ans.}$$

The sum of the fractions $= \frac{27}{16}$, or $1\frac{11}{16}$, which added to the sum of the whole numbers $= 34\frac{11}{16}$.

Example 2.—What is the sum of 17, $13\frac{3}{16}$, $\frac{9}{32}$, and $3\frac{1}{4}$?

Solution.—The least common denominator is 32. $13\frac{3}{16} = 13\frac{6}{32}$, $3\frac{1}{4} = 3\frac{8}{32}$.

$$\begin{array}{rr} & 17 \\ & 13\frac{6}{32} \\ & \frac{9}{32} \\ & 3\frac{8}{32} \\ \hline \textit{sum} & 33\frac{23}{32} \end{array} \quad \text{Ans.}$$

37. Rule.—I. *Make the fractions similar; write the sum of the numerators over the least common denominator.*

II. *When there are integers or mixed numbers, add them separately and then add the results.*

EXAMPLES FOR PRACTICE.

38. Find the sum of:

(*a*)	$\frac{4}{6}, \frac{7}{24}, \frac{5}{8}$.	(*a*)	$1\frac{7}{12}$.
(*b*)	$\frac{2}{3}, \frac{5}{15}, \frac{24}{45}$.	(*b*)	$1\frac{8}{15}$.
(*c*)	$\frac{1}{2}, \frac{3}{8}, \frac{5}{16}$.	(*c*)	$1\frac{3}{16}$.
(*d*)	$\frac{5}{6}, \frac{11}{72}, \frac{13}{18}$.	Ans. (*d*)	$1\frac{17}{24}$.
(*e*)	$\frac{10}{11}, \frac{6}{33}, \frac{23}{66}$.	(*e*)	$1\frac{29}{66}$.
(*f*)	$\frac{23}{45}, \frac{11}{15}, \frac{14}{45}$.	(*f*)	$1\frac{5}{9}$.
(*g*)	$\frac{4}{11}, \frac{7}{22}, \frac{14}{22}$.	(*g*)	$1\frac{7}{22}$.
(*h*)	$\frac{3}{7}, \frac{14}{49}, \frac{2}{7}$.	(*h*)	1.

SUBTRACTION OF FRACTIONS.

39. *Fractions cannot be subtracted without first reducing them to a common denominator.* This can be shown in the same manner as in the case of addition of fractions. (Art. **34.**)

EXAMPLE.—Subtract $\frac{3}{8}$ from $\frac{13}{16}$.

SOLUTION.—The least common denominator is 16.

$$\tfrac{3}{8} = \tfrac{6}{16}.\quad \tfrac{13}{16} - \tfrac{6}{16} = \frac{13-6}{16} = \tfrac{7}{16}.\quad \text{Ans.}$$

40. EXAMPLE.—From 7 take $\frac{5}{8}$.

SOLUTION.— $1 = \frac{8}{8}$; therefore, since $7 = 6+1$, $7 = 6+\frac{8}{8} = 6\frac{8}{8}$, and $6\frac{8}{8} - \frac{5}{8} = 6\frac{3}{8}$. Ans.

41. EXAMPLE.—What is the difference between $17\frac{9}{16}$ and $9\frac{15}{32}$?

SOLUTION.—The least common denominator of the fraction is 32. $17\frac{9}{16} = 17\frac{18}{32}$.

minuend	$17\frac{18}{32}$	
subtrahend	$9\frac{15}{32}$	
difference	$8\frac{3}{32}$	Ans.

42. EXAMPLE.—From $9\frac{1}{4}$ take $4\frac{7}{16}$.

SOLUTION.—The least common denominator of the fractions is 16. $9\frac{1}{4} = 9\frac{4}{16}$.

minuend	$9\frac{4}{16}$	or	$8\frac{20}{16}$	
subtrahend	$4\frac{7}{16}$		$4\frac{7}{16}$	
remainder	$4\frac{13}{16}$		$4\frac{13}{16}$	Ans.

EXPLANATION.—As the fraction in the subtrahend is greater than the fraction in the minuend, it cannot be subtracted; therefore, borrow 1, or $\frac{16}{16}$, from the 9 in the minuend and add it to the $\frac{4}{16}$; $\frac{4}{16}+\frac{16}{16} = \frac{20}{16}$. $\frac{7}{16}$ from $\frac{20}{16} = \frac{13}{16}$. Since 1 was borrowed from 9, 8 remains; 4 from $8 = 4$; $4+\frac{13}{16} = 4\frac{13}{16}$.

43. EXAMPLE.—From 9 take $8\frac{3}{16}$.

SOLUTION.—

minuend	9	or	$8\frac{16}{16}$	
subtrahend	$8\frac{3}{16}$		$8\frac{3}{16}$	
difference	$\frac{13}{16}$		$\frac{13}{16}$	Ans.

EXPLANATION.—As there is no fraction in the minuend from which to take the fraction in the subtrahend, borrow 1, or $\frac{16}{16}$, from 9. $\frac{3}{16}$ from $\frac{16}{16} = \frac{13}{16}$. Since 1 was borrowed from 9, only 8 is left. 8 from $8 = 0$.

44. Rule.—I. *Reduce the given fractions to fractions having the least common denominator. Subtract one numerator from the other and place the remainder over the common denominator.*

II. *When there are mixed numbers, subtract the fractions and whole numbers separately.*

III. *When the fraction in the subtrahend is greater than the fraction in the minuend, borrow 1 from the whole number in the minuend, and add it to the fraction in the minuend, from which subtract the fraction in the subtrahend.*

IV. *When the minuend is a whole number, borrow 1 from it; reduce the 1 to a fraction whose denominator is the same as the denominator of the fraction in the subtrahend, and then subtract.*

EXAMPLES FOR PRACTICE.

45. Subtract:

			Ans.
(*a*)	$\frac{10}{24}$ from $\frac{11}{12}$.	(*a*)	$\frac{1}{2}$.
(*b*)	$\frac{7}{14}$ from $\frac{17}{28}$.	(*b*)	$\frac{3}{28}$.
(*c*)	$\frac{4}{30}$ from $\frac{5}{10}$.	(*c*)	$\frac{11}{30}$.
(*d*)	$\frac{15}{35}$ from $\frac{45}{70}$.	(*d*)	$\frac{3}{14}$.
(*e*)	$\frac{15}{16}$ from $\frac{57}{48}$.	(*e*)	$\frac{1}{4}$.
(*f*)	$13\frac{1}{4}$ from $30\frac{1}{2}$.	(*f*)	$17\frac{1}{4}$.
(*g*)	$12\frac{1}{8}$ from 27.	(*g*)	$14\frac{7}{8}$.
(*h*)	$5\frac{1}{4}$ from 30.	(*h*)	$24\frac{3}{4}$.

MULTIPLICATION OF FRACTIONS.

46. In multiplication of fractions it is not necessary to reduce the given fractions to fractions having a common denominator.

47. *Multiplying the numerator or dividing the denominator multiplies the fraction.*

EXAMPLE.—Multiply $\frac{3}{4}$ by 4.

SOLUTION.— $\frac{3}{4}\times 4 = \frac{3\times 4}{4} = \frac{12}{4} = 3$. Ans.

Or, $\frac{3}{4}\times 4 = \frac{3}{4\div 4} = \frac{3}{1} = 3$. Ans.

The word "of" in multiplication of fractions means the same as $\times$, or times. Thus,

$$\frac{3}{4} \text{ of } 4 = \frac{3}{4}\times 4 = 3.$$
$$\frac{1}{8} \text{ of } \frac{5}{16} = \frac{1}{8}\times\frac{5}{16} = \frac{5}{128}.$$

EXAMPLE.—Multiply 2 by $\frac{3}{8}$.

SOLUTION.— $2 \times \frac{3}{8} = \frac{3 \times 2}{8} = \frac{6}{8} = \frac{3}{4}$. Ans.

Or $2 \times \frac{3}{8} = \frac{3}{8 \div 2} = \frac{3}{4}$. Ans.

48. EXAMPLE.—What is the product of $\frac{4}{16}$ and $\frac{7}{8}$?

SOLUTION.— $\frac{4}{16} \times \frac{7}{8} = \frac{4 \times 7}{16 \times 8} = \frac{28}{128} = \frac{7}{32}$. Ans.

Or, by cancelation, $\frac{\cancel{4} \times 7}{\underset{4}{\cancel{16}} \times 8} = \frac{7}{4 \times 8} = \frac{7}{32}$. Ans.

49. EXAMPLE.—What is $\frac{4}{8}$ of $\frac{3}{4}$ of $\frac{16}{32}$?

SOLUTION.— $\frac{\cancel{4} \times 3 \times \cancel{16}}{8 \times \cancel{4} \times \underset{2}{\cancel{32}}} = \frac{3}{8 \times 2} = \frac{3}{16}$. Ans.

50. EXAMPLE.—What is the product of $9\frac{3}{4}$ and $5\frac{5}{8}$?

SOLUTION.— $9\frac{3}{4} = \frac{39}{4}$; $5\frac{5}{8} = \frac{45}{8}$.

$$\frac{39}{4} \times \frac{45}{8} = \frac{39 \times 45}{4 \times 8} = \frac{1755}{32} = 54\frac{27}{32}. \text{ Ans.}$$

51. EXAMPLE.—Multiply $15\frac{7}{8}$ by 3.

SOLUTION.—

$$\begin{array}{r} 15\frac{7}{8} \\ 3 \\ \hline 47\frac{5}{8} \end{array} \qquad \begin{array}{l} 15\frac{7}{8} \\ \quad\;\; 3 \\ \hline 45 + \frac{21}{8} = 45 + 2\frac{5}{8} = 47\frac{5}{8}. \text{ Ans.} \end{array}$$

52. Rule.—I. *Divide the product of the numerators by the product of the denominators. All factors common to the numerators and denominators should first be cast out by cancelation.*

II. *To multiply one mixed number by another, reduce them both to improper fractions.*

III. *To multiply a mixed number by a whole number, first multiply the fractional part by the multiplier, and if the product is an improper fraction, reduce it to a mixed number, and add the whole-number part to the product of the multiplier and the whole number.*

EXAMPLES FOR PRACTICE.

53. Find the product of:

(*a*)	$7 \times \frac{3}{19}$.	Ans. (*a*)	$1\frac{2}{19}$.
(*b*)	$14 \times \frac{5}{16}$.	(*b*)	$4\frac{3}{8}$.
(*c*)	$\frac{21}{32} \times \frac{5}{14}$.	(*c*)	$\frac{15}{64}$.
(*d*)	$\frac{16}{27} \times 4$.	(*d*)	$2\frac{10}{27}$.
(*e*)	$\frac{19}{18} \times 7$.	(*e*)	$7\frac{7}{18}$.
(*f*)	$17\frac{18}{21} \times 7$.	(*f*)	125.
(*g*)	$\frac{105}{224} \times 32$.	(*g*)	15.
(*h*)	$\frac{15}{28} \times 14$.	(*h*)	$7\frac{1}{2}$.

54. Short Methods of Multiplying by a Mixed Number.—In all business transactions, the multiplication of a mixed number by an integer, an integer by a mixed number, or a mixed number by a mixed number, is of very frequent occurrence. Unless the numbers are quite small, which is not usually the case, it is very inconvenient to reduce the mixed numbers to improper fractions, multiply, and then reduce the product to a mixed number. A better way is to use one of the methods given below:

55. EXAMPLE.—Multiply $126\frac{7}{8}$ by 27.

SOLUTION.—

$$\begin{array}{r} 126\frac{7}{8} \\ 27 \\ \hline 3\frac{3}{8} \\ 18\frac{18}{8} \\ 882\phantom{\frac{1}{8}} \\ 252\phantom{0\frac{1}{8}} \\ \hline 3425\frac{5}{8} \end{array} \text{ Ans.}$$

EXPLANATION.—First multiply the $\frac{7}{8}$ by 27; this may be conveniently done as follows: Multiply $\frac{1}{8}$ by 27; to do this all that is necessary is to divide 27 by 8 (this is evidently correct, since $\frac{1}{8} \times 27 = \frac{27}{8} = 27 \div 8$), obtaining $3\frac{3}{8}$. Now multiply the result just obtained ($3\frac{3}{8}$) by the numerator of the fraction less 1, or in this case, by $7 - 1 = 6$, getting $18\frac{18}{8}$ for the product, which write under $3\frac{3}{8}$, as shown. Then multiply 126 by 27 in the usual manner, placing the unit figure under the unit figures of the two mixed numbers, which may be

regarded as partial products. That this method is correct is readily seen. For $3\frac{3}{8}$ is $\frac{1}{8}$ of 27, and $18\frac{18}{8}$ is $\frac{6}{8}$ of 27; therefore, $3\frac{3}{8} + 18\frac{18}{8} = \frac{1}{8}$ of $27 + \frac{6}{8}$ of $27 = \frac{7}{8}$ of 27.

56. Example.—Multiply 825 by $29\frac{3}{4}$.

Solution.—*First Method.*

$$\begin{array}{r} 825 \\ \underline{29\frac{3}{4}} \\ 4\overline{)2475} \\ 618\frac{3}{4} \\ 7425 \\ \underline{1650} \\ 24543\frac{3}{4} \text{ Ans.} \end{array}$$

Second Method.

$$\begin{array}{r} 825 \\ \underline{29\frac{3}{4}} \\ 206\frac{1}{4} \\ 412\frac{2}{4} \\ 7425 \\ \underline{1650} \\ 24543\frac{3}{4} \text{ Ans.} \end{array}$$

Explanation.—*First Method:* Since $\frac{3}{4}$ of 825 is the same as $\frac{1}{4}$ of 3 times 825, we first find 3 times 825 and take $\frac{1}{4}$ of the product. The remainder of the operation needs no explanation. The *second method* is similar to that used in the preceding example.

57. Example.—Multiply $89\frac{2}{3}$ by $75\frac{3}{8}$.

Solution.—

First Method.

$$\begin{array}{r} 89\frac{2}{3} \\ \underline{75\frac{3}{8}} \\ 8)267\phantom{\frac{1}{1}} \\ \underline{3)150\phantom{\frac{1}{1}}} \\ 33\frac{3}{8} \\ 50\frac{1}{4} \\ 445\phantom{\frac{1}{1}} \\ \underline{623\phantom{0\frac{1}{1}}} \\ 6758\frac{5}{8} \text{ Ans.} \end{array}$$

$$\frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\underset{4}{\cancel{8}}} = \frac{1}{4}$$

Second Method.

$$\begin{array}{r} 89\frac{2}{3} \\ \underline{75\frac{3}{8}} \\ 11\frac{1}{8} \\ 22\frac{2}{8} \\ 25\phantom{\frac{1}{1}} \\ 25\phantom{\frac{1}{1}} \\ 445\frac{1}{4} \\ \underline{623\phantom{0\frac{1}{1}}} \\ 6758\frac{5}{8} \text{ Ans.} \end{array}$$

Explanation.—*First Method:* In this example there are four operations. (1) To multiply the fraction by the fraction. (2) To multiply the upper number by the lower fraction. (3) To multiply the lower number by the upper fraction. (4) To multiply the whole number by the whole number. We first multiply 89 by $\frac{3}{8}$, but in order to save space the division of 3 times 89 by 8 is merely indicated for the present. $\frac{2}{3}$ is multiplied by 75 in the same manner, multiplying

75 by 2 and indicating the division by 3. 267 (i. e., 3×89) is now divided by 8, obtaining $33\frac{3}{8}$, which is written as shown. 150 (i. e., 2×75) is divided by 3, obtaining 50, which is written under the $33\frac{3}{8}$. The two fractions are multiplied and the product, $\frac{1}{4}$, placed alongside of the 50. The two integers are now multiplied and all the separate products added together, as shown.

Second Method: This is a combination of the method of Art. **55,** and the second method of Art. **56,** and should be understood without further explanation. The product of the two fractions, $\frac{1}{4}$, is written alongside of the 445 and added to the other two fractions, $\frac{1}{8}$ and $\frac{2}{8}$, as shown.

58. EXAMPLE.—At $3\frac{5}{8}$ cents per pound, what will $66\frac{1}{2}$ pounds of sugar cost?

SOLUTION.—*First Method.*

$$\begin{array}{rr} & 66\frac{1}{2} \\ & 3\frac{5}{8} \\ \hline 8) & 330 \\ 2) & 3 \\ \hline & 41\frac{1}{4} \\ & 1\frac{1}{2} \\ & 198\frac{5}{16} \\ \hline & 241\frac{1}{16} \end{array} \text{ cents. Ans.} \qquad \frac{1}{2} \times \frac{5}{8} = \frac{5}{16}$$

Second Method.

$$\begin{array}{r} 66\frac{1}{2} \\ 3\frac{5}{8} \\ \hline 8\frac{1}{4} \\ 33 \\ 1\frac{1}{2} \\ 198\frac{5}{16} \\ \hline 241\frac{1}{16} \end{array}$$

EXPLANATION.—First multiply 66 by $\frac{5}{8}$, then 3 by $\frac{1}{2}$, then 66 by 3, and finally $\frac{1}{2}$ by $\frac{5}{8}$. The $\frac{5}{16}$ is written under and added to the other fractions.

EXAMPLES FOR PRACTICE.

59. Solve the following examples:

1. Find the cost of $89\frac{2}{3}$ yards of silk velvet at $\$4\frac{3}{4}$ per yard.
Ans. $\$425\frac{11}{12}$.

2. How much must be paid for $83\frac{1}{2}$ tons of hay at $\$16\frac{5}{8}$ a ton?
Ans. $\$1,388\frac{3}{16}$.

3. How far can a man ride on a bicycle in $14\frac{5}{6}$ hours at the rate of $9\frac{3}{4}$ miles per hour? Ans. $144\frac{5}{8}$ miles.

4. Find the following products: (*a*) $28\frac{7}{8} \times 17\frac{2}{3}$. (*b*) $44\frac{7}{9} \times 16\frac{5}{8}$. (*c*) $127\frac{4}{5} \times 69\frac{2}{7}$. (*d*) $86\frac{5}{12} \times 78\frac{4}{5}$. (*e*) $53\frac{3}{4} \times 27\frac{1}{2}$.
Ans. (*a*) $510\frac{1}{8}$. (*b*) $744\frac{31}{72}$. (*c*) $8,854\frac{5}{7}$. (*d*) $6,809\frac{19}{30}$. (*e*) $1,478\frac{1}{8}$.

DIVISION OF FRACTIONS.

60. In division of fractions it is not necessary to reduce the given fractions to fractions having a common denominator.

61. *Dividing the numerator or multiplying the denominator of a fraction, divides the fraction.*

EXAMPLE.—Divide $\frac{6}{8}$ by 3.

SOLUTION.—When *dividing* the *numerator*, we have

$$\tfrac{6}{8} \div 3 = \frac{6 \div 3}{8} = \tfrac{2}{8} = \tfrac{1}{4}. \quad \text{Ans.}$$

When *multiplying* the *denominator*, we have

$$\tfrac{6}{8} \div 3 = \frac{6}{8 \times 3} = \tfrac{6}{24} = \tfrac{1}{4}. \quad \text{Ans.}$$

EXAMPLE.—Divide $\frac{3}{16}$ by 2.

SOLUTION.— $\tfrac{3}{16} \div 2 = \dfrac{3}{16 \times 2} = \tfrac{3}{32}$. Ans.

EXAMPLE.—Divide $\frac{14}{32}$ by 7.

SOLUTION.— $\tfrac{14}{32} \div 7 = \dfrac{14 \div 7}{32} = \tfrac{2}{32} = \tfrac{1}{16}$. Ans.

62. To **invert** a fraction is to *turn it upside down*, that is, make the numerator and denominator change places.

Invert $\frac{3}{4}$ and it becomes $\frac{4}{3}$.

63. EXAMPLE.—Divide $\frac{9}{16}$ by $\frac{3}{16}$.

SOLUTION.—1. The fraction $\frac{3}{16}$ is contained in $\frac{9}{16}$, 3 times, for the denominators are the same, and one numerator is contained in the other 3 times. 2. If we now invert the divisor, $\frac{3}{16}$, and multiply, the solution is

$$\frac{9}{16} \times \frac{16}{3} = \frac{\overset{3}{\cancel{9}} \times \cancel{16}}{\cancel{16} \times \cancel{3}} = 3. \quad \text{Ans.}$$

This gives the same quotient as in the first case.

64. EXAMPLE.—Divide $\frac{3}{8}$ by $\frac{1}{4}$.

SOLUTION.—We cannot divide $\frac{3}{8}$ by $\frac{1}{4}$, as in the first case above, for the denominators are not the same; therefore, we must solve as in the second case.

$$\frac{3}{8} \div \frac{1}{4} = \frac{3}{8} \times \frac{4}{1} = \frac{3 \times \cancel{4}}{\underset{2}{\cancel{8}} \times 1} = \frac{3}{2} \text{ or } 1\tfrac{1}{2}. \quad \text{Ans.}$$

65. EXAMPLE.—Divide 5 by $\frac{10}{16}$.

SOLUTION.— $\frac{10}{16}$ inverted becomes $\frac{16}{10}$.

$$5 \times \frac{16}{10} = \frac{\cancel{5} \times \overset{8}{\cancel{16}}}{\underset{2}{\cancel{10}}} = 8. \quad \text{Ans.}$$

66. EXAMPLE.—How many times is $3\frac{3}{4}$ contained in $7\frac{7}{16}$?

SOLUTION.— $3\frac{3}{4} = \frac{15}{4}$; $7\frac{7}{16} = \frac{119}{16}$.

$\frac{15}{4}$ inverted equals $\frac{4}{15}$.

$$\frac{119}{16} \times \frac{4}{15} = \frac{119 \times \cancel{4}}{\underset{4}{\cancel{16}} \times 15} = \frac{119}{60} = 1\tfrac{59}{60}. \quad \text{Ans.}$$

67. **Rule.**—*Invert the divisor and proceed as in multiplication.*

68. We have learned that a line placed between two numbers indicates that the number above the line is to be divided by the number below it. Thus, $\frac{18}{3}$ denotes that 18 is to be divided by 3. This is also true if a fraction or a fractional expression be placed above or below a line.

$\dfrac{9}{\frac{3}{8}}$ means that 9 is to be divided by $\frac{3}{8}$; $\dfrac{3 \times 7}{\frac{8+4}{16}}$ means that 3×7 is to be divided by the value of $\dfrac{8+4}{16}$.

$\dfrac{\frac{1}{4}}{\frac{3}{8}}$ is the same as $\frac{1}{4} \div \frac{3}{8}$.

69. It will be noticed that there is a heavy line between the 9 and the $\frac{3}{8}$. This is necessary, since otherwise there would be nothing to show whether 9 is to be divided by $\frac{3}{8}$, or

$\frac{9}{3}$ is to be divided by 8. Whenever a heavy line is used, as shown here, it indicates that *all above the line* is to be divided by *all below it.*

EXAMPLES FOR PRACTICE.

70. Divide:

(*a*)	15 by $6\frac{3}{7}$.	(*a*)	$2\frac{1}{3}$.
(*b*)	30 by $\frac{6}{8}$.	(*b*)	40.
(*c*)	172 by $\frac{4}{5}$.	(*c*)	215.
(*d*)	$\frac{14}{18}$ by $1\frac{7}{16}$.	Ans. (*d*)	$\frac{112}{207}$.
(*e*)	$\frac{103}{6}$ by $14\frac{2}{3}$.	(*e*)	$1\frac{15}{88}$.
(*f*)	$\frac{142}{27}$ by $17\frac{1}{9}$.	(*f*)	$\frac{71}{231}$.
(*g*)	$\frac{14}{18}$ by $\frac{145}{72}$.	(*g*)	$\frac{56}{145}$.
(*h*)	$\frac{128}{18}$ by $72\frac{1}{3}$.	(*h*)	$\frac{64}{651}$.

COMPLEX FRACTIONS.

71. Whenever an expression like one of the three following is obtained, it may always be simplified by transposing the denominator from *above* to *below* the line, or from *below* to *above*, as the case may be, taking care, however, to indicate that the denominator, when so transferred, is a multiplier. These expressions are called **complex fractions.**

1. $\dfrac{\frac{3}{4}}{9} = \dfrac{3}{9 \times 4} = \frac{3}{36} = \frac{1}{12}$; for, regarding the fraction above the heavy line as the numerator of a fraction whose denominator is 9, $\dfrac{\frac{3}{4} \times 4}{9 \times 4} = \dfrac{3}{9 \times 4}$, as before.

2. $\dfrac{9}{\frac{3}{4}} = \dfrac{9 \times 4}{3} = 12$. The proof is the same as in the first case.

3. $\dfrac{\frac{5}{9}}{\frac{3}{4}} = \dfrac{5 \times 4}{3 \times 9} = \frac{20}{27}$; for, regarding $\frac{5}{9}$ as the numerator of a fraction whose denominator is $\frac{3}{4}$, $\dfrac{\frac{5}{9} \times 9}{\frac{3}{4} \times 9} = \dfrac{5}{\frac{3 \times 9}{4}}$; and

$$\dfrac{\dfrac{5}{3 \times 9} \times 4}{\dfrac{3 \times 9}{4} \times 4} = \dfrac{5 \times 4}{3 \times 9} = \frac{20}{27}, \text{ as above.}$$

This principle may be used to great advantage in cases like $\frac{\frac{1}{4}\times 310\times\frac{27}{12}\times 72}{40\times 4\frac{1}{2}\times 5\frac{1}{6}}$. Reducing the mixed numbers to fractions, the expression becomes $\frac{\frac{1}{4}\times 310\times\frac{27}{12}\times 72}{40\times\frac{9}{2}\times\frac{31}{6}}$. Now, transferring the denominators of the fractions and canceling,

$$\frac{1\times310\times27\times72\times2\times6}{40\times9\times31\times4\times12} = \frac{1\times\overset{\cancel{10}}{\cancel{310}}\times\overset{3}{\cancel{27}}\times\overset{\overset{3}{\cancel{6}}}{\cancel{72}}\times\cancel{2}\times\overset{3}{\cancel{6}}}{\underset{\underset{\cancel{2}}{\cancel{4}}}{\cancel{40}}\times\cancel{9}\times\cancel{31}\times\underset{2}{\cancel{4}}\times\cancel{12}} = \frac{27}{2} = 13\tfrac{1}{2}.$$

Greater exactness in results can usually be obtained by using this principle than can be obtained by reducing the fractions to decimals. The principle, however, should not be employed *if a sign of addition or subtraction occurs either above or below the dividing line.*

EXAMPLES FOR PRACTICE.

72. Simplify the following:

(*a*) $\frac{4\frac{2}{3}\times 10\frac{2}{5}\times 26\frac{1}{4}}{8\frac{2}{5}\times 4\frac{7}{8}\times 12\frac{3}{5}}$. (*b*) $\frac{10\frac{2}{3}\times 12\frac{4}{7}\times 15\frac{3}{4}}{4\frac{7}{12}\times\frac{24}{35}\times 8\frac{2}{5}}$.

(*c*) $\frac{20\frac{2}{5}\times 34\frac{2}{3}\times 8\frac{5}{8}}{14\frac{1}{6}\times 3\frac{9}{10}\times 18\frac{2}{5}}$. (*d*) $\frac{15\frac{3}{11}\times 32\frac{2}{3}\times 25\frac{3}{5}}{28\frac{4}{5}\times 38\frac{1}{4}\times 17\frac{7}{9}}$.

Ans. (*a*) $2\frac{38}{81}$. (*b*) 80. (*c*) 6. (*d*) $\frac{5488}{8415}$.

ARITHMETIC.

DECIMALS.

NOTATION AND NUMERATION.

1. A **decimal,** or a **decimal fraction,** is a fraction whose denominator is 10, 100, 1,000, etc.

2. The denominator is always 10 or a power of 10, and is not expressed as in common fractions, by writing it under the numerator, with a line between them; as $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$. The denominator is always understood, the numerator consisting of the figures on the right of the *unit* figure of the number. In order to distinguish the unit figure, a period (.), called the **decimal point,** is placed between the unit figure and the next figure on the right. The decimal point may be regarded in two ways: first, as indicating that the number on the right is the numerator of a fraction whose denominator is 10, 100, 1,000, etc.; and, second, as a part of the Arabic system of notation, each figure on the right being 10 times as large as the next succeeding figure, and 10 times as small as the next preceding figure, serving merely to point out the unit figure.

3. The reading of a *decimal* depends upon the number of decimal places in it; i. e., upon the number of figures to the *right* of the unit figure.

The first figure to the right of the unit figure expresses *tenths.*

The second figure to the right of the unit figure expresses *hundredths.*

The third figure to the right of the unit figure expresses *thousandths.*

The fourth figure to the right of the unit figure expresses *ten-thousandths.*

The fifth figure to the right of the unit figure expresses *hundred-thousandths.*

The sixth figure to the right of the unit figure expresses *millionths.*

Thus:

$$
\begin{aligned}
.3 &= \tfrac{3}{10} &&= 3 \text{ tenths.}\\
.03 &= \tfrac{3}{100} &&= 3 \text{ hundredths.}\\
.003 &= \tfrac{3}{1000} &&= 3 \text{ thousandths.}\\
.0003 &= \tfrac{3}{10000} &&= 3 \text{ ten-thousandths.}\\
.00003 &= \tfrac{3}{100000} &&= 3 \text{ hundred-thousandths.}\\
.000003 &= \tfrac{3}{1000000} &&= 3 \text{ millionths.}
\end{aligned}
$$

The first figure to the right of the unit figure is called the *first decimal place;* the second figure, the *second decimal place*, etc. We see in the above that the number of decimal places in a decimal equals the *number of ciphers to the right of the figure 1 in the denominator of its equivalent fraction.* This fact kept in mind will be of much assistance in reading and writing decimals.

Whatever may be written to the *left* of a decimal point is a whole number. The decimal point affects only the figures to its *right.*

When a whole number and decimal are written together, the expression is a *mixed number.* Thus, 8.12 and 17.25 are mixed numbers.

The relation of decimals and whole numbers to each other is clearly shown by the following table:

hundreds of millions.	tens of millions.	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.	decimal point.	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	ten-millionths.	hundred-millionths.
9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9

The figures to the left of the decimal point represent whole numbers; those to the right are decimals.

In both the decimals and whole numbers, the *units* place is made the starting point of notation and numeration. The *decimals decrease* on the scale of ten to the right, and the *whole numbers increase* on the scale of ten to the left. The first figure to the left of units is *tens*, and the first figure to the right of units is *tenths*. The second figure to the left of units is *hundreds*, and the second figure to the right is *hundredths*. The third figure to the left is *thousands*, and the third to the right is *thousandths*, and so on. The figures equally distant from units place correspond in name, but the decimals have the ending *ths*, which distinguishes them from whole numbers. The following is the numeration of the number in the above table: nine hundred eighty-seven million, six hundred fifty-four thousand, three hundred twenty-one, and twenty-three million, four hundred fifty-six thousand, seven hundred eighty-nine hundred-millionths.

The decimals increase to the left, on the scale of ten, the same as whole numbers; for, beginning at, say, 4 thousandths, in the table, the next figure to the left is hundredths, which is ten times as great, and the next tenths, or ten times the hundredths, and so on through both decimals and whole numbers.

PRINCIPLES OF DECIMALS.

4. I. *The value of a decimal is not changed by annexing or rejecting a cipher at the right.*

Thus, $.5 = .50$. For $.5 = \frac{5}{10} = \frac{1}{2}$, and $.50 = \frac{50}{100} = \frac{1}{2}$.

II. *A decimal is divided by 10 by inserting a cipher after the decimal point.*

Thus, $.5 = \frac{5}{10}$; $\frac{5}{10} \div 10 = \frac{5}{100} = .05$.

III. *A decimal is multiplied by 10 by rejecting a cipher from its left.*

Thus, $.05 = \frac{5}{100}$; $\frac{5}{100} \times 10 = \frac{5}{10} = .5$.

EXAMPLES FOR PRACTICE.

5. Read the following numbers:

1.	.00707.	5.	.004001008.
2.	.100707.	6.	75040.0742.
3.	114.75206.	7.	234.542000.
4.	40032.890001.	8.	961724.009.

Write the following numbers:

9. Five hundred forty-six ten-millionths.

10. Three thousand, four, and three thousand four hundred seventeen hundred-thousandths.

11. Five hundred four, and three-tenths.

12. Nineteen thousand thirteen, and one hundred four thousand, five hundred one ten-billionths.

13. Six hundred thirty-eight million, four hundred twenty-five thousand, six hundred seventy-two, and thirty-two million, six hundred seventy-two thousand, five hundred forty-five hundred-millionths.

ADDITION OF DECIMALS.

6. To add decimals, they must be written so that the units of the same order are in the same column. That this may be, it is necessary only to see that the decimal points are in the same vertical column.

	whole numbers			*decimals*			*mixed decimals*	
	342			.342			342.032	
	4234			.4234			4234.5	
	26			.26			26.6782	
	3			.03			3.06	
sum	4605	Ans.	*sum*	1.0554	Ans.	*sum*	4606.2702	Ans.

7. EXAMPLE.—What is the sum of 242, .36, 118.725, 1.005, 6. and 100.1?

SOLUTION.—

	242.
	.36
	118.725
	1.005
	6.
	100.1
sum	468.190 Ans.

8. Rule.—*Place the numbers to be added so that the decimal points shall be directly under each other. Add as in whole numbers, and place the decimal point in the sum directly under the decimal points above.*

EXAMPLES FOR PRACTICE.

9. Find the sum of:

(*a*)	.2143, .105, 2.3042, and 1.1417.	Ans. (*a*)	3.7652.
(*b*)	783.5, 21.473, .2101, and .7816.	(*b*)	805.9647.
(*c*)	21.781, 138.72, 41.8738, .72, and 1.413.	(*c*)	204.5078.
(*d*)	.3724, 104.15, 21.417, and 100.042.	(*d*)	225.9814.
(*e*)	200.172, 14.105, 12.1465, .705, and 7.2.	(*e*)	234.3285.
(*f*)	1,427.16, .244, .32, .032, and 10.0041.	(*f*)	1,437.7601.
(*g*)	2,473.1, 41.65, .7243, 104.067, and 21.073.	(*g*)	2,640.6143.
(*h*)	4,107.2, .00375, 21.716, 410.072, and .0345.	(*h*)	4,539.02625.

SUBTRACTION OF DECIMALS.

10. For the same reason as in addition of decimals, the numbers are placed so that the decimal points shall be in the same vertical column.

EXAMPLE.—Subtract .132 from .3063.

SOLUTION.—

minuend	.3 0 6 3
subtrahend	.1 3 2
difference	.1 7 4 3 Ans.

11. EXAMPLE.—What is the difference between 7.895 and .725 ?

SOLUTION.—

minuend	7.8 9 5
subtrahend	.7 2 5
difference	7.1 7 0 or 7.17 Ans.

12. EXAMPLE.—Subtract .625 from 11.

SOLUTION.—

minuend	1 1.0 0 0
subtrahend	.6 2 5
difference	1 0.3 7 5 Ans.

13. **Rule.**—*Place the subtrahend under the minuend, so that the decimal points shall be in the same vertical column. Subtract as in whole numbers, and place the decimal point in the remainder directly under the decimal points above.*

When there are more decimal places in the subtrahend than in the minuend, place ciphers in the minuend above them, and subtract as before.

EXAMPLES FOR PRACTICE.

14. From:

(*a*)	407.385 take 235.0004.	(*a*)	172.3846.
(*b*)	22.718 take 1.7042.	(*b*)	21.0138.
(*c*)	1,368.17 take 13.6817.	(*c*)	1,354.4883.
(*d*)	70.00017 take 7.000017.	(*d*)	63.000153.
(*e*)	630.630 take .6304.	Ans. (*e*)	629.9996.
(*f*)	421.73 take 217.162.	(*f*)	204.568.
(*g*)	1.000014 take .00001.	(*g*)	1.000004.
(*h*)	.783652 take .542314.	(*h*)	.241338.

MULTIPLICATION OF DECIMALS.

15. In multiplication of decimals, no attention is paid for the time being to the decimal points. Write the multiplier under the multiplicand, so that the right-hand figure of the one is under the right-hand figure of the other, and proceed *exactly as in multiplication of whole numbers.* After multiplying, *count the number of decimal places in both multiplicand and multiplier, and point off the same number in the product.*

EXAMPLE.—Multiply .825 by 13.

SOLUTION.—

multiplicand	.8 2 5	
multiplier	1 3	
	2 4 7 5	
	8 2 5	
product	1 0.7 2 5	Ans.

In this example there are 3 decimal places in the multiplicand and none in the multiplier; therefore, $3 + 0 = 3$ decimal places are pointed off in the product.

16. EXAMPLE.—What is the product of 426 and the decimal .005?

SOLUTION.—

multiplicand	4 2 6	
multiplier	.0 0 5	
product	2.1 3 0 or 2.13	Ans.

In this example there are 3 decimal places in the multiplier and none in the multiplicand; therefore, 3 decimal places are pointed off in the product.

17. It is not necessary to multiply by the ciphers on the *left* of a decimal; they merely *determine the number of decimal places.* Ciphers to the *right* of a decimal should be removed, as they only make more figures to deal with, and do not change the value.

18. EXAMPLE.—Multiply 1.205 by 1.15.

SOLUTION.—

multiplicand	1.2 0 5
multiplier	1.1 5
	6 0 2 5
	1 2 0 5
	1 2 0 5
product	1.3 8 5 7 5 Ans.

In this example there are 3 decimal places in the multiplicand, and 2 in the multiplier; therefore, 3 + 2, or 5 decimal places must be pointed off in the product.

19. EXAMPLE.—Multiply .232 by .001.

SOLUTION.—

multiplicand	.2 3 2
multiplier	.0 0 1
product	.0 0 0 2 3 2 Ans.

In this example we multiply the multiplicand by the digit in the multiplier, which makes 232 in the product, but since there are 3 decimal places in the multiplier and 3 in the multiplicand, we must prefix 3 ciphers to the 232, to make 3 + 3, or 6 decimal places in the product.

20. Rule.—*Place the multiplier under the multiplicand, disregarding the position of the decimal points. Multiply as in whole numbers, and in the product point off as many decimal places as there are decimal places in both multiplier and multiplicand, prefixing ciphers if necessary.*

EXAMPLES FOR PRACTICE.

21. Find the product of:

(*a*)	.000492 × 4.1418.	(*a*)	.0020377656.
(*b*)	4,003.2 × 1.2.	(*b*)	4,803.84.
(*c*)	78.6531 × 1.03.	(*c*)	81.012693.
(*d*)	.3685 × .042.	(*d*)	.015477.
(*e*)	178,352 × .01.	Ans. (*e*)	1,783.52.
(*f*)	.00045 × .0045.	(*f*)	.000002025.
(*g*)	.714 × .00002.	(*g*)	.00001428.
(*h*)	.00004 × .008.	(*h*)	.00000032.

DIVISION OF DECIMALS.

22. In division of decimals we pay no attention to the decimal point until after the division is performed. Divide exactly as in whole numbers. *If the divisor contains more decimal places than the dividend, annex ciphers to the dividend until the number of decimal places in the dividend equals the number of decimal places in the divisor, before dividing. Subtract the number of decimal places in the divisor from the number of decimal places in the dividend, and point off as many decimal places in the quotient as there are units in the remainder thus found.*

23. Example.—Divide .625 by 25.

```
              divisor dividend quotient
Solution.—       25 ) .625 ( .025  Ans.
                       50
                       ---
                       125
                       125
                       ---
              remainder  0
```

In this example there are no decimal places in the divisor, and 3 decimal places in the dividend; therefore, there are 3 minus 0, or 3 decimal places in the quotient. One cipher has to be prefixed to the 25, to make the 3 decimal places.

24. Example.—Divide 6.035 by .05.

```
                   divisor dividend
Solution.—           .05 ) 6.035
                          ------
               quotient   120.7   Ans.
```

In this example we divide by 5, as if the cipher were not before it. There is one more decimal place in the dividend

than in the divisor; therefore, one decimal place is pointed off in the quotient.

25. EXAMPLE.—Divide .125 by .005.

SOLUTION.—

```
  divisor  dividend
    .005 ) .125
 quotient   25   Ans.
```

In this example there are the same number of decimal places in the dividend as in the divisor; therefore, the quotient has no decimal places, and is a whole number.

26. EXAMPLE.—Divide 326 by .25.

SOLUTION.—

```
   divisor  dividend  quotient
     .25 ) 326.00 ( 1304   Ans.
           25
           ---
            76
            75
            ---
             100
             100
             ---
remainder      0
```

In this problem two ciphers were annexed to the dividend to make the number of decimal places equal to the number in the divisor. The quotient is a whole number.

27. EXAMPLE.—Divide .0025 by 1.25.

SOLUTION.—

```
     1.25 ) .00250 ( .002   Ans.
               250
               ---
remainder        0
```

EXPLANATION.—In this example we are to divide .0025 by 1.25. Consider the dividend as a whole number or 25 (disregarding the two ciphers at its left, for the present); also, consider the divisor as a whole number, or 125. It is evident that the dividend 25 will not contain the divisor, 125; we must, therefore, annex one cipher to the 25, thus making the dividend 250. 125 is contained twice in 250, so we place the figure 2 in the quotient. In pointing off the decimal places in the quotient, it must be remembered that there were only four decimal places in the dividend; but one cipher was annexed, making 4 + 1, or 5, decimal places. Since there are 5 decimal places in the dividend and 2 decimal places in the

divisor, we must point off $5-2$, or 3, decimal places in the quotient. In order to point off 3 decimal places, two ciphers must be prefixed to the figure 2, thereby making .002 the quotient. It is not necessary to consider the ciphers at the left of a decimal when dividing, except when determining the position of the decimal point in the quotient.

28. **Rule.—I.** *Place the divisor to the left of the dividend, and proceed as in division of whole numbers; in the quotient, point off as many decimal places as the number of decimal places in the dividend exceeds those in the divisor, prefixing ciphers to the quotient, if necessary.*

II. *If in dividing one number by another there is a remainder, the remainder can be placed over the divisor, as a fractional part of the quotient, but it is generally better to annex ciphers to the remainder, and continue dividing until there are 3 or 4 decimal places in the quotient, and then if there is still a remainder, terminate the quotient by the plus sign (+), to show that the division can be carried farther.*

29. Example.—What is the quotient of 199 divided by 15?

Solution.—
```
          15)199(13+4/15 Ans.
             15
             --
              49
              45
              --
remainder      4

Or, 15)199.000(13.266+ Ans.
       15
       --
        49
        45
        --
         40
         30
         ---
         100
          90
          ---
          100
           90
           --
remainder  10
```

$13\frac{4}{15} = 13.266+$

$\frac{4}{15} = .266+$

30. It frequently happens, as in the above example, that the division will never terminate. In such cases, decide how many decimal places are desired in the quotient and then carry the work one place farther. If the last figure of the quotient thus obtained is 5 or a greater number, increase the preceding figure by 1, and write after it the minus sign (—), thus indicating that the quotient is not quite so great as indicated; if the figure thus obtained is less than 5, write the plus sign (+) after the quotient, thus indicating that the number is slightly greater than indicated. In the last example, had it been desired to obtain the answer correct to four decimal places, the work would have been carried to five places, obtaining 13.26666, and the answer would have been given as 13.2667—. This remark applies to any other calculation involving decimals, when it is desired to omit some of the figures in the decimal. Thus, if it is desired to retain three decimal places in the number .2471253, it would be expressed as .247+; if it were desired to retain five decimal places, it would be expressed as .24713—. Both the + and — signs are frequently omitted; they are seldom used in this connection outside of arithmetic, except in exact calculations, when it is desired to call particular attention to the fact that the result obtained is not *quite* exact.

EXAMPLES FOR PRACTICE.

31. Divide:

(*a*)	101.6688 by 2.36.	Ans.	(*a*)	43.08.
(*b*)	187.12264 by 123.107.		(*b*)	1.52.
(*c*)	.08 by .008.		(*c*)	10.
(*d*)	.0003 by 3.75.		(*d*)	.00008.
(*e*)	.0144 by .024.		(*e*)	.6.
(*f*)	.00375 by 1.25.		(*f*)	.003.
(*g*)	.004 by 400.		(*g*)	.00001.
(*h*)	.4 by .008.		(*h*)	50.

TO REDUCE A FRACTION TO A DECIMAL.

32. Example.— $\frac{3}{4}$ equals what decimal?

Solution.— $4\overline{)3.0\,0}$, $.7\,5$ or $\frac{3}{4} = .75$. Ans.

EXAMPLE.—What decimal is equivalent to $\frac{7}{8}$?

SOLUTION.— $\begin{array}{r} 8\,)\,7.0\,0\,0 \\ \hline .8\,7\,5 \end{array}$ or $\frac{7}{8} = .875$. Ans.

33. **Rule.**—*Annex ciphers to the numerator and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed.*

EXAMPLES FOR PRACTICE.

34. Reduce the following common fractions to decimals:

(*a*) $\frac{15}{32}$.		(*a*) .46875.
(*b*) $\frac{7}{8}$.		(*b*) .875.
(*c*) $\frac{21}{32}$.		(*c*) .65625.
(*d*) $\frac{51}{64}$.	Ans.	(*d*) .796875.
(*e*) $\frac{4}{25}$.		(*e*) .16.
(*f*) $\frac{5}{8}$.		(*f*) .625.
(*g*) $\frac{10}{200}$.		(*g*) .05.
(*h*) $\frac{4}{1000}$.		(*h*) .004.

35. **To reduce inches to decimal parts of a foot.**

EXAMPLE.—What decimal part of a foot is 9 inches?

SOLUTION.—Since there are 12 inches in one foot, 1 inch is $\frac{1}{12}$ of a foot, and 9 inches is $9 \times \frac{1}{12}$ or $\frac{9}{12}$ of a foot. This reduced to a decimal by the above rule, shows what decimal part of a foot 9 inches is.

$$\begin{array}{l} 1\,2\,)\,9.0\,0\,(\,.7\,5 \text{ of a foot. Ans.} \\ \quad\;\; \underline{8\,4} \\ \quad\;\;\; 6\,0 \\ \quad\;\;\; \underline{6\,0} \end{array}$$

36. **Rule.—I.** *To reduce inches to a decimal part of a foot, divide the number of inches by 12.*

II. *Should the resulting decimal be an unending one and it is desired to terminate the division at some point, say the fourth decimal place, carry the division one place farther, and if the fifth figure is 5 or greater, increase the fourth figure by 1. Omit the signs + and —.*

EXAMPLES FOR PRACTICE.

37. Reduce to the decimal part of a foot:

(*a*) 3 in.		(*a*) .25.
(*b*) $4\frac{1}{2}$ in.		(*b*) .375.
(*c*) 5 in.	Ans.	(*c*) .4167.
(*d*) $6\frac{5}{8}$ in.		(*d*) .5521.
(*e*) 11 in.		(*e*) .9167.

TO REDUCE A DECIMAL TO A FRACTION.

38. EXAMPLE.—Reduce .125 to a fraction.

SOLUTION.— $.125 = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8}$.

EXAMPLE.—Reduce .875 to a fraction.

SOLUTION.— $.875 = \frac{875}{1000} = \frac{35}{40} = \frac{7}{8}$. Ans.

39. Rule.—*Under the figures of the decimal, place 1 followed by as many ciphers at its right as there are decimal places in the decimal, and reduce the resulting fraction to its lowest terms.*

EXAMPLES FOR PRACTICE.

40. Reduce the following to common fractions:

(*a*) .125.		(*a*) $\frac{1}{8}$.
(*b*) .625.		(*b*) $\frac{5}{8}$.
(*c*) .3125.		(*c*) $\frac{5}{16}$.
(*d*) .04.		(*d*) $\frac{1}{25}$.
(*e*) .06.	Ans.	(*e*) $\frac{3}{50}$.
(*f*) .75.		(*f*) $\frac{3}{4}$.
(*g*) .15625.		(*g*) $\frac{5}{32}$.
(*h*) .875.		(*h*) $\frac{7}{8}$.

41. To express a decimal approximately as a fraction having a given denominator.

EXAMPLE.—Express .5827 in 64ths.

SOLUTION.— $.5827 \times \frac{64}{64} = \frac{37.2928}{64}$, say, $\frac{37}{64}$.

Hence, $.5827 = \frac{37}{64}$, nearly. Ans.

EXAMPLE.—Express .3917 in 12ths.

SOLUTION.— $.3917 \times \frac{12}{12} = \frac{4.7004}{12}$, say, $\frac{5}{12}$.

Hence, $.3917 = \frac{5}{12}$, nearly. Ans.

42. **Rule.**—*Reduce 1 to a fraction having the given denominator. Multiply the given decimal by the fraction so obtained, and the result will be the fraction required.*

EXAMPLES FOR PRACTICE.

43. Express:

(*a*)	.625 in 8ths.	Ans. (*a*)	$\frac{5}{8}$.
(*b*)	.3125 in 16ths.	(*b*)	$\frac{5}{16}$.
(*c*)	.15625 in 32ds.	(*c*)	$\frac{5}{32}$.
(*d*)	.77 in 64ths.	(*d*)	$\frac{49}{64}$.
(*e*)	.81 in 48ths.	(*e*)	$\frac{39}{48}$.
(*f*)	.923 in 96ths.	(*f*)	$\frac{89}{96}$.

UNITED STATES MONEY.

44. The sign for dollars is \$. It is read dollars. \$25 is read 25 dollars.

Since there are 100 cents in a dollar, one cent is 1 one-hundredth of a dollar; the first two figures of a decimal part of a dollar represent *cents*. Since a mill is $\frac{1}{10}$ of a cent, or $\frac{1}{1000}$ of a dollar, the third figure represents mills. Thus, \$25.16 is read twenty-five dollars and sixteen cents; \$25.168 is read twenty-five dollars, sixteen cents, and eight mills.

45. To change dollars to cents, move the decimal point *two* places to the *right;* to change dollars to mills, move the decimal point *three* places to the right. Thus, to change \$143.75 to cents, we have \$143.75 = 14,375 cents, or 143,750 mills. The decimal point is always understood as following the unit figure, whether written or not; hence, to change \$100 to cents, write it thus, \$100. ; to move the decimal point two places to the right, it is necessary to annex two ciphers, thus, 10000; in other words, \$100 = 10,000 cents.

Moving the decimal point two places to the right is evidently the same thing as multiplying by 100, since it changes the unit figure from the first order to the third order. Thus, in 143.75, 3 is a figure of the first order; but, in 14375., 3 is a figure of the third order; and, since all the other figures have also been advanced two orders, the number has been multiplied by 100.

46. To change cents to dollars, move the decimal point *two* places to the *left;* or, to change mills to dollars, move the decimal point *three* places to the left. Thus, to change 143,750 mills to dollars, move the decimal point (understood to follow the 0) three places to the left, obtaining \$143.750 = \$143.75. Similarly, 14,375 cents = \$143.75, and 10,000 cents = \$100.00.

REPEATING DECIMALS.

47. Consider the fraction $\frac{2}{3}$. Reducing this to a decimal, it becomes .66666+, and it is evident that, no matter how far the operation is carried, the process will never terminate, and that all succeeding figures will be 6's. Now, instead of writing the result as above, it may be written $.\dot{6}$, the dot over the 6 indicating that the 6 *repeats;* that is, that $.\dot{6}$ is equivalent to .666666+. Again, consider the fraction $\frac{10}{11}$; this, reduced to a decimal, becomes .90909090+, in which two figures, 9 and 0, repeat. This fact may be indicated by placing a dot over the 9 and over the 0, thus, $.\dot{9}\dot{0}$, whence, $\frac{10}{11} = .909090+ = .\dot{9}\dot{0}$. Had the fraction been $\frac{1}{11}$ instead of $\frac{10}{11}$, the result would have been $\frac{1}{11} = .090909+ = .\dot{0}\dot{9}$. Again, $\frac{1}{7}$ equals .142857142857+. Here it will be noticed that the figures 142857 repeat; this is indicated by placing a dot over the 1 and another dot over the 7, thus, $.\dot{1}4285\dot{7}$; that is, $\frac{1}{7} = .\dot{1}4285\dot{7}$.

48. Any decimal containing figures that repeat at regular intervals is called a **repeating decimal,** a **circulating decimal,** or a **circulate.**

The repeating part of the decimal is called the **repetend.** When the repetend begins with the first figure to the right of the decimal point, it is called a **pure repetend.** All of the above repetends are pure repetends.

49. Any fraction whose denominator contains only the prime factors, 2 and 5, will not repeat when converted into a decimal; that is, the division will be exact if carried far enough. But if the denominator contains any other prime factors than 2 and 5, as 3, 7, 11, etc., the fraction will produce a repeating decimal.

50. If the denominator contains the prime factor 2 or 5, or both, and other prime factors also, and the fraction itself is reduced to its lowest terms, the first figure to the right of the decimal point will not be a part of the repetend; the resulting decimal is then called a **mixed repetend.** Thus, $5 \times 3 = 15$; whence, $\frac{8}{15} = .53333+ = .5\dot{3}$, a mixed repetend. Similarly, $2 \times 11 = 22$, and $\frac{5}{22} = .22727+ = .2\dot{2}\dot{7}$.

51. Since most fractions will not reduce to exact decimals, it is frequently convenient to express the resulting decimal as a common fraction, or as a decimal and common fraction combined. This may always be done; for, any repetend may be converted into a common fraction.

Since $\frac{1}{9} = .11111+ = .\dot{1}$, $\frac{1}{99} = .010101+ = .\dot{0}\dot{1}$, $\frac{1}{999} = .001001001+ = .\dot{0}0\dot{1}$, etc. $.\dot{1} = \frac{1}{9}$, $.\dot{0}\dot{1} = \frac{1}{99}$, $.\dot{0}0\dot{1} = \frac{1}{999}$, etc.; hence, we may derive the denominator of any circulate from its relation to these given circulates. To illustrate, reduce $.\dot{5}$, $.\dot{4}\dot{5}$, and $.\dot{3}2\dot{4}$ to common fractions. Since $.\dot{1} = \frac{1}{9}$, $.\dot{5}$ which is 5 times $.\dot{1}$ = 5 times $\frac{1}{9} = \frac{5}{9}$. $.\dot{0}\dot{1} = \frac{1}{99}$ and $.\dot{4}\dot{5}$ which is 45 times $.\dot{0}\dot{1} = \frac{45}{99}$, and, reduced to its lowest terms, $= \frac{5}{11}$. In the same way $.\dot{3}2\dot{4}$ may be reduced to the fraction $\frac{12}{37}$. Hence, the following rule:

52. Rule.—*Write the repetend as the numerator of a fraction, and for the denominator, write as many nines as there are different figures that repeat.*

53. Example.—Express (*a*) $.\dot{6}$; (*b*) $.\dot{9}\dot{0}$; (*c*) $.\dot{0}\dot{9}$; (*d*) $.\dot{1}4285\dot{7}$ as common fractions.

Solution.—(*a*) Applying the rule, $.\dot{6} = \frac{6}{9} = \frac{2}{3}$. Ans.
(*b*) $.\dot{9}\dot{0} = \frac{90}{99} = \frac{10}{11}$. Ans.
(*c*) $.\dot{0}\dot{9} = \frac{9}{99} = \frac{1}{11}$. Ans.
(*d*) $.\dot{1}4285\dot{7} = \frac{142857}{999999} = \frac{1}{7}$. Ans.

In the case of a mixed repetend, apply the rule to the part that repeats.

Example.—Reduce (*a*) $.5\dot{3}$ and (*b*) $.2\dot{2}\dot{7}$ to a common fraction.

Solution.—(*a*) $.5\dot{3} = .5\frac{3}{9} = .5\frac{1}{3}$, and $\frac{5\frac{1}{3}}{10} = \frac{5\frac{1}{3} \times 3}{10 \times 3} = \frac{16}{30} = \frac{8}{15}$. Ans.

(*b*) $.2\dot{2}\dot{7} = .2\frac{27}{99} = .2\frac{3}{11} = \frac{2\frac{3}{11}}{10} = \frac{2\frac{3}{11} \times 11}{10 \times 11} = \frac{25}{110} = \frac{5}{22}$. Ans.

54. When a decimal and a common fraction are used in combination, *the fraction belongs to the order of the figure next preceding it.* Thus, in $.37\frac{1}{2}$, the fraction belongs to the second order, and not to the third. In other words,

$$.37\tfrac{1}{2} = \frac{37\frac{1}{2}}{100} = \frac{37.5}{100} = \frac{375}{1,000} = .375.$$

So, also, $.0\frac{1}{2} = \dfrac{\frac{1}{2}}{10} = \frac{1}{20}$, and $0.\frac{1}{2} = .\frac{1}{2} = \frac{1}{2}$. The decimal point serves only to separate the order of units from that of tenths. In all cases in which a decimal and a common fraction are used in combination, as, for example, $.67\frac{5}{8}$, if it is desired to express the common fraction as a decimal, proceed as if there were no decimal fraction before it, and then annex the figures of the result to the original decimal. Thus, in $.67\frac{5}{8}$, $\frac{5}{8} = .625$; hence, $.67\frac{5}{8} = .67625$. Proof: $.67\frac{5}{8} = \dfrac{67\frac{5}{8}}{100}$ $= \dfrac{541}{800} = .67625$. For the want of a better name, we shall call such fractions as $.67\frac{5}{8}$ **combined fractions.**

Combined fractions frequently occur in business. They possess nearly all of the advantages of decimals, and at the same time give the absolutely exact results obtainable by common fractions. If a repeating decimal has a mixed repetend, it may be converted into a combined fraction by means of the rule given in Art. **52.**

55. Repeating decimals frequently arise in ordinary division. Thus, $1,188 \div 56 = 21.2142857142857+$. Here it is seen that the six figures 142857 form a repetend. We can place the first dot over any figure of the repetend we choose; then look to the right *five* figures further, and place the second dot over the fifth figure. Thus, in the above expression, suppose that we desired a combined fraction having three figures in the decimal part. Place the first dot over the fourth figure from the decimal point, and the second dot over the ninth figure. Thus, $21.214\dot{2}8571\dot{4}$. Reducing this to a combined fraction it becomes $21.214\frac{2}{7}$.

SYMBOLS OF AGGREGATION.

56. The **vinculum** ——, **parenthesis** ( ), **brackets** [ ], and **brace** { } are called **symbols of aggregation,** and are used to include numbers that are to be considered together; thus, $13 \times \overline{8-3}$, or $13 \times (8-3)$ shows that 13 is to be multiplied by the difference between 8 and 3.

$$13 \times (8-3) = 13 \times 5 = 65. \quad \text{Ans.}$$

$$13 \times \overline{8-3} = 13 \times 5 = 65. \quad \text{Ans.}$$

When the vinculum or parenthesis is not used, we have

$$13 \times 8 - 3 = 104 - 3 = 101. \quad \text{Ans.}$$

57. In any series of numbers connected by the signs $+$, $-$, $\times$, and $\div$, the operations indicated by the signs must be performed in order from left to right, *except* that no addition or subtraction may be performed if a sign of multiplication or division *follows* the number on the *right* of a sign of addition or subtraction, until the indicated multiplication or division has been performed. In all cases the sign of multiplication takes the precedence, the reason being that when two or more numbers or expressions are connected by the sign of multiplication, the numbers thus connected are regarded as factors of the product indicated, and not as separate numbers.

EXAMPLE.—What is the value of $4 \times 24 - 8 + 17$?

SOLUTION.—Performing the operations in order from left to right, $4 \times 24 = 96$; $96 - 8 = 88$; $88 + 17 = 105$. Ans.

58. EXAMPLE.—What is the value of the following expression: $1{,}296 \div 12 + 160 - 22 \times 3\frac{1}{2}$?

SOLUTION.— $1{,}296 \div 12 = 108$; $108 + 160 = 268$; here we cannot subtract 22 from 268 because the sign of multiplication *follows* 22; hence multiplying 22 by $3\frac{1}{2}$, we get 77, and $268 - 77 = 191$. Ans.

Had the above expression been written $1{,}296 \div 12 + 160 - 22 \times 3\frac{1}{2} \div 7 + 25$, it would have been necessary to divide $22 \times 3\frac{1}{2}$ by 7 before subtracting, and the final result would have been $22 \times 3\frac{1}{2} = 77$; $77 \div 7 = 11$; $268 - 11 = 257$; $257 + 25 = 282$. Ans. In other words, it is necessary to

perform *all* of the multiplication or division included between the signs + and —, or — and +, before adding or subtracting. Also, had the expression been written $1{,}296 \div 12 + 160 - 24\frac{1}{2} \div 7 \times 3\frac{1}{2} + 25$, it would have been necessary to multiply $3\frac{1}{2}$ by 7 before dividing $24\frac{1}{2}$, since the sign of multiplication takes the precedence, and the final result would have been $3\frac{1}{2} \times 7 = 24\frac{1}{2}$; $24\frac{1}{2} \div 24\frac{1}{2} = 1$; $268 - 1 = 267$; $267 + 25 = 292$. Ans.

It likewise follows that if a succession of multiplication and division signs occur, the indicated operations must not be performed in order, from left to right—the multiplication must be performed first. Thus, $24 \times 3 \div 4 \times 2 \div 9 \times 5 = \frac{1}{5}$. Ans. In order to obtain the same result that would be obtained by performing the indicated operations in order, from left to right, symbols of aggregation must be used. Thus, by using two vinculums, the last expression becomes $24 \times \overline{3 \div 4} \times \overline{2 \div 9} \times 5 = 20$, the same result that would be obtained by performing the indicated operations in order, from left to right.

EXAMPLES FOR PRACTICE.

59. Find the values of the following expressions:

		Ans.
(*a*)	$(8 + 5 - 1) \div 4$.	(*a*) 3.
(*b*)	$5 \times 24 - 32$.	(*b*) 88.
(*c*)	$5 \times 24 \div 15$.	(*c*) 8.
(*d*)	$144 - 5 \times 24$.	(*d*) 24.
(*e*)	$(1{,}691 - 540 + 559) \div 3 \times 57$.	(*e*) 10.
(*f*)	$2{,}080 + 120 - 80 \times 4 - 1{,}670$.	(*f*) 210.
(*g*)	$(\overline{90 + 60} \div 25) \times 5 - 29$.	(*g*) 1.
(*h*)	$\overline{90 + 60} \div 25 \times 5$.	(*h*) 1.2.

ALIQUOT PARTS.

60. An **aliquot part** of a number is a number that will divide it without a remainder. For example, 15 is an aliquot part of 60, because 15 is an exact divisor of 60. For the same reason, 5, 6, 10, 12, etc. are also aliquot parts of 60.

61. About the only case of any importance to the business man is that concerning the aliquot parts of 100, which are given in the following table:

$2\frac{1}{2} = \frac{1}{40}$	$12\frac{1}{2} = \frac{1}{8}$	$50 = \frac{1}{2}$
$3\frac{1}{3} = \frac{1}{30}$	$16\frac{2}{3} = \frac{1}{6}$	$62\frac{1}{2} = \frac{5}{8}$
$4 = \frac{1}{25}$	$20 = \frac{1}{5}$	$66\frac{2}{3} = \frac{2}{3}$
$5 = \frac{1}{20}$	$25 = \frac{1}{4}$	$75 = \frac{3}{4}$
$6\frac{1}{4} = \frac{1}{16}$	$33\frac{1}{3} = \frac{1}{3}$	$87\frac{1}{2} = \frac{7}{8}$
$10 = \frac{1}{10}$	$37\frac{1}{2} = \frac{3}{8}$	

The aliquot parts given in the above table should be carefully memorized; in many cases calculations may be shortened by using them. Thus, in order to multiply any number by one of the numbers in the above table:

62. Rule.—*Move the decimal point of the multiplicand two places to the right, and divide the multiplicand by the denominator of the fraction opposite the multiplier in the table. Then multiply the result by the numerator of the fraction.*

63. Example.—Multiply 478.4 by 25.

Solution.—Applying the rule in Art. **62**, the fraction opposite 25 is $\frac{1}{4}$. Hence, $478.4 \times 25 = 47{,}840 \div 4 \times 1 = 11{,}960$. Ans.

The result may be proved to be true by actual multiplication.

64. The reasoning on which the rule is based is this: The last example required the multiplication of 478.4 by 25. But 25 is $\frac{1}{4}$ of 100; that is, $25 = \frac{100}{4}$. Hence, in multiplying by 25, $\frac{100}{4}$ can be used as a multiplier, and the operation becomes $478.4 \times \frac{100}{4} = \frac{47{,}840}{4} = 11{,}960$. Since, in order to multiply by 100, it is necessary only to move the decimal point two places to the right, the correctness of the rule is evident.

Example.—Multiply (*a*) 50.64 by $16\frac{2}{3}$; (*b*) 1,894 by $37\frac{1}{2}$.

Solution.—(*a*) Since $16\frac{2}{3}$ is $\frac{1}{6}$ of 100, $50.64 \times 16\frac{2}{3} = \frac{5{,}064}{6} = 844$. Ans.

(*b*) Since $37\frac{1}{2} = \frac{3}{8}$ of 100, $1{,}894 \times 37\frac{1}{2} = \frac{189{,}400}{8} \times 3 = 23{,}675 \times 3 = 71{,}025$. Ans.

65. It matters not where the decimal point is placed in the number denoting the aliquot part—the principle can still be applied with a slight modification. Thus, suppose it is required to multiply 72 by 625. Now, $62\frac{1}{2}$, or 62.5, is $\frac{5}{8}$ of 100; that is, $62.5 = 100 \times \frac{5}{8} = \frac{500}{8}$. It is an axiom in mathematics that if equals be multiplied or divided by equals, the result will be equal; that is, if $4 = 4$, and both 4's be multiplied or divided by the same number, the results will be equal. Thus, multiplying by 10, $40 = 40$; dividing by 10, $.4 = .4$. Hence, if in the expression $62.5 = \frac{500}{8}$, both numbers are multiplied by 10, the result is $625 = \frac{5,000}{8}$, or $625 = 1,000 \times \frac{5}{8}$. Therefore, to multiply 72 by 625, move the decimal point three places to the right, divide by 8 and multiply by 5. (It makes no difference whether we divide by the denominator or multiply by the numerator first.) Then, $72 \times 625 = \frac{72,000}{8} \times 5 = 45,000$. Had it been required to multiply 72 by .0625, we note that $62.5 = 100 \times \frac{5}{8}$, and that moving the decimal point in 62.5 three places to the left will give .0625. Hence, moving the decimal point in the other number (the 100) three places to the left, we have $.0625 = .1 \times \frac{5}{8} = \frac{.5}{8}$; whence, $72 \times .0625 = \frac{72}{8} \times .5 = 4.5$.

To multiply any number by 5, multiply by 10 and divide by 2.

66. To divide by one of the aliquot parts of 100, simply reverse the rule. Thus:

Rule.—*Move the decimal point two places to the left, multiply by the denominator of the equivalent fraction in the table, and divide by the numerator.*

67. Example.—Divide 1,844 by $16\frac{2}{3}$.

Solution.—Since $16\frac{2}{3} = 100 \times \frac{1}{6}$, $1,844 \div 16\frac{2}{3} = 1,844 \times \frac{6}{100} = 110.64$. Ans.

Or, applying the rule, $1,844 \div 16\frac{2}{3} = 18.44 \times 6 \div 1 = 110.64$. Ans.

EXAMPLE.—Divide 71,025 by 37.5.

SOLUTION.—Applying the rule, $71,025 \div 37.5 = 710.25 \times 8 \div 3 = 5,682 \div 3 = 1,894$. Ans.

68. To divide by a number having all the figures of the aliquot part, but with the decimal point in a different place, proceed in the same manner as in Art. **65.** Thus, to divide 45,000 by 625, we have $62.5 = 100 \times \frac{5}{8}$; hence, $625 = 1,000 \times \frac{5}{8} = \frac{5,000}{8}$, and $45,000 \div 625 = 45,000 \div \frac{5,000}{8} = \overset{9}{\cancel{45},\cancel{000}} \times \frac{8}{\cancel{5},\cancel{000}} = 72$. Or, $45,000 \times \frac{8}{5,000} = 45,000 \times \frac{1}{1,000} \times \frac{8}{5} = \overset{9}{\cancel{45}} \times \frac{8}{\cancel{5}} = 72$. Ans.

69. The student will find the principle of aliquot parts extremely convenient for accurate and rapid work. Such numbers as $12\frac{1}{2}$, $16\frac{2}{3}$, 25, $33\frac{1}{3}$, $37\frac{1}{2}$, $62\frac{1}{2}$, 75, and $87\frac{1}{2}$ are of very frequent occurrence in business accounts, and the method can be readily employed in such cases. If these numbers are given as $.12\frac{1}{2}$, $.16\frac{2}{3}$, etc., as is usually the case, the example becomes even easier, as they are then equivalent to $\frac{1}{8}$, $\frac{1}{6}$, etc.

EXAMPLE.—What will be the cost of 48 yards of carpeting at $\$1.33\frac{1}{3}$ per yard?

SOLUTION.—Since $.33\frac{1}{3} = \frac{1}{3}$, $\$1.33\frac{1}{3} = 1\frac{1}{3}$, and $48 \times 1\frac{1}{3} = \$64$. Ans.

$$\begin{array}{r} 48 \\ \underline{1\frac{1}{3}} \\ 16 \\ \underline{48} \\ \$64 \end{array}$$

EXAMPLES FOR PRACTICE.

70. Multiply:

(*a*)	5,427 by 25.	(*a*)	135,675.
(*b*)	6,301 by $12\frac{1}{2}$.	(*b*)	78,762.5.
(*c*)	42,078 by .0375.	Ans. (*c*)	1,577.925.
(*d*)	750 by $6.6\frac{2}{3}$.	(*d*)	5,000.
(*e*)	89.14 by 8,750.	(*e*)	779,975.

Divide:

(*f*) 542.7 by 75.		(*f*) 7.236.
(*g*) 90,309 by 125.		(*g*) 722.472.
(*h*) 3.1416 by $66\frac{2}{3}$	Ans.	(*h*) .047124.
(*i*) 20,412 by 50.		(*i*) 408.24.
(*j*) 1,729 by 875.		(*j*) 1.976.

1. Find the cost of 12 dozen hats at \$4.12$\frac{1}{2}$ per dozen. Ans. \$49.50.

2. Find the cost of 24 boxes of note paper at 16$\frac{2}{3}$ cents per box. Ans. \$4.

3. Find the cost of 75 books at 25 cents each. Ans. \$18.75.

4. Find the cost of 30.19 hundredweight of bran at 62$\frac{1}{2}$ cents per hundredweight. Ans. \$18.86$\frac{7}{8}$.

5. Find the cost of 36 pairs of shoes at \$2.25 per pair. Ans. \$81.

6. Find the cost of 87 pounds of sugar at 5 cents per pound. Ans. \$4.35.

ARITHMETIC.

COMPOUND NUMBERS.

1. Numbers may be expressed according to a *uniform* or a *varying scale.* By **scale** is meant the relation of a unit of one order to the unit of the next higher or lower order. When the relation is the same for any two consecutive orders, the scale is said to be **uniform**; otherwise, it is **varying.** For example, the scale by which numbers are expressed in the Arabic notation is a uniform scale, since a unit of any order is 10 times as great as the unit of the next lower order; for 100 is 10 times 10, and 10 is 10 times 1, etc. The Arabic notation, the metric system, and United States money are the leading examples of the application of a uniform scale. All other numbers in commercial use in English-speaking countries require the use of a varying scale. Thus, to express 4 yards 2 feet 7 inches, it is necessary to write the words *yards*, *feet*, and *inches*, or their abbreviations, since 12 inches equal 1 foot, and 3 feet equal 1 yard.

2. A **simple number** is one which expresses one or more units of the same name or denomination; as, 5, 6 yards, etc.

3. A **compound number** is one which expresses units of two or more denominations of the *same kind*, the denominations increasing or decreasing according to a varying scale; as, 4 yards 2 feet 7 inches. But 4 yards and 5 ounces is not a compound number, since there is no relation between yards and ounces; that is, no number of ounces can equal a yard. Compound numbers are also frequently called **denominate numbers.**

4. Compound numbers form a very important section of arithmetic. It is necessary for the student to obtain a clear idea of their use, and to be able to add, subtract, multiply, and divide them. The only real difficulty that arises is the memorizing of the tables. The best way to do this is to read them over very carefully several times, and then, after carefully studying the sections on Reduction, Addition, etc., work the examples (all of them), constantly referring to the tables for help. But, before leaving the subject to take up the next paper, he should thoroughly memorize *all* of the tables, so that when any one asks him how many square rods there are in an acre, or a similar question, he can answer instantly, without being obliged to stop and think.

5. A **measure** is a *standard unit* established by law or custom, by which the length, surface, capacity, and weight of things are estimated.

6. Measures are of six kinds, as follows:

(1) Extension.	(4) Time.
(2) Weight.	(5) Angles or Arcs.
(3) Capacity.	(6) Money or Value.

We shall consider them in the above order.

MEASURES OF EXTENSION.

7. Measures of extension are used in measuring lengths (distances), surfaces (areas), and solids (volumes), and are divided, accordingly, into *linear measure*, *square measure*, and *cubic measure*.

LINEAR MEASURE.

8. The standard to which all measures of extension are referred is the *yard*, which is the distance between two points on a brass bar kept at Washington. The yard is subdivided into feet and inches; and multiples of the yard are termed rods and miles. The relations between the different units are shown in the following table; the letters in Italics are the abbreviations of the names of the units.

LINEAR MEASURE.

TABLE I.

12	inches (*in.*)	= 1 foot	*ft.*
3	feet	= 1 yard	*yd.*
$5\frac{1}{2}$	yards	= 1 rod	*rd.*
320	rods	= 1 mile	*mi.*

in.	ft.	yd.	rd.	mi.
12 =	1			
36 =	3 =	1		
198 =	$16\frac{1}{2}$ =	$5\frac{1}{2}$ =	1	
63,360 =	5,280 =	1,760 =	320 =	1

9. The inch is usually divided into halves, quarters, eighths, and sixteenths; by civil engineers and scientists, into tenths, hundredths, thousandths, etc., and in other ways. In measuring cloth, ribbons, and other goods that are sold by the yard, the yard is divided into halves, quarters, eighths, and sixteenths.

A furlong is one-eighth of a mile, or 40 rods. The mile of 5,280 feet is a **statute mile,** so called to distinguish it from the geographical or sea mile, which equals 6,081 feet.

10. Another abbreviation frequently used for inches and feet is (″) and (′). Thus, instead of writing 4 feet 6 inches as 4 ft. 6 in., it may be written 4′ 6″; but when so written, it is customary to place a dash between the feet and inches; thus, 4′–6″. Still another way of writing the above is 4 ft. 6″.

SURVEYOR'S LINEAR MEASURE.

TABLE II.

7.92 inches (*in.*)	= 1 link	*li.*
25 links	= 1 rod	*rd.*
4 rods / 100 links	= 1 chain	*ch.*
80 chains	= 1 mile	*mi.*

in.	li.	ft.	rd.	ch.	mi.
7.92 =	1				
198 =	25 =	$16\frac{1}{2}$ =	1		
792 =	100 =	66 =	4 =	1	
63,360 =	8,000 =	5,280 =	320 =	80 =	1

11. Surveyor's linear measure is used in measuring land, roads, etc. The unit is a steel chain 66 feet long, and made of 100 links, all of equal length; therefore, the length of a link is $66 \times 12 \div 100 = 7.92$ inches. For railroad surveying and other purposes, civil engineers use a steel tape 100 feet long, the feet being divided into tenths and hundredths. In computations, the links are written as so many hundredths of a chain.

SQUARE MEASURE.

TABLE III.

144	square inches (*sq. in.*)	= 1	square foot	*sq. ft.*
9	square feet	= 1	square yard	*sq. yd.*
$30\frac{1}{4}$	square yards	= 1	square rod	*sq. rd.*
160	square rods	= 1	acre	*A.*
640	acres	= 1	square mile	*sq. mi.*

sq. in.		sq. ft.		sq. yd.		sq. rd.		A.		sq. mi.
144	=	1								
1,296	=	9	=	1						
39,204	=	$272\frac{1}{4}$	=	$30\frac{1}{4}$	=	1				
6,272,640	=	43,560	=	4,840	=	160	=	1		
4,014,489,600	=	27,878,400	=	3,097,600	=	102,400	=	640	=	1

12. Square measure is used in estimating the *area* of surfaces. In commercial use, the square yard is the largest unit employed, the square rod, acre, and square mile being used for measuring land. The unit of square measure is a *square* whose sides are equal in length to the linear unit. The units of square measure are *derived units;* that is, they depend for their value upon the values of some other units, which, in this case, are the units of linear measure. If the large square in Fig. 1 measures 1 foot on each side, as indicated

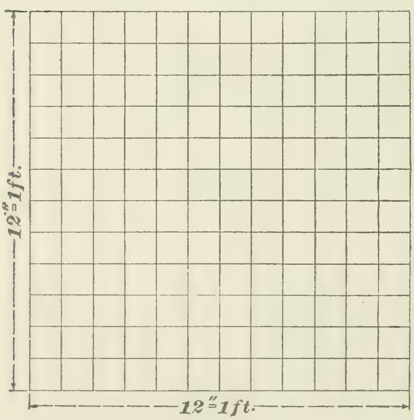

FIG. 1.

by the dimension lines, then the space which the square covers, on a flat surface, is *1 square foot.* If horizontal and vertical lines be drawn 1 inch apart, as shown, the small squares so formed will measure 1 inch on each side; and if these small squares be counted, they will be found to number 144. Hence, there are 144 square inches in 1 square foot. In the same way, it can be shown that 1 square yard contains 9 square feet. The *acre* is the only unit that forms an exception—it cannot be expressed as an exact square of any unit. A piece of land 208.71 feet square contains almost exactly an acre.

Roofers, plasterers, and carpenters frequently call 100 square feet a **square.**

SURVEYOR'S SQUARE MEASURE.

TABLE IV.

625 square links (*sq. li.*)	= 1 square rod....	*sq. rd.*
16 square rods	= 1 square chain .	*sq. ch.*
10 square chains	= 1 acre..........	*A.*
640 acres...........................	= 1 square mile...	*sq. mi.*
36 square miles (6 miles square)....	= 1 township	*Tp.*

13. **Surveyor's square measure** is used only by civil engineers and surveyors. For this reason, no further remarks will be made concerning this measure.

CUBIC MEASURE.

TABLE V.

1,728 cubic inches (*cu. in.*)........	= 1 cubic foot......	*cu. ft.*
27 cubic feet....................	= 1 cubic yard	*cu. yd.*

128 cubic feet = 1 cord of wood.

cu. in.		cu. ft.		cu. yd.
1,728	=	1		
46,656	=	27	=	1

14. **Cubic measure** is used in measuring the volumes of solids or bodies which have length, breadth, and thickness. The units of cubic measure are also derived units, since they depend upon linear measurements for their values.

The unit of cubic measure is a cube whose edges are equal in length to the corresponding linear unit. Fig. 2 represents a cube whose sides are all 3 feet long. By dividing it into equal parts, as shown, it is readily seen that 27 small cubes, measuring 1 foot on each edge, will be formed; hence, 1 cubic yard contains 27 cubic feet. In a similar manner, it can be shown that 1 cubic foot contains 1,728 cubic inches.

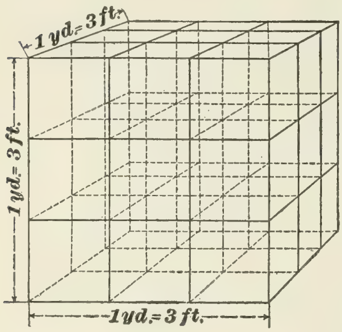

FIG. 2.

15. The **cord** is used in measuring wood. A pile of wood 8 feet long, 4 feet wide, and 4 feet high contains 1 cord, since $8 \times 4 \times 4 = 128$ cubic feet. A **cord foot** is 1 foot in length of such a pile; that is, it is 1 foot long, 4 feet wide, and 4 feet high. The cord foot contains $1 \times 4 \times 4 = 16$ cubic feet.

16. Masons use what is termed a perch. A **perch** of masonry is 1 rod ($16\frac{1}{2}$ ft.) long, $1\frac{1}{2}$ feet thick, and 1 foot high, and contains $16\frac{1}{2} \times 1\frac{1}{2} \times 1 = 24\frac{3}{4}$ cubic feet. The perch is going out of use, the cubic yard being used instead; but, when used, it is generally considered to be 25 cubic feet.

MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT.

TABLE VI.

16 ounces (*oz.*)	= 1 pound	*lb.*
100 pounds	= 1 hundredweight	*cwt.*
20 hundredweight / 2,000 pounds	= 1 ton	*T.*

oz.		lb.		cwt.		T.
16	=	1				
1,600	=	100	=	1		
32,000	=	2,000	=	20	=	1

17. Avoirdupois weight is used in nearly all commercial transactions. The ton of 2,000 pounds is always understood, unless the *long* or *gross* ton is specified. Formerly, the long ton of 2,240 pounds was used exclusively, and is still used in Great Britain, by U. S. Custom Houses, in ocean freights, and by wholesale dealers in coal and iron, and iron ores. The **long ton table** is as follows:

LONG TON TABLE.

TABLE VII.

16 ounces (*oz.*)	= 1 pound	*lb.*
28 pounds	= 1 quarter	*qr.*
4 quarters	= 1 hundredweight	*cwt.*
20 hundredweight / 2,240 pounds	= 1 ton	*T.*

oz.		lb.		qr.		cwt.		T.
16	=	1						
448	=	28	=	1				
1,792	=	112	=	4	=	1		
35,840	=	2,240	=	80	=	20	=	1

18. Formerly, the ounce was divided into 16 parts, called drams. The dram is now seldom or never used.

TROY WEIGHT.

TABLE VIII.

24 grains (*gr.*)	= 1 pennyweight	*pwt.*
20 pennyweights	= 1 ounce	*oz.*
12 ounces	= 1 pound	*lb.*

gr.		pwt.		oz.		lb.
24	=	1				
480	=	20	=	1		
5,760	=	240	=	12	=	1

19. Troy weight is used by jewelers, and for the weighing of gold, silver, coins, and jewels. The Troy pound contains 5,760 grains, while the avoirdupois pound contains

7,000 grains. Hence, the Troy ounce contains $5,760 \div 12 = 480$ grains, and the avoirdupois ounce, $7,000 \div 16 = 437\frac{1}{2}$ grains. The student will find it useful to remember these facts. When it is stated that an ounce of gold is worth, say, twenty dollars, the Troy ounce, which contains 480 grains, is meant.

APOTHECARIES' WEIGHT.

TABLE IX.

20 grains (*gr.*)	= 1 scruple	*sc.* or ℈
3 scruples	= 1 dram	*dr.* or ʒ
8 drams	= 1 ounce	*oz.* or ℥
12 ounces	= 1 pound	*lb.* or ℔

gr.		℈		ʒ		℥		℔.
20	=	1						
60	=	3	=	1				
480	=	24	=	8	=	1		
5,760	=	288	=	96	=	12	=	1

20. Apothecaries' weight is used by physicians and druggists in prescribing and compounding medicines not in the liquid state. When the *symbols* are used instead of the abbreviations, they are placed before the figures denoting the number of units. Thus, ʒ5 ℈2 means 5 drams 2 scruples. The pound, ounce, and grain are the same as in Troy weight.

Drugs and medicines are sold, when in large quantities, by avoirdupois weight.

MEASURES OF CAPACITY.

LIQUID MEASURE.

TABLE X.

4 gills (*gi.*)	= 1 pint	*pt.*
2 pints	= 1 quart	*qt.*
4 quarts	= 1 gallon	*gal.*
$31\frac{1}{2}$ gallons	= 1 barrel	*bbl.*
2 barrels / 63 gallons	= 1 hogshead	*hhd.*

gi.	pt.	qt.	gal.	bbl.	hhd.
4 =	1				
8 =	2 =	1			
32 =	8 =	4 =	1		
1,008 =	252 =	126 =	$31\frac{1}{2}$ =	1	
2,016 =	504 =	252 =	63 =	2 =	1

21. Liquid measure is used for measuring liquids. The standard gallon used in the United States is the *wine gallon*, so called to distinguish it from the *beer gallon.* The beer gallon was formerly used for measuring beer, milk, etc., but has now passed out of use. The wine gallon contains 231 cubic inches; the beer gallon contained 282 cubic inches. The gallon used in Great Britain is called the British imperial gallon; it contains 277.274 cubic inches. One wine gallon of water weighs 8.355 pounds, and 1 cubic foot contains 7.481 wine gallons. One British imperial gallon weighs 10 pounds. The student should carefully memorize these facts; he will find them very useful.

22. The barrel and hogshead are used in estimating the capacity of tanks, cisterns, reservoirs, etc. The gallon is the unit most commonly used when estimating in large quantities. The ordinary barrels and hogsheads used in commerce vary greatly in size, and their contents can be determined only by gauging or actual measurement.

APOTHECARIES' FLUID MEASURE.

TABLE XI.

60 minims, or drops, (♏)	= 1 fluidrachm	f ʒ.
8 fluidrachms	= 1 fluidounce	f ℥.
16 fluidounces	= 1 pint	*O.*
8 pints	= 1 gallon	*Cong.*

23. Apothecaries' fluid measure is used in prescribing and compounding medicines. The gallon and pint are the same as the wine gallon and pint. As in apothecaries' weight, the symbols precede the numbers to which they refer. For example, Cong. 2 O. 7 f℥ 12 means 2 gallons 7 pints 12 fluidounces. *Cong.* is the abbreviation of the Latin word *congius*, gallon; *O.* is the abbreviation of the Latin word *octavius*, one-eighth.

DRY MEASURE.

TABLE XII.

2 pints (*pt.*)	= 1 quart	*qt.*
8 quarts	= 1 peck	*pk.*
4 pecks	= 1 bushel	*bu.*

pt.		qt.		pk.		bu.
2	=	1				
16	=	8	=	1		
64	=	32	=	4	=	1

24. Dry measure, as its name implies, is used in measuring dry articles, as fruits, vegetables, grain, etc. The unit is the so called Winchester bushel, which contains 2,150.42 cubic inches, or very nearly 9.31 wine gallons. A box 14 inches long, 12.8 inches wide, and 12 inches deep (all inside measurements) contains almost exactly 1 bushel.

25. It is becoming the custom to sell by weight many articles that would ordinarily be sold by dry measure. The number of pounds of various commodities that are taken as equivalent to one bushel, varies greatly in different states; but the Boards of Trade of the principal cities of the United States use the equivalents given in the following table:

AVOIRDUPOIS POUNDS IN A BUSHEL.

TABLE XIII.

Commodities.	Lb.	Commodities.	Lb.	Commodities.	Lb.
Barley	48	Corn (shelled)	56	Potatoes	60
Beans	60	Corn (in the ear)	70	Rye	56
Buckwheat	48	Malt	34	Timothy Seed	45
Clover Seed	60	Oats	32	Wheat	60

26. The following units are also in commercial use:

1 Quintal of fish	=	100 lb.
1 Barrel of flour	=	196 lb.
1 Barrel of pork or beef	=	200 lb.
1 Gallon of petroleum	=	$6\frac{1}{2}$ lb.
1 Keg of nails	=	100 lb.

27. Stricken measure means measuring the vessel even full, and striking off the surplus with a stick. Grain, seeds, berries, etc. are sold by stricken measure. The standard bushel mentioned above is a stricken bushel. The **heaped bushel** means the contents of the measuring vessel heaped up in the form of a cone. Corn in the ear, large fruits and vegetables, coal, lime, and other bulky articles, are sold by the heaped bushel. It is customary to allow 5 stricken bushels for 4 heaped ones.

In San Francisco and some other markets, produce is bought and sold by the cental, a **cental** being 100 pounds. In computing freight charges the hundredweight of 100 pounds is taken as the unit.

MEASURES OF TIME.

TABLE XIV.

60 seconds (*sec.*)	= 1 minute	*min.*
60 minutes	= 1 hour	*hr.*
24 hours	= 1 day	*da.*
7 days	= 1 week	*wk.*
4 weeks	= 1 month	*mo.*
12 months	= 1 year	*yr.*
100 years	= 1 century	*C.*

sec.	min.	hr.	da.	wk.	yr.
60 =	1				
3,600 =	60 =	1			
86,400 =	1,440 =	24 =	1		
604,800 =	10,080 =	168 =	7 =	1	
31,556,936 =	525,948 =	8,765 =	365 =	52 =	1

28. The divisions of time are peculiar. The only units that may be called *natural* are the day, the lunar month, and the year, the other divisions being artificial. The time in which the earth makes one complete revolution around the sun is called a **solar year,** and it equals 365 days, 5 hours, 48 minutes, 49.7 seconds, or $365\frac{1}{4}$ days, very nearly. A **solar day** is the interval between two consecutive returns of the sun to the meridian. On account of the earth moving in an

elliptical path around the sun, the length of the solar day varies; hence, for civil purposes, the average of all the days in the year is taken as a unit. The months contain from 28 to 31 days.

In order to make the *calendar*, or *civil*, year agree as nearly as possible with the solar year, it is customary to add 1 day to the year (making 366 days) every fourth year. Years containing 366 days are called **leap years.** Every leap year is exactly divisible by 4. Thus 1896, which was a leap year, is divisible by 4 without a remainder. Any year which ends with two or more ciphers, as 1800, 1900, 2000, etc., is called a **secular year.** Since the year falls short of $365\frac{1}{4}$ days by 11 minutes and 10.3 seconds, the addition of 1 day every 4 years makes about $\frac{3}{4}$ of a day too much in a century; hence, to correct this, secular years are not leap years except when exactly divisible by 400. Thus, 1900 will not be a leap year but 2000 will. This last correction makes an error of about 1 day in 5,400 years.

29. The following is a list of months, in regular order, with the number of days which each contains:

	Days.		*Days.*
1. January (Jan.)	31	7. July	31
2. February (Feb.)	28	8. August (Aug.)	31
3. March (Mar.)	31	9. September (Sept.)	30
4. April (Apr.)	30	10. October (Oct.)	31
5. May	31	11. November (Nov.)	30
6. June	30	12. December (Dec.)	31

In leap years, one day is added to February, giving it 29 days. The following lines will assist the student in remembering the number of days in each month:

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Save February, which alone
Hath but twenty-eight, in fine,
But leap year gives it twenty-nine."

In many business transactions, the year is regarded as 360 days, or 12 months of 30 days each.

MEASURES OF ANGLES OR ARCS.

CIRCULAR MEASURE.

TABLE XV.

60 seconds (″)	=	1 minute	′
60 minutes	=	1 degree	°
360 degrees	=	1 circle	⊙

$$60'' = 1'$$
$$3{,}600'' = 60' = 1^\circ$$
$$1{,}296{,}000'' = 21{,}600' = 360^\circ = 1\odot$$

30. A **quadrant** is one-fourth of a circle, or 90°; a **sextant** is one-sixth of a circle, or 60°. A right angle (∟) contains 90°. The unit of measurement is the degree, or $\frac{1}{360}$ of the circumference of a circle.

31. **Circular** or **angular measure** is used principally by surveyors, navigators, astronomers, and by technical men generally, for measuring angles and arcs of circles.

LONGITUDE AND TIME.

32. The earth is very nearly spherical in shape, and has a rotary movement about an imaginary line (called the axis) which is assumed to pass through the center. The two points where the axis emerges from the earth are called **poles**—one the **north pole,** the other the **south pole.** The time required for the earth to make one rotation, or complete turn, about this axis, is called one day (see Art. **28**). Midway between the poles, the earth is supposed to have a circle passing around it, called the **equator.** The equator is divided into 360 equal parts called degrees (see Table XV); each degree is divided into 60 equal parts, called minutes; and each minute is again divided into 60 equal parts, called seconds. Through each point of division a circle, called a **meridian of longitude,** is supposed to pass, the meridian also passing through the poles, as shown in Fig. 3. Other circles, called **parallels of latitude,** are supposed to be drawn parallel to the equator, and between it and the poles. By means of these imaginary circles the position of any place on the surface of the earth may be determined; this will be made plain by observing where the

parallels and meridians intersect in Fig. 3. It is evident that if we know what parallel and what meridian pass through the place, the position of the place is determined by their point of intersection. The distance between the equator and either pole is divided into 90°. The **latitude** of a place is its distance from the equator, measured in degrees,

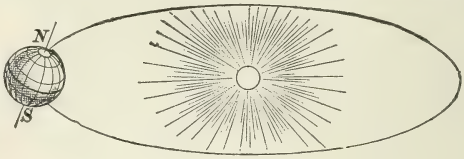

FIG. 3.

minutes, and seconds along the meridian passing through the place. The **longitude** of a place is its distance from some fixed meridian (called the **prime meridian**) in degrees, minutes, and seconds, measured along the parallel passing through the place. There are three prime meridians: that of Greenwich, that of Paris, and that of Washington, that of Greenwich being the one principally used by navigators. Longitudes reckoned east of the meridian passing through Greenwich to 180° are called **east longitudes,** and those reckoned west, **west longitudes.** Latitudes reckoned north from the equator are called **north latitudes,** and those reckoned south from the equator, **south latitudes.**

The astronomical day begins at noon, when the meridian of any place comes in direct line with the sun; but, for business and other reasons, the civil day begins twelve hours earlier, bringing what we call **noon** in the middle of the day. A little consideration of these facts will make it evident to the student that for any place situated on a meridian east of where he lives, noon will occur earlier, and for any place west of him, later, than for the place in which he is. If the longitudes of any two places are known, their difference of time, as it is called, is easily determined. For, since the earth rotates from *west* to *east* 360° in 24 hours, the sun appears to go from *east* to *west* $\frac{1}{24}$ of 360°, or 15°, in 1 hour of time. Hence, in 1 minute of time it seems to go $\frac{1}{60}$ of 15°, or 15′, and in 1 second of time, $\frac{1}{60}$ of 15′, or 15″. This is shown in the following table:

360° in longitude correspond to 24 hours of time
15° in longitude correspond to 1 hour of time.
15′ in longitude correspond to 1 minute of time.
15″ in longitude correspond to 1 second of time.

33. Rule.—*To find the difference of time between two places, find the difference in longitude* in ° ′ ″, and divide by 15; the result will be their difference in time in hours, minutes, and seconds.*

34. Rule.—*To find the difference in longitude of two places, multiply their difference in time (solar time) expressed in hours, minutes, and seconds by 15; the result will be their difference in longitude in ° ′ ″.*

35. In the following table is given the difference in time between some important cities and Greenwich; also, the longitudes of those cities reckoned from the prime meridian passing through Greenwich:

	hr.	min.	sec.	Longitudes.
Albany	4	54	59.2	73° 44′ 48″ W.
Ann Arbor	5	34	55.1	83° 43′ 46.5″ W.
Boston	4	44	15.3	71° 3′ 49.5″ W.
Berlin	0	53	34.9	13° 23′ 43.5″ E.
Calcutta	5	55	20.7	88° 50′ 10.5″ E.
New York	4	55	53.6	Columbia College, 73° 58′ 24″ W.
New Orleans	6	0	12.9	U. S. Mint. 90° 3′ 13.9″ W.
Paris	0	9	20.9	2° 20′ 13.5″ E.
Philadelphia	5	0	38.5	75° 9′ 37.5″ W.
Rome	0	49	54.7	12° 28′ 40.5″ E.
Cincinnati	5	37	41.3	84° 25′ 19.5″ W.
Chicago	5	50	26.7	87° 36′ 40.5″ W.
Jefferson City	6	8	36	92° 9′ 0″ W.
London	0	0	22.5	5′ 37.5″ W.
City of Mexico	6	36	26.7	99° 6′ 40.5″ W.
Richmond	5	9	44	77° 26′ 0″ W.
San Francisco	8	9	38.1	122° 24′ 31.5″ W.
St. Paul	6	12	20	93° 5′ 0″ W.
St. Louis	6	0	41.1	90° 10′ 16.5″ W.
Washington	5	8	12	77° 3′ 0″ W.

Examples relating to the above will be given later. See Arts. **82–84.**

*To add, subtract, multiply, or divide compound numbers, see rules, Arts. **60, 63, 73,** and **78.**

MEASURES OF MONEY.

UNITED STATES MONEY.

TABLE XVI.

10 mills (*m.*) .... = 1 cent .... *ct.*
10 cents .... = 1 dime .... *d.*
10 dimes .... = 1 dollar .... *$.*
10 dollars .... = 1 eagle .... *E.*

m.		ct.		d.		$		E.
10	=	1						
100	=	10	=	1				
1,000	=	100	=	10	=	1		
10,000	=	1,000	=	100	=	10	=	1

36. The unit of value is the gold dollar, which weighs 25.8 grains. Since gold and silver are so soft that they would rapidly lose in weight if circulated as money in their natural state, they are alloyed with 1 part of alloy to every 9 parts of the pure metal. In other words, a gold or silver coin contains only .9 of its weight of pure gold or silver. Since 9 parts in 10 are equivalent to 900 parts in every thousand, a gold dollar is stated to contain 25.8 grains of gold 900 *fine*, the word "fine" meaning the number of parts of the pure metal in 1,000 parts by weight of the coin or alloyed metal. In business operations, the terms dollar and cent only are used, the terms eagle and dime being merely names of coins. The mill is not coined, and is rarely used except in referring to tax rates.

37. The term **legal tender** is applied to money which may be legally offered in payment of debts. All coins are legal tender up to a certain amount, depending upon the value of the coins. By legal tender is meant that if the debtor offers to pay his obligation in legal-tender money, the creditor *must* accept it. All gold coins are legal tender for their face value to any amount, provided their weight has not diminished more than $\frac{1}{200}$. Silver dollars are also legal tender to any amount; but silver coins of lower denomination than one dollar are legal tender only for sums not exceeding $10. Nickel and copper coins are legal tender for sums not exceeding 25 cents.

38. The legal coins of the United States are:

GOLD.

	Weight in grains.
1-dollar piece,	25.8
$2\frac{1}{2}$-dollar piece, or quarter-eagle,	64.5
3-dollar piece,	77.4
5-dollar piece, or half-eagle,	129.0
10-dollar piece, or eagle,	258.0
20-dollar piece, or double-eagle,	516.0

SILVER.

	Weight.
Standard dollar,	$412\frac{1}{2}$ grains.
Half-dollar, or 50-cent piece,	192.9 grains, or $12\frac{1}{2}$ grams.
Quarter-dollar, or 25-cent piece,	96.45 grains, or $6\frac{1}{4}$ grams.
Dime, or 10-cent piece,	38.58 grains, or $2\frac{1}{2}$ grams.

COPPER AND NICKEL.

5-cent piece,	77.16 grains, or 5 grams.
3-cent piece,	30.00 grains.
1-cent piece,	48.00 grains.

39. United States money is expressed and read decimally. Eagles and dimes are never read; only dollars and hundredths, or dollars and cents are read. Thus, 5 eagles 4 dollars 7 dimes and 3 cents would be written $54.73 and read "fifty-four dollars and seventy-three cents." If there are more than two decimal places, the extra ones are read as decimal parts of a cent. Thus, $54.7329 is read "fifty-four dollars and seventy-three and twenty-nine hundredths cents."

When the number of cents is less than ten, a cipher must be written in the first place to the right of the decimal point. Thus, four dollars and five cents is written $4.05.

In checks, notes, drafts, etc., to prevent forgery and mistakes, the cents are written as hundredths of a dollar in the form of a fraction. Thus, $13.47 would be written $\$13\frac{47}{100}$.

ENGLISH MONEY.

TABLE XVII.

4 farthings (*far.*)	= 1 penny	*d.*
12 pence	= 1 shilling	*s.*
20 shillings	= 1 pound, or sovereign	£.

far.		d.		s.		£.
4	=	1				
48	=	12	=	1		
960	=	240	=	20	=	1

40. The unit of English money is the **pound sterling,** the value of which in United States money is \$4.8665. The fineness of English silver is .925; of the gold coins, $.916\frac{2}{3}$. What is called sterling silver when applied to *solid* silver articles has the same fineness. Hence the name—sterling silver.

The other coins of Great Britain are the *florin* (= 2 shillings), the *crown* (= 5 shillings), the *half-crown* (= $2\frac{1}{2}$ shillings), and the *guinea* (= 21 shillings). The largest silver coin is the crown, and the smallest, the threepence. The shilling is worth 25 cents (24.3+ cents) in United States money. The guinea is no longer coined. The abbreviation £ is written before the number, while s. and d. follow. Thus, £25 4 s. 6 d. = 25 pounds 4 shillings 6 pence.

MISCELLANEOUS TABLES.

41. The following table is used in counting certain articles:

TABLE XVIII.

12 of anything	= 1 dozen	*doz.*
12 dozen	= 1 gross	*gr.*
12 gross	= 1 great gross	*g. gr.*
20 of anything	= 1 score.	

units		doz.		gr.		g. gr.
12	=	1				
144	=	12	=	1		
1,728	=	144	=	12	=	1

42. The following table is used in the paper trade:

TABLE XIX.

24 sheets..........	= 1 quire..........	*qr.*
20 quires..........	= 1 ream..........	*rm.*
2 reams..........	= 1 bundle........	*bdl.*
5 bundles........	= 1 bale...........	*B.*

sheets		qr.		rm.		bdl.		B.
24	=	1						
480	=	20	=	1				
960	=	40	=	2	=	1		
4,800	=	200	=	10	=	5	=	1

It is now becoming customary to consider 500 sheets as a ream, and to discard the higher denominations.

REDUCTION OF COMPOUND NUMBERS.

43. Reduction of compound numbers is the process of changing their denomination without changing their value. Reduction is divided into two cases. In one case, we reduce the number to units lower than the highest named in the number; in the other case, we reduce units of a low denomination to a higher one. The first case is called *reduction descending*, and the second, *reduction ascending*.

REDUCTION DESCENDING.

44. Reduction descending, or reducing units of a higher to those of a lower denomination, is performed by multiplication. Thus, to reduce 4 miles (4 mi.) to rods, yards, feet, and inches, successively, it is known that there are 320 rd. in 1 mi. (see Table I); hence, in 4 mi., there are evidently $4 \times 320 = 1{,}280$ rd. Again, since there are $5\frac{1}{2}$ yd. in 1 rd., in 1,280 rd. there are $1{,}280 \times 5\frac{1}{2} = 7{,}040$ yd. In 1 yd. there are 3 ft.; hence, in 7,040 yd., there are $7{,}040 \times 3$

= 21,120 ft. And, since in 1 ft. there are 12 in., in 21,120 ft., there are $21,120 \times 12 = 253,440$ in. All examples in reduction descending are performed in the same manner.

45. If more than one unit is given, reduce the highest unit to the next lower denomination mentioned, and then add the units of this next lower denomination to the result previously obtained. So proceed until the lowest denomination is reached. An example, which will serve as a model for the student, is here given:

EXAMPLE.—Reduce 5 mi. 47 rd. 3 yd. $1\frac{1}{2}$ ft. to feet.

SOLUTION.—

$$\begin{array}{rl}
\text{mi.}\quad \text{rd.}\quad \text{yd.}\quad \text{ft.} & \\
5\quad 47\quad 3\quad 1\frac{1}{2} & \\
320 & \\
\hline
1600 & \text{rd.} \\
47 & \\
\hline
1647 & \text{rd.} \\
5\frac{1}{2} & \\
\hline
823\frac{1}{2} & \\
8235\phantom{\frac{1}{2}} & \\
\hline
9058\frac{1}{2} & \text{yd.} \\
3\phantom{\frac{1}{2}} & \\
\hline
9061\frac{1}{2} & \text{yd.} \\
3\phantom{\frac{1}{2}} & \\
\hline
1\frac{1}{2} & \\
27183\phantom{\frac{1}{2}} & \\
\hline
27184\frac{1}{2} & \text{ft.} \\
1\frac{1}{2} & \\
\hline
27186 & \text{ft.}\quad \text{Ans.}
\end{array}$$

EXPLANATION.—Since there are 320 rd. in 1 mi., multiplying 5 by 320 and adding 47 to the product will give the number of rods in 5 mi. 47 rd., or 1,647 rd. Since there are $5\frac{1}{2}$ yd. in 1 rd., multiplying 1,647 by $5\frac{1}{2}$ and adding 3 to the product will give the number of yards in 5 mi. 47 rd. 3 yd., or $9,061\frac{1}{2}$ yd. Since there are 3 ft. in 1 yd. multiplying

9,061½ by 3, and adding the 1½ to the product, will give the required result, or 27,186 ft. Ans.

46. Rule.—*Multiply the number of units of the highest denomination in the given compound number by the number of units of the lower denomination required to make one unit of the higher denomination, and to the product add the given number of units of the lower denomination. Proceed in this manner until the given compound number is reduced to the required denomination.*

47. In order to avoid mistakes, if any denomination be omitted between the highest and lowest denominations of the given number, represent it by a cipher.

EXAMPLE.—Reduce 40 A. 21 sq. yd. 6 sq. ft. 93 sq. in. to square inches.

SOLUTION.—

```
                  A.   sq. rd.   sq. yd.   sq. ft.   sq. in.
                  40     0         21        6         93
                 160
                ----
                6400 sq. rd.
                 30¼
                ----
                1600
              192000
              ------
              193600 sq. yd.
                  21
              ------
              193621 sq. yd.
                   9
             -------
             1742589 sq. ft.
                   6
             -------
             1742595 sq. ft.
                 144
             -------
             6970380
            6970380
           1742595
           ---------
           250933680 sq. in.
                  93
           ---------
           250933773 sq. in.   Ans.
```

The absence of any square rods is denoted by the cipher, as shown.

EXAMPLES FOR PRACTICE.

48. Reduce:

(*a*)	7 mi. 4 rd. 2 yd. 2 ft. to feet.	(*a*)	37,034 ft.
(*b*)	4 bu. 2 pk. 2 qt. to quarts.	(*b*)	146 qt.
(*c*)	8 lb. 4 oz. 6 pwt. 12 gr. to grains.	(*c*)	48,156 gr.
(*d*)	52 hhd. 24 gal. 1 pt. to pints.	(*d*)	26,401 pt.
(*e*)	$5 \odot 16° 20' 46''$ to seconds.	(*e*)	$6{,}538{,}846''$.
(*f*)	14 bu. to quarts.	(*f*)	448 qt.
(*g*)	11 T. 9 cwt. 57 lb. to pounds.	(*g*)	22,957 lb.
(*h*)	£9 13 s. 10 d. to pence.	(*h*)	2,326 d.
(*i*)	16 cd. 112 cu. ft. to cubic feet.	(*i*)	2,160 cu. ft.
(*j*)	97 sq. rd. to square feet. Ans.	(*j*)	$26{,}408\frac{1}{4}$ sq. ft.
(*k*)	4 yr. 8 mo. 1 wk. 3 da. to days. (Count 12 mo. to the year, and 4 wk. to the month.)	(*k*)	1,578 da.
(*l*)	19 perches to cubic feet.	(*l*)	$470\frac{1}{4}$ cu. ft.
(*m*)	21 cu. yd. to cubic feet.	(*m*)	567 cu. ft.
(*n*)	Cong.2 O.6 f℥10 fʒ5 to fluidrachms.	(*n*)	fʒ2,901.
(*o*)	13 long tons to pounds.	(*o*)	29,120 lb.
(*p*)	℔1 ℥9 ʒ6 ℈2 to grains.	(*p*)	10,480 gr.
(*q*)	Find the actual number of days in example (*k*). (Count 1 leap year and 30 days to the month.)	(*q*)	1,711 da.

REDUCTION ASCENDING.

49. Reduction ascending, or reducing units of a lower to those of a higher denomination, is performed by division. Thus, to reduce 253,440 in. to feet, we must evidently divide by 12, since there are 12 in. in 1 ft. That is, 253,440 in. $= 253{,}440 \div 12 = 21{,}120$ ft. To reduce this last number to yards, we divide by 3; or, 21,120 ft. $= 21{,}120 \div 3 = 7{,}040$ yd. Continuing the process, 7,040 yd. $= 7{,}040 \div 5\frac{1}{2} = 1{,}280$ rd. $= 1{,}280 \div 320 = 4$ mi. It will be noticed that this is the reverse of the reduction of 4 mi. to inches, as in Art. **44.**

50. If, in reducing a number, as in the last article, to a higher denomination there is a remainder, reserve it and divide the quotient obtained by the division by the number of units required to make one unit of the next higher denomination. Or, carry the division farther by annexing ciphers to the remainder, thus obtaining a decimal part of the next higher unit.

EXAMPLE.—Reduce 27,186 feet to miles, rods, etc.

SOLUTION.—Instead of giving all the work of division, we shall use the short method of division to express the various steps. This will save space and make the work plainer.

$$\begin{array}{r|l} 3\,) & \underline{2\,7\,1\,8\,6} \text{ ft.} \\ 5\tfrac{1}{2}\,) & \underline{9\,0\,6\,2} \text{ yd.} \\ 3\,2\,0\,) & \underline{1\,6\,4\,7} \text{ rd.} + 3\tfrac{1}{2} \text{ yd.} \\ & \quad 5 \text{ mi.} + 47 \text{ rd.} \end{array}$$

$3\frac{1}{2}$ yd. = 3 yd. $1\frac{1}{2}$ ft.

Hence, 27,186 ft. = 5 mi. 47 rd. 3 yd. $1\frac{1}{2}$ ft. Ans. (See Art. **45**.)

EXPLANATION.—Since there are 3 ft. in 1 yd., divide 27,186 by 3 and obtain 9,062 yd. Dividing this by $5\frac{1}{2}$ to reduce yards to rods, the result is, 1,647 rd. and $3\frac{1}{2}$ yd. remaining. Write as shown. Dividing 1,647 rd. by 320, the number of rods in a mile, the result is 5 mi. and 47 rd. remaining. Hence, 27,186 ft. = 5 mi. 47 rd. $3\frac{1}{2}$ yd. But $\frac{1}{2}$ yd. = $\frac{1}{2} \times 3$ = $1\frac{1}{2}$ ft.; consequently, instead of the above, 5 mi. 47 rd. 3 yd. $1\frac{1}{2}$ ft. may be written.

51. When dividing a whole number by a mixed number, as in the above example, where 9,062 was divided by $5\frac{1}{2}$, a simple method is the following: *Multiply both dividend and divisor by the denominator of the fraction, and then divide the new dividend by the new divisor.* This will make the divisor a whole number and avoid decimals. In the above case, the denominator of the fraction was 2; $5\frac{1}{2} \times 2 = 11$, and $9{,}062 \times 2 = 18{,}124$. Then, $18{,}124 \div 11 = 1{,}647 + 7$ remainder. This remainder, however, is 2 times too large; since, when the dividend was multiplied by 2, any remainder that might occur in the division was also multipled by 2. Therefore, the true remainder is $7 \div 2 = 3\frac{1}{2}$, as may be proved by dividing 9,062 by 5.5. That this method of division is correct may be easily shown. Thus, $9{,}062 \div 5\frac{1}{2} = \dfrac{9{,}062}{5\frac{1}{2}}$, which corresponds exactly to a fraction having a mixed number for a denominator. Since multiplying both numerator and denominator by the same number does not alter the *value* of the fraction, it follows that the method is correct.

52. Instead of retaining the lower units, the preceding number may be reduced to miles and decimals of a mile, as in the following example:

EXAMPLE.—Reduce 27,186 feet to miles and decimals of a mile.

SOLUTION.—The process is essentially the same as the preceding.

$$
\begin{array}{r@{\,}l}
3\,) & \underline{2\,7\,1\,8\,6} \text{ ft.} \\
5\tfrac{1}{2}\,) & \underline{9\,0\,6\,2} \\
1\,1\,) & \underline{1\,8\,1\,2\,4} \\
3\,2\,0\,) & \underline{1\,6\,4\,7.6\,3\,\dot{6}\,\dot{3}} \\
 & \quad 5.1\,4\,8\,8\,6\,\tfrac{4}{11} \text{ mi. Ans.}
\end{array}
$$

EXPLANATION.—The work should be evident from what has preceded. Before dividing by $5\frac{1}{2}$ we multiply both dividend and divisor by 2. (See Art. **51.**) When obtaining a decimal it is not necessary to remember that the remainder 7 is $\frac{7}{2}$ yd. We simply annex a cipher to the 7 and continue the division. This is evidently correct; for $\frac{\frac{7}{2}}{5\frac{1}{2}}$ is the same as $\frac{7}{11}$. The quotient contains the repeating decimal $\dot{6}\dot{3}$; and, when dividing by 320, we bring down first a 6 and next a 3, etc. instead of ciphers.

53. Rule.—*Divide the number of units of the denomination given by the number of units of that denomination that are required to make one unit of the next higher denomination. The remainder (if any) will be of the same denomination, but the quotient will be of the next higher denomination. Divide this quotient by the number of units of its denomination that are required to make one unit of the next higher denomination. Continue thus until the required denomination is reached. If it is desired to obtain a decimal fraction of the required denomination, instead of a series of remainders of lower denomination, proceed as explained in Art.* **52.**

54. The rule will be illustrated by two examples.

EXAMPLE.—Reduce 250,933,773 sq. in. to higher denominations.

SOLUTION.—

$$
\begin{array}{r l}
144\,)\,\underline{250933773} & \text{sq. in.} \\
9\,)\,\underline{1742595} & \text{sq. ft.} + 93 \text{ sq. in.} \\
30\tfrac{1}{4}\,)\,\underline{193621} & \text{sq. yd.} + 6 \text{ sq. ft.} \\
121\,)\,\underline{774484} & \\
160\,)\,\underline{6400} & \text{sq. rd.} + \tfrac{84}{4} \text{ sq. yd.} = 21 \text{ sq. yd.} \\
40 \text{ A. } 21 \text{ sq. yd. } 6 \text{ sq. ft. } 93 \text{ sq. in.} & \text{Ans.}
\end{array}
$$

EXPLANATION.—Multiplying the third divisor and the third dividend by 4 to get rid of the fraction, the remainder, after division, is 84; that is, it is $\frac{84}{4}$ sq. yd. = 21 sq. yd. The remainder of the work should be clear from the preceding explanations.

EXAMPLE.—Reduce f ʒ 2,901 to gallons.

SOLUTION.—Understanding by the wording of the example that gallons and decimals of a gallon are required, the solution is as follows:

8) f ʒ 2 9 0 1
16) f ℥ 3 6 2.6 2 5
8) O. 2 2.6 6 4 0 6 2 5
Cong. 2.8 3 3 0 0 7 8 1 2 5, say Cong. 2.833. Ans.

EXAMPLES FOR PRACTICE.

55. Reduce all answers to the "Examples for Practice," in Art. **48**, to units of higher denomination. Also solve the following, reducing as required, and expressing the result in decimals, as in Arts. **52** and **54** (second example).

Reduce the following:

(*a*)	875 pt. to gallons.	(*a*)	109.375 gal.
(*b*)	10,000 pt. to bushels.	(*b*)	156.25 bu.
(*c*)	147,368 cu. in. to cubic yards.	(*c*)	3.1586+ cu. yd.
(*d*)	10,000 gr. to Troy pounds.	(*d*)	1.7361 lb.
(*e*)	8,375 d. to pounds.	(*e*)	£34.8958+.
(*f*)	28,140 sq. yd. to acres.	(*f*)	5.814+ A.
(*g*)	49,175 in. to miles.	(*g*)	0.776+ mi.
(*h*)	380,421″ to degrees.	(*h*)	105.6725°.

Ans.

56. In solving examples (*a*) to (*h*), Art. **55**, the table of equivalents, which accompanies every table of measures,

will be found very useful. Thus, in solving (*g*), for instance, it will be seen, by referring to Table I (lower half), that 1 mi. contains 63,360 in. Hence, (*g*) may be solved by simply dividing 49,175 by 63,360. Similarly, to solve (*f*) we find, by Table III, that 1 A. contains 4,840 sq. yd.; hence, divide 28,140 sq. yd. by 4,840. The student should practise both methods.

ADDITION OF COMPOUND NUMBERS.

57. Addition of compound numbers is similar in every respect to addition of whole numbers or of decimals, so far as the principles involved are concerned. The points of difference arise from the use of a varying scale of notation instead of a uniform scale of 10. An example will serve to show the process.

EXAMPLE.—Find the sum of 4 T. 3 cwt. 46 lb. 12 oz.; 8 cwt. 12 lb. 13 oz.; 2 T. 12 cwt. 50 lb. 13 oz.; 1 T. 27 lb. 4 oz.

SOLUTION.—

	T.	cwt.	lb.	oz.	
	4	3	46	12	
		8	12	13	
	2	12	50	13	
	1		27	4	
	7	23	135	42	
or	8 T.	4 cwt.	37 lb.	10 oz.	Ans.

EXPLANATION.—Write the units of the same denomination in the same column, as shown, with the abbreviation of the name of the unit at the head of each column, and decreasing in order from left to right. Beginning with the right-hand column, the sum of the numbers in that column is 42, i. e., 42 oz. The sum of the numbers in the second column is 135 lb.; in the third column, 23 cwt.; in the fourth column, 7 T. Now, since 42 oz. are more than 1 lb., reduce the 42 oz. to pounds and ounces, obtaining 2 lb. 10 oz. Reserve the 10 oz., add the 2 lb. to the 135 lb. in the second column, obtaining 137 lb.; this reduced to hundredweight and pounds gives 1 cwt. 37 lb. The 1 cwt. added to the 23 cwt. in the next column gives 24 cwt., or 1 T. 4 cwt. Finally, 7 T. + 1 T. = 8 T. Hence, the sum is 8 T. 4 cwt. 37 lb. 10 oz.

58. That the process just described is the same in all respects as the addition of simple numbers will be manifest after a little consideration. Thus, suppose that we represent *thousands* by *thds.*, *hundreds* by *h.*, *tens* by *t.*, and *units* by *u.*; then the addition of 2,046, 812, 2,151, and 1,707 might be performed as follows:

	thds.	h.	t.	u.	
	2		4	6	
		8	1	2	
	2	1	5	1	
	1	7		7	
	5	16	10	16	
or	6 thds.	7 h.	1 t.	6 u.,	which equals 6,716.

It is easily seen that the same principles govern both cases.

59. Instead of writing the sum of each column separately and then reducing, as in the preceding example (Art. **57**), it is customary to add the right-hand column, and if the sum contains more units than are required to make one unit of the next higher denomination, to reduce it to the next higher denomination, placing the remainder (if any) under the right-hand column and carrying the quotient to the next column. The student will recognize this as corresponding exactly to the ordinary process of addition.

EXAMPLE.—What is the sum of 2 rd. 3 yd. 2 ft. 5 in.; 6 rd. 1 ft. 10 in.; 17 rd. 1 yd. 11 in.; 1 rd. 4 yd. 1 ft.?

SOLUTION.—	rd.	yd.	ft.	in.	
	2	3	2	5	
	6		1	10	
	17	1		11	
	1	4	1		
	27	$4\frac{1}{2}$	0	2	
or	27 rd.	4 yd.	1 ft.	8 in.	Ans.

EXPLANATION.—The sum of the units in the first column is 26 in., or 2 ft. 2 in. Writing the 2 in. under the right-hand column, and carrying the 2 ft. to the next column, the result is 2 (carried) $+ 1 + 1 + 2 = 6$ ft. $=$ 2 yd. 0 ft. Carrying the 2 yd. to the third column, the sum is 10 yd. $=$ 1 rd. $4\frac{1}{2}$ yd. Carrying the 1 rd. to the fourth column, the sum is 27 rd.

Now, to avoid fractions, the $\frac{1}{2}$ yd. is reduced to feet and inches, giving 1 ft. 6 in., which added to the 0 ft. and 2 in. gives for the answer 27 rd. 4 yd. 1 ft. 8 in.

60. Rule.—*Place the numbers in vertical columns so that like denominations are under each other. Begin at the right-hand column, and add. If the sum contains more units than are necessary to make one unit of the next higher denomination, reduce the sum to the next higher denomination, placing the remainder under the column added, and carrying the unit (or units) of the next higher denomination so obtained to the second column. Continue in this manner until the required denomination is reached. Should fractions of a unit be obtained, or should they occur in the numbers to be added, reduce them to units of lower denomination and add them to the sum first obtained.*

EXAMPLES FOR PRACTICE.

61. Find the sum of the following:

(*a*) 25 lb. 7 oz. 15 pwt. 23 gr.; 17 lb. 16 pwt.; 15 lb. 4 oz. 12 pwt.; 18 lb. 16 gr.; 10 lb. 2 oz. 11 pwt. 16 gr.

(*b*) 9 mi. 13 rd. 4 yd. 2 ft.; 16 rd. 5 yd. 1 ft. 5 in.; 16 mi. 2 rd. 3 in.; 14 rd. 1 yd. 9 in.

(*c*) £16 5 s. 4 d.; £12 8 s. 9 d.; £13 14 s. 8 d.; £42 7 d.; 18 s. 6 d.

(*d*) 13 cwt. 46 lb. 12 oz.; 12 cwt. $9\frac{1}{2}$ lb.; $2\frac{1}{4}$ cwt. $21\frac{5}{8}$ lb.

(*e*) 4 bu. 3 pk. 6 qt. 1 pt.; 10 bu. 2 pk. 7 qt. 1 pt.; 11 bu. 3 pk. 1 qt. 1 pt.; 9 bu. 2 pk. 5 qt. 1 pt.

(*f*) 10 yr. 8 mo. 5 wk. 3 da.; 42 yr. 6 mo. 7 da.; 7 yr. 5 mo. 18 wk. 4 da.; 17 yr. 17 da.

(*g*) 14 sq. yd. 8 sq. ft. 19 sq. in.; 105 sq. yd. 16 sq. ft. 240 sq. in.; 42 sq. yd. 28 sq. ft. 165 sq. in.

(*h*) 16 gal. 3 qt. 1 pt.; 45 gal. 2 qt.; 17 gal. 1 qt. 1 pt.; 4 gal. 3 qt.; 15 gal. 1 pt.; 24 gal. 3 qt. 1 pt.

(*i*) 16 hr. 43 min. 48 sec.; 3 hr. 12 min. 40 sec.; 1 hr. 49 min. 13 sec.; 5 hr. 19 sec.

Ans.
- (*a*) 86 lb. 3 oz. 16 pwt. 7 gr.
- (*b*) 25 mi. 47 rd. 1 ft. 5 in.
- (*c*) £85 7 s. 10 d.
- (*d*) 1 T. 8 cwt. 2 lb. 14 oz.
- (*e*) 37 bu. 5 qt.
- (*f*) 78 yr. 1 mo. 3 wk. 3 da.
- (*g*) 5 sq. rd. 15 sq. yd. 7 sq. ft. 100 sq. in.
- (*h*) 124 gal. 2 qt.
- (*i*) 26 hr. 46 min.

SUBTRACTION OF COMPOUND NUMBERS.

62. The operation of subtraction is the same in principle for compound numbers as for simple numbers, the difference being due wholly to the varying scales.

EXAMPLE.—From 21 rd. 2 yd. 2 ft. $6\frac{1}{2}$ in. take 9 rd. 4 yd. $10\frac{1}{4}$ in.

SOLUTION.—

rd.	yd.	ft.	in.
21	2	2	$6\frac{1}{2}$
9	4		$10\frac{1}{4}$
11	$3\frac{1}{2}$	1	$8\frac{1}{4}$

or 11 rd. 3 yd. 2 ft. $14\frac{1}{4}$ in. = 11 rd. 4 yd. $2\frac{1}{4}$ in. Ans.

EXPLANATION.—Writing the numbers as for addition, with units of like denomination under each other, we begin with the right-hand column and subtract. Since $10\frac{1}{4}$ inches cannot be subtracted from $6\frac{1}{2}$ inches, 1 foot (= 12 inches) is borrowed from the column of feet, reduced to inches, and added to the inches in the minuend, giving $18\frac{1}{2}$ inches. Then, $18\frac{1}{2}$ in. $-$ $10\frac{1}{4}$ in. = $8\frac{1}{4}$ in. Since 1 foot was borrowed from the 2 feet, only 1 foot remains, and, as there are no feet in the subtrahend, the 1 foot is brought down in the remainder. As 4 yards cannot be taken from 2 yards, 1 rod is borrowed from the rod column, reduced to yards, and added to the 2 yards, giving $7\frac{1}{2}$ yards; then, $7\frac{1}{2}$ yd. $-$ 4 yd. = $3\frac{1}{2}$ yd. Finally, 20 rd. $-$ 9 rd. = 11 rd., and the remainder is 11 rd. $3\frac{1}{2}$ yd. 1 ft. $8\frac{1}{4}$ in. Reducing the $\frac{1}{2}$ yd., the remainder becomes 11 rd. 3 yd. 2 ft. $14\frac{1}{4}$ in.; or, since $14\frac{1}{4}$ in. = 1 ft. $2\frac{1}{4}$ in., and 2 ft. + 1 ft. = 3 ft. = 1 yd., the remainder is 11 rd. 4 yd. $2\frac{1}{4}$ in.

63. Rule.—*Place the less quantity under the greater quantity, with units of like denomination under each other. Beginning at the right, subtract successively the units in the subtrahend in each denomination from those above, and place the several remainders beneath, as in simple subtraction. If the units of any denomination in the minuend are fewer than units of the same denomination in the subtrahend, borrow one unit from the next higher denomination in the minuend, reduce it to the next lower denomination, and add it to the proper number in the minuend; then subtract as before. Reduce fractional results in the remainder if there are any.*

64. Example.—From ℔4 ℈1 take ℔1 ℥5 ℈2 16 gr.

Solution.—

	℔	℥	ʒ	℈	gr.	
	4	0	0	1	0	
	1	5	0	2	16	
	2	6	7	1	4	Ans.

Explanation.—Since 16 grains cannot be taken from 0 grain, ℈1 is borrowed from the second column. ℈1 = 20 gr., and 20 gr. − 16 gr. = 4 gr. As ℈2 cannot be taken from ℈0, and there are no drams or ounces, ℔1 is borrowed from the fifth column. ℔1 = ℥11 ʒ7 ℈3; hence, subtracting ℈2 from ℈3, ʒ0 from ʒ7, ℥5 from ℥11, and ℔1 from ℔3, the remainder, or answer, is ℔2 ℥6 ʒ7 ℈1 4 gr.

65. The only case of subtraction that need cause any trouble is the finding of the difference between two dates. If great exactness is not required, put down the year, the number of the month, and the day of the month in the case of both minuend and subtrahend. Then, counting 30 days to the month, subtract as above.

Example.—How many years, months, and days between September 28, 1868, and June 15, 1891?

Solution.—

yr.	mo.	da.	
1891	6	15	
1868	9	28	
22	8	17	Ans.

Explanation.—June of the minuend is the 6th month, and September of the subtrahend is the 9th month. Writing the minuend and subtrahend as shown, we subtract as above. The result is 22 years, 8 months, 17 days. Ans.

66. Had it been required to find the exact number of days between the two given dates, the above method would not give the correct result.

67. The following method is employed by most banks and by the United States government:

From Sept. 28, 1868, to Sept. 28, 1890 = 22 yr.
From Sept. 28, 1890, to June 15, 1891, in days is
Sept. Oct. Nov. Dec. Jan. Feb. Mar. Apr. May June
2 + 31 + 30 + 31 + 31 + 28 + 31 + 30 + 31 + 15 = 260 da.

Ans. $\left\{\begin{array}{l}22 \text{ yr.}\\ 260 \text{ da.}\end{array}\right.$

Explanation.—The exact whole number of years between the dates is first found; it equals 22 years. The remaining days are then found as shown.

68. The following table will be of great assistance in determining the actual number of days between two dates. The table gives the number of days between the same dates of any two months. Thus, to find the number of days between Mar. 12 and Sept. 12 of any year, we find opposite Mar. in the left-hand column and in the column headed Sept. the number 184, the required number of days. Had it been required to find the number of days between Mar. 12 and

TABLE XX.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
Apr.	275	306	334	365	30	61	91	122	153	183	214	244
May.	245	276	304	334	365	31	61	92	123	153	184	214
June.	214	245	273	304	334	365	30	61	92	122	153	183
July.	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

Sept. 25, we should have found the number of days between Mar. 12 and Sept. 12, or 184 days; then subtracting 12 from 25, the difference, 13, must be added to 184, obtaining 197 days, the number of days between Mar. 12 and Sept. 25. Had it been required to find the number of days between

Sept. 25 and Mar. 12, we should find opposite Sept., and in the column headed Mar., 181; then subtracting 12 from 25, we subtract the difference from 181, because 181 days is the number of days between Sept. 25 and Mar. 25, instead of Mar. 12, which occurs 13 days earlier. Hence, there are $181 - 13 = 168$ days between Sept. 25 and Mar. 12. Had Mar. 12 occurred in a leap year, there would have been one day more, or 169 days between the two dates, on account of Feb. 29.

The table will also be useful in those cases where a certain number of days is to be added to a given date. Thus, to find the date of 90 days after Feb. 18, we see, on referring to the table, that 89 days after Feb. 18 is May 18; hence, 90 days after Feb. 18 is May 19, or, if it is a leap year, May 18. Again, 127 days after Feb. 19 is June 26; because, referring to the table, 120 days after Feb. 19 is June 19, and $127 - 120 + 19 = 26$.

If it is desired to subtract a certain number of days from a given date, the process is simply reversed. To find, for example, the date 120 days previous to Sept. 21, we look down the column headed Sept. and find opposite May the number 123; hence, from May 21 to Sept. 21 is 123 days, and therefore from May 24 to Sept. 21 is 120 days.

EXAMPLES FOR PRACTICE.

69. Solve the following examples:

(*a*) From £10 6 s. 4 d. take £8 15 s. 3 d.

(*b*) From 125 lb. 8 oz. 14 pwt. 18 gr. take 96 lb. 9 oz. 10 pwt. 4 gr.

(*c*) From 16 yr. 8 mo. 10 da. take 12 yr. 5 mo. 8 da.

(*d*) From 126 hhd. 27 gal. take 104 hhd. 14 gal. 1 qt. 1 pt.

(*e*) Find the exact number of days between Sept. 20, 1895, and Mar. 17, 1897, inclusive.

(*f*) From 65 T. 14 cwt. 64 lb. 10 oz. take 16 T. 11 cwt. 14 oz.

(*g*) From 148 sq. yd. 16 sq. ft. 102 sq. in. take 132 sq. yd. 136 sq. in.

(*h*) Subtract 28 bu. 2 pk. 5 qt. 1 pt. from 100 bu.

(*i*) Subtract 3 mi. 27 rd. 11 yd. 4 ft. 10 in. from 14 mi. 34 rd. 16 yd. 13 ft. 11 in.

(*j*) Subtract 27° 19′ 47″ from 126° 37′ 23″.

(*k*) Referring to Art. **35,** find difference of longitude between Boston and San Francisco.

(*l*) Find difference of longitude between Albany and Calcutta.

Answers.—(*a*) £1 11s. 1d.; (*b*) 28 lb. 11 oz. 4 pwt. 14 gr.; (*c*) 4 yr. 3 mo. 2 da.; (*d*) 22 hhd. 12 gal. 2 qt. 1 pt.; (*e*) 545 da.; (*f*) 49 T. 3 cwt. 63 lb. 12 oz.; (*g*) 16 sq. yd. 15 sq. ft. 110 sq. in.; (*h*) 71 bu. 1 pk. 2 qt. 1 pt.; (*i*) 11 mi. 8 rd. 2 yd. 1 ft. 7 in.; (*j*) 99° 17′ 36″; (*k*) 51° 20′ 42″; (*l*) 162° 34′ 58.5″.

MULTIPLICATION OF COMPOUND NUMBERS.

70. The multiplication of compound numbers is similar in all respects to multiplication of simple numbers. The process will be illustrated by an example.

Example.—A merchant divided his syrup into 12 equal parts, each containing 2 gal. 2 qt. $1\frac{1}{2}$ pt.; how much did he have altogether?

Solution.—He evidently had 12 times 2 gal. 2 qt. $1\frac{1}{2}$ pt, or

$$\begin{array}{ccc} \text{gal.} & \text{qt.} & \text{pt.} \\ 2 & 2 & 1\frac{1}{2} \\ & & 12 \\ \hline 24 & 24 & 18 \end{array} = 32 \text{ gal. } 1 \text{ qt. Ans.}$$

Explanation.—Multiplying the units of each denomination by 12, and reducing the units of each denomination of the products to higher denominations, the result is 32 gal. 1 qt.

71. Such examples are usually solved as follows:

$$\begin{array}{ccc} \text{gal.} & \text{qt.} & \text{pt.} \\ 2 & 2 & 1\frac{1}{2} \\ & & 12 \\ \hline 32 & 1 & 0 \end{array} \text{ Ans.}$$

Explanation.— $1\frac{1}{2}$ pt. $\times 12 = 18$ pt. $= 9$ qt.; reserve this, and add to the quarts product. 2 qt. $\times 12 + 9$ qt. $= 33$ qt. $= 8$ gal. 1 qt. Write the 1 qt. and reserve the 8 gal. 2 gal. $\times 12 + 8$ gal. $= 32$ gal. Hence, the answer is 32 gal. 1 qt.

72. When the multiplier contains a fraction, it is usually easier to reduce the multiplicand to the *lowest* denomination before multiplying, and then reduce the product to higher denominations. In any case, the method used in Art. **70** is to be preferred to that given in Art. **71,** when the multiplier contains a fraction. All three methods will be applied to an example to illustrate the point.

EXAMPLE.—Multiply 7 lb. 5 oz. 13 pwt. 15 gr. by $5\frac{1}{2}$.

SOLUTION.—*First Method—*

	lb.	oz.	pwt.	gr.	
	7	5	13	15	
				$5\frac{1}{2}$	
	$38\frac{1}{2}$	$27\frac{1}{2}$	$71\frac{1}{2}$	$82\frac{1}{2}$	
or	38	33	81	$94\frac{1}{2}$	
or	41 lb.	1 oz.	4 pwt.	$22\frac{1}{2}$ gr.	Ans.

Second Method—	lb.	oz.	pwt.	gr.	
	7	5	13	15	
				$5\frac{1}{2}$	
	$40\frac{1}{2}$	$6\frac{1}{2}$	$14\frac{1}{2}$	$10\frac{1}{2}$	
or	40	12	24	$22\frac{1}{2}$	
or	41 lb.	1 oz.	4 pwt.	$22\frac{1}{2}$ gr.	Ans.

Third Method— 7 lb. 5 oz. 13 pwt. 15 gr. = 43,047 gr. (if the lowest denomination given had not been grains, we should still have reduced to grains). Hence, 43,047 gr. $\times$ 5.5 = 236,758.5 gr. = 41 lb. 1 oz. 4 pwt. $22\frac{1}{2}$ gr. Ans.

73. Rule.—*Multiply the units of the lowest denomination by the multiplier and reduce the product to the next higher denomination. Multiply the units of the next higher denomination by the multiplier, and add to the product the result obtained by the first operation. So continue with the remaining units. The last result, together with the various remainders, is the entire product. Should the entire product contain fractions, reduce the fractions of a unit to lower denominations, as in Art.* **72.**

Or, reduce the multiplicand to its lowest denomination; perform the multiplication, and then reduce the product to higher denominations.

EXAMPLES FOR PRACTICE.

74. Solve the following:

(*a*) Multiply £17 10 s. 8 d. by 7; by 9; by 15.

(*b*) How many cords of wood in 12 loads, each load containing 2 cd. 108 cu. ft.?

(*c*) Find the weight of 2 dozen silver spoons, each spoon weighing 1 oz. 13 pwt. What would they cost at 6 cents per pennyweight?

(*d*) If 15 men perform a certain piece of work in 3 da. 16 hr. 52 min., how long would it take one man to perform the work?

(*e*) Multiply 3 T. 15 cwt. 90 lb. by 5.

(*f*) Multiply 4 hhd. 3 gal. 1 qt. 1 pt. by 12.

(*g*) At $2.16 per gallon what would be the cost of Cong.2 O.6 f℥10 of a certain drug?

(*h*) What would be the cost of 5 bu. 3 pk. 6 qt. of potatoes at 48 cents per bushel?

(*i*) Multiply 6 A. 114 sq. rd. 19 sq. yd. 53 sq. ft. by 13.

Ans.
(*a*) £122 14 s. 8 d.; £157 16 s.; £263.
(*b*) 34 cd. 16 cu. ft.
(*c*) 3 lb. 3 oz. 12 pwt.; $47.52.
(*d*) 55 da. 13 hr.
(*e*) 18 T. 19 cwt. 50 lb.
(*f*) 48 hhd. 1 bbl. 9 gal.
(*g*) $6.11.
(*h*) $2.85.
(*i*) 87 A. 52 sq. rd. 21 sq. yd. ½ sq. ft.

DIVISION OF COMPOUND NUMBERS.

75. There are two cases of division of compound numbers. In the first case, the divisor is an abstract number; in the second case, the divisor is itself a compound number. When the divisor is an abstract number, the division may be conveniently performed as in the following examples, Arts. **76–78,** inclusive.

76. Example.—Divide 48 lb. 11 oz. 6 pwt. by 8.

Solution.—

	lb.	oz.	pwt.	gr.	
8)	48	11	6	0	
	6 lb.	1 oz.	8 pwt.	6 gr.	Ans.

Explanation.—After placing the quantities as above, proceed as follows: 8 is contained in 48 six times without a remainder. 8 is contained in 11 oz. once with 3 oz. remaining. $3 \times 20 = 60$; $60 + 6 = 66$ pwt.; 66 pwt. $\div 8 = 8$ pwt. and 2 pwt. remaining; 2×24 gr. $= 48$ gr.; 48 gr. $\div 8 = 6$ gr. Therefore, the entire quotient is 6 lb. 1 oz. 8 pwt. 6 gr. Ans.

Example.—A silversmith melted up 2 lb. 8 oz. 10 pwt. of silver, which he made into 6 soup ladles; what was the weight of each?

Solution.—

	lb.	oz.	pwt.	
6)	2	8	10	
		5 oz.	8 pwt. 8 gr.	Ans.

Explanation.—Since we cannot divide 2 lb. by 6, we reduce them to ounces. 2 lb. = 24 oz., and 24 oz. + 8 oz. = 32 oz.; 32 oz. ÷ 6 = 5 oz. and 2 oz. over. 2 oz. = 40 pwt. 40 pwt. + 10 pwt. = 50 pwt., and 50 pwt. ÷ 6 = 8 pwt. and 2 pwt. over. 2 pwt. = 48 gr., and 48 gr. ÷ 6 = 8 gr. Hence, each ladle weighs 5 oz. 8 pwt. 8 gr. Ans.

77. Example.—Divide 820 rd. 4 yd. 2 ft. by 112.

Solution.—

rd.
112) 820 (7 rd.
36 rd.
$5\frac{1}{2}$
198 yd.
4
112) 202 yd. (1 yd.
90 yd.
3
270 ft.
2
112) 272 ft. (2 ft.
48 ft.
12
112) 576 in. ($5\frac{16}{112} = 5\frac{1}{7}$ in.
16 in.

Explanation.—We divide as in long division, using the short method. The first quotient is 7 rd. with a remainder of 36 rd., which = 198 yd. 198 yd. + 4 yd. = 202 yd.; 202 yd. ÷ 112 = 1 yd. with a remainder of 90 yd., which = 270 ft. 270 ft. + 2 ft. = 272 ft.; 272 ft. ÷ 112 = 2 ft. with a remainder of 48 ft., which = 576 in. 576 in. ÷ 112 = $5\frac{1}{7}$ in.

If desired, $\frac{1}{7}$ in. may be reduced to a decimal in the manner already explained, = .1428+ in. The common fractional form is, however, better than the decimal form, since if the quotient be multiplied by the divisor the result will then be *exactly* the same as the dividend.

78. Rule.—*Find how many times the divisor is contained in the first or highest denomination of the dividend. Reduce the remainder (if any) to the next lower denomination, and add to it the number in the dividend expressing that denomination. Divide this new dividend by the divisor. The quotient will be the next denomination in the quotient required. Continue in this manner until the lowest denomination is reached. The successive quotients will constitute the entire quotient.*

EXAMPLES FOR PRACTICE.

79. Divide:

(*a*) 376 mi. 276 rd. by 22; (*b*) 1,137 bu. 3 pk. 4 qt. 1 pt. by 10; (*c*) 84 cwt. 48 lb. 49 oz. by 16; (*d*) 78 sq. yd. 18 sq. ft. 41 sq. in. by 18; (*e*) 148 mi. 64 rd. 24 yd. by 12; (*f*) 100 T. 16 cwt. 18 lb. 11 oz. by 15; (*g*) 36 lb. 18 oz. 18 pwt. 14 gr. by 8; (*h*) 112 mi. 48 rd. by 100.

Ans.
(*a*) 17 mi. 41 rd. 3 yd. 1 ft. 6 in.
(*b*) 113 bu. 3 pk. 1 qt. $\frac{1}{2}$ pt.
(*c*) 5 cwt. 28 lb. $3\frac{1}{16}$ oz.
(*d*) 4 sq. yd. 4 sq. ft. $2\frac{5}{18}$ sq. in.
(*e*) 12 mi. 112 rd. 2 yd.
(*f*) 6 T. 14 cwt. 41 lb. $3\frac{14}{15}$ oz.
(*g*) 4 lb. 8 oz. 7 pwt. $7\frac{3}{4}$ gr.
(*h*) 1 mi. 38 rd. 4 yd. 2 ft. 6.24 in.

80. When the divisor is a compound number, the easiest and best way to perform the division is to reduce both numbers to the lowest denomination given in either the dividend or divisor; then divide as in whole numbers.

81. Example.—How many bottles, each holding 1 qt. $\frac{1}{2}$ pt., can be filled from a cask holding 21 gal. 3 qt.?

Solution.—In 1 qt. $\frac{1}{2}$ pt. there are $2\frac{1}{2}$ pt.; in 21 gal. 3 qt. there are 174 pt. Then, 174 pt. $\div 2\frac{1}{2}$ pt. $=$ 69 bottles and $1\frac{1}{2}$ pt. left over. Ans.

LONGITUDE AND TIME—(Continued.)

82. We are now prepared to find the difference of time, or the difference of longitude, between two places. For this purpose we use the rules given in Arts. **33** and **34**.

EXAMPLE.—What is the difference of time between New York and Chicago?

SOLUTION.—In Art. **35** the longitude of New York is stated to be 73° 58′ 24″ W., and of Chicago 87° 36′ 40½″ W. Applying the rule given in Art. **33**,

87°	36′	40½″ W.
73°	58′	24 ″ W.
13°	38′	16½″ W.

Dividing 13° 38′ 16½″ by 15 the result is

15) 13°	38′	16.5″
	54 min.	33.1 sec.

EXAMPLE.—Find the difference of time between Albany and Rome.

SOLUTION.—Applying rule, Art. **33**, difference of longitude equals

73°	44′	48″ W.
12°	28′	40.5″ E.
86°	13′	28.5″

15) 86°	13′	28.5″	
5 hr.	44 min.	53.9 sec.	Ans.

We must evidently *add* to find the difference, since Rome is *east* of Greenwich, or the prime meridian, and Albany is *west*. Were both places on the *same* side, both east or both west, we should subtract.

83. The longitude of a place is determined by means of a very accurate watch, called a *chronometer*, and by observation of the sun or stars. The watch is set for Greenwich time, and an observation of the sun is taken for the place whose longitude it is desired to find. By aid of suitable instruments it can be determined when it is exactly noon at any place, and by looking at the watch, the difference of time between noon and the time indicated by the watch will be the difference of time between the place at which the observation is taken and Greenwich. If the watch appears to be *slow*, the longitude is *east;* if *fast*, it is *west*. Knowing the difference of time, the longitude is easily found by the rule given in Art. **34.**

84. EXAMPLE.—A watch set to Greenwich time appeared to be 6 hr. 10 min. 41 sec. fast at a certain place. What was the longitude of the place?

SOLUTION.—Applying rule, Art. **34,**

6 hr.	10 min.	41 sec.
		15
92°	40′	15″

Since the watch appeared fast, the longitude was 92° 40′ 15″ W.

EXAMPLES FOR PRACTICE.

85. Find the difference of time between:

(*a*) London and City of Mexico. Ans. (*a*) 6 hr. 36 min. $4\frac{1}{5}$ sec.

(*b*) Paris and Philadelphia. Ans. (*b*) 5 hr. 9 min. $59\frac{3}{5}$ sec.

(*c*) Richmond, Va., and San Francisco.

Ans. (*c*) 2 hr. 59 min. $54\frac{1}{10}$ sec.

(*d*) Boston and Ann Arbor. Ans. (*d*) 50 min. $39\frac{4}{5}$ sec.

(*e*) London and Calcutta. Ans. (*e*) 5 hr 55 min. $43\frac{1}{5}$ sec.

Find the longitude when the watch is apparently

(*f*) Fast 7 hr. 43 min 11 sec.

(*g*) Slow 1 hr. 0 min. 49 sec.

(*h*) Slow 4 hr. 37 min. 6 sec.

(*i*) Fast 8 hr. 19 min. 24 sec.

Ans. $\begin{cases} (f)\ 115° 47' 45'' \text{ W.} \\ (g)\ 15° 12' 15'' \text{ E.} \\ (h)\ 69° 16' 30'' \text{ E.} \\ (i)\ 124° 51' \text{ W.} \end{cases}$

The student should verify the table of times given in the table, Art. **35.**

ARITHMETIC.

THE METRIC SYSTEM.

1. In the metric system, a uniform scale of 10 is used throughout, as in the ordinary scale of numbers and in United States money. The name is derived from the meter (from the Greek word *metron*, a measure), the unit from which all the other units are derived. The use of the metric system was made legal in the United States in 1866, but has not yet been made compulsory; it is used by scientists throughout the world.

2. The **meter** is very nearly one ten-millionth part of the distance from the equator to the pole, and has been officially declared by the United States Government to equal 39.37 inches, which corresponds very nearly to the distance above mentioned.

3. The metric system has three principal units, the only ones we shall consider, which are: the **meter** (pronounced meeter), the unit of length; the **liter** (pronounced leeter), the unit of capacity; and the **gram,** the unit of weight. Each of the units has its multiples and subdivisions.

4. The names of the denominations higher than the leading unit are obtained by prefixing to the name of the unit the Greek names *dek'a*, *hek'to*, *kil'o*, and *myr'ia*. Thus, taking the meter as the unit, we write

Dek'a me'ter,	10 meters,	deka meaning	10;
Hek'to me'ter,	100 meters,	hekto meaning	100;
Kil'o me'ter,	1,000 meters,	kilo meaning	1,000;
Myr'ia me'ter,	10,000 meters,	myria meaning	10,000.

5. The names of the denominations lower than the leading unit are obtained by prefixing to the name of the unit the Latin names *dec'i*, *cent'i*, and *mil'li*. Thus,

Dec'i me'ter,	$\frac{1}{10}$ meter,	deci meaning	$\frac{1}{10}$;
Cent'i me'ter,	$\frac{1}{100}$ meter,	centi meaning	$\frac{1}{100}$;
Mil'li me'ter,	$\frac{1}{1000}$ meter,	milli meaning	$\frac{1}{1000}$.

The prefixes and their meanings, as given in this and the preceding article, should be carefully committed to memory.

MEASURES OF EXTENSION.

MEASURES OF LENGTH.

TABLE I.

10 millimeters (*mm.*)	= 1 centimeter....	*cm.*	=	.3937 in.
10 centimeters	= 1 decimeter.....	*dm.*	=	3.937 in.
10 decimeters.......	= 1 meter.........	*m.*	=	3.28 ft.
10 meters...........	= 1 dekameter....	*Dm.*	=	32.8 ft.
10 dekameters......	= 1 hektometer...	*Hm.*	=	328.09 ft.
10 hektometers.....	= 1 kilometer.....	*Km.*	=	.62137 mi.
10 kilometers.......	= 1 myriameter...	*Mm.*	=	6.2137 mi.

It will be noticed that the abbreviations of the names of the units of the higher denominations begin with a *capital* letter, and those of the lower denominations, with a small letter. A decimeter divided into centimeters and millimeters is shown in the figure below.

6. The decimeter, dekameter, hektometer, and myriameter are rarely used. The meter is used for those measurements that would ordinarily be made in feet and yards; the centimeter and millimeter for those ordinarily made in inches and hundredths of an inch, and the kilometer for long distances, such as we ordinarily express in miles.

7. The *approximate* length of the meter is 40 inches, or, more closely, $39\frac{3}{8}$ inches (39.4 inches is a closer approximation). When great exactness is not required, 40 inches is near enough. Hence, 1 decimeter equals $40 \times \frac{1}{10} = 4$ inches; 1 centimeter = .4 inch, and 1 millimeter = .04 inch. The length of 1 kilometer = $\frac{5}{8}$ mile, approximately.

8. Since meters, centimeters, and millimeters have a decimal scale like dollars, cents, and mills, they may be read in a similar way. Thus, 41.457 meters may be read as 41 and 457-thousandths meters, or as 41 meters 45 centimeters 7 millimeters. It might also be read 41 meters 45 and 7-tenths centimeters, or 41 meters 457 millimeters.

9. To express metric numbers decimally in terms of a given unit:

Example.—Express 4 Km. 3 Hm. 1 m., 5 cm. and 9 mm. in meters.

Solution.—We write the meters in units place, at the left of the decimal point; then, writing the other units in order, supplying the missing denominations (if any) with ciphers, we have 4301.059 meters. Ans.

Had it been required to express the above in kilometers, the result would have been 4.301059 Km.

10. Rule.—*Write the number of given units; then, the numbers of the higher denominations on the left, as integers, and those of the lower denominations on the right, as decimals, supplying any missing denomination with a cipher.*

11. To reduce a metric number to higher or lower denominations, all that is necessary is to move the decimal point. Thus, 74.1026 Km. = 741.026 Hm. = 7,410.26 Dm. = 74,102.6 m. = 741,026 dm. = 7,410,260 cm. = 74,102,600 mm. Also, to reduce 97,452 cm. to kilometers, write 97,452 cm. Now, beginning with the decimal point (which, of course, follows the 2), count centi, deci, meter, deka, hekto, kilo, placing the decimal point after the last-named unit, and supplying any missing ones with ciphers. The result is 0.97452 Km.

EXAMPLES FOR PRACTICE.

12. Solve the following:

(*a*) Reduce 25.7 Km. to meters.
(*b*) Reduce 43.4 m. to millimeters.
(*c*) Reduce 4,823.6 m. to hektometers.
(*d*) Reduce 48,639 cm. to meters.
(*e*) Reduce 738.4 Dm. to centimeters.

Read the following:

(*f*) 14.5 m.
(*g*) 47.3 Dm.
(*h*) 568 Hm.
(*i*) 434.5 Km.
(*j*) 27.4 mm.
(*k*) 92.76 cm.

Answers.—(*a*) 25,700 m.; (*b*) 43,400 mm.; (*c*) 48.236 Hm.; (*d*) 486.39 m.; (*e*) 738,400 cm.; (*f*) 14 and 5-tenths meters; (*g*) 47 and 3-tenths dekameters; (*h*) 568 hektometers; (*i*) 434 and 5-tenths kilometers; (*j*) 27 and 4-tenths millimeters; (*k*) 92 and 76-hundredths centimeters.

SQUARE MEASURE.

TABLE II.

100 square millimeters (*sq. mm.* or *mm²*.)	=	1 square centimeter ..	*sq. cm.* or *cm²*.
100 square centimeters....	=	1 square decimeter ...	*sq. dm.* or *dm²*.
100 square decimeters.....	=	1 square meter ..... or centare..........	*sq. m.* or *m²*. *ca.*
100 square meters (*100 centares*)	=	1 square dekameter or are..............	*sq. Dm.* or *Dm²*. *A.*
100 square dekameters (*100 ares*) ..	=	1 square hektometer or hektare	*sq. Hm.* or *Hm²*. *Ha.*
100 square hektometers (*100 hektares*) ..	=	1 square kilometer....	*sq. Km.* or *Km²*.

COMMON EQUIVALENTS.

1 sq. cm. = 0.1550 sq. in.
1 sq. dm. = 0.1076 sq. ft.
1 sq. m. = 1.1960 sq. yd. = 10.7637 sq. ft.
1 are = 3.954 sq. rd.
1 hektare = 2.471 A.
1 sq. Km. = 0.3861 sq. mi.

13. In square measure, the scale is 100 (10×10); hence, in reducing units of any denomination to a lower or higher denomination, the decimal point must be moved two places for each denomination. Thus, to reduce 42.09872 sq. m. to square centimeters, begin at the decimal point, point off two places to the right and say square decimeters, then two more places and say square centimeters, obtaining 420,987.2 sq. cm. Had square millimeters been desired, it would have been necessary to annex a cipher; thus, 42.09872 sq. m. = 42,098,720 sq. mm. Reduction to higher denominations is performed in the same manner, moving the decimal point to the left. Thus, 42,098,720 mm^2. = 0.4209872 ares.

14. The square meter is used in measuring floors, ceilings, and other ordinary surfaces; the are and hektare in measuring land, and the square kilometer in measuring states and territories.

EXAMPLES FOR PRACTICE.

15. Solve the following:

(*a*) Write 78.29 ares as centares; also as hektares.
(*b*) Write 9 m^2. as square decimeters; also as square centimeters.
(*c*) In 3,246 ca. how many ares?
(*d*) Express 7,041.6 sq. dm. in ares.

Answers.—(*a*) 7,829 ca. or .7829 Ha.; (*b*) 900 dm^2. or 90,000 cm^2.; (*c*) 32.46 A.; (*d*) .70416 A.

CUBIC MEASURE.

TABLE III.

1,000 cubic millimeters (*cu. mm.* or mm^3.) =	1 cubic centimeter....	*cu. cm.* or cm^3.
1,000 cubic centimeters...... =	1 cubic decimeter.....	*cu. dm.* or dm^3.
1,000 cubic decimeters....... =	1 cubic meter..........	*cu. m.* or m^3.

COMMON EQUIVALENTS.

1 cu. cm. = .06102 cu. in.
1 cu. dm. = 61.023 cu. in.
1 cu. m. = 61,023.4 cu. in. = 35.3145 cu. ft. = 1.308 cu. yd.

In measuring wood, the cubic meter is called a *stere*.

16. Units higher than the cubic meter are not used, except in denoting the volume of planets.

In cubic measure, the scale is 1,000 ($10\times10\times10$); hence, to reduce units from one denomination to another, apply the method given in Art. **13,** moving the decimal point three places each time, instead of two places, as in square measure.

MEASURES OF CAPACITY.

TABLE IV.

10 mil'li li'ters (*ml.*)	= 1 cen'ti li'ter	*cl.*
10 centiliters	= 1 dec'i li'ter	*dl.*
10 deciliters	= 1 li'ter	*l.*
10 liters	= 1 dek'a li'ter	*Dl.*
10 dekaliters	= 1 hek'to li'ter	*Hl.*
10 hektoliters	= 1 kil'o li'ter	*Kl.*
10 kiloliters	= 1 myr'ia li'ter	*Ml.*

COMMON EQUIVALENTS.

1 liter	= 61.023 cu. in.
1 liter	= 1.0567 liquid quarts.
1 liter	= 0.9078 dry quarts.
1 hektoliter	= 3.53144 cu. ft.
1 hektoliter	= 26.417 gallons.
1 hektoliter	= 2.8378 bushels.

17. The **liter** is equal in volume to 1 cubic decimeter, i. e., to a cube whose edges measure 1 decimeter on a side. The liter is the principal unit in measures of capacity, and is used for both *dry* and *liquid* measure; it is very nearly equal to a liquid quart.

18. One milliliter is equal in volume to 1 cubic centimeter, since 1 cu. cm. = $\frac{1}{1000}$ cu. dm. = $\frac{1}{1000}$ liter = 1 milliliter.

The centiliter is a little more than $\frac{1}{12}$ gill; it is used for measuring small quantities of liquids, as medicines. The liter is used for the same purposes as the quart, and the hektoliter for the same purposes as the gallon and the bushel.

The units are reduced from one denomination to another in the same way as measures of length. See Art. **11.**

EXAMPLES FOR PRACTICE.

19. Solve the following:

(*a*) Express 8.53 l. as centiliters; as deciliters.
(*b*) Express 4.64 Kl. as liters; as hektoliters.
(*c*) How many deciliters in 8 liters? In 9.35 liters?
(*d*) How many liters in 6.358 cl.?
(*e*) In 8,500 liters how many kiloliters?

Answers.—(*a*) 853 cl. or 85.3 dl.; (*b*) 4,640 l. or 46.4 Hl.; (*c*) 80 dl.; 93.5 dl.; (*d*) .06358 l.; (*e*) 8.5 Kl.

MEASURES OF WEIGHT.

TABLE V.

10 milligrams (*mg.*)	=	1 centigram	*cg.*
10 centigrams	=	1 decigram	*dg.*
10 decigrams	=	1 gram	*g.*
10 grams	=	1 dekagram	*Dg.*
10 dekagrams	=	1 hektogram	*Hg.*
10 hektograms	=	1 kilogram, or kilo	*Kg.* or *K.*
10 kilograms	=	1 myriagram	*Mg.*
100 myriagrams	=	1 tonneau, or ton	*T.*

COMMON EQUIVALENTS.

1 gram	=	1 cu. cm., or 1 ml. of water.
1 kilogram	=	1 cu. dm., or 1 liter of water.
1 metric ton.	=	1 cu. m., or 1 kiloliter of water.
1 gram	=	15.432 gr. Troy.
1 gram	=	0.03527 oz. avoirdupois.
1 kilogram	=	2.2046 lb. avoirdupois.
1 metric ton.	=	1.1023 tons of 2,000 lb.

20. The gram is the principal unit of weight; it is the weight of 1 cubic centimeter of pure distilled water at its temperature of maximum density, or 39.2° Fahrenheit. The gram is used in weighing gold, silver, letters (for postage), and in mixing medicines.

The kilogram is generally called the *kilo*, and is used for the same purposes as the pound avoirdupois. The tonneau

(usually called the *metric ton*) is nearly equal in weight to the long ton of 2,240 pounds.

21. Referring to Art. **38,** § 4, it will be noticed that the 5-cent piece (nickel) weighs 5 grams, that the half-dollar weighs $12\frac{1}{2}$ grams, etc.

22. The units of weight most commonly used are the milligram (in chemical analysis), the gram, the kilo, and the ton. It will be noticed that 1,000 mg. = 1 g., 1,000 g. = 1 K., and 1,000 K. = 1 ton. The milligram is, approximately, equal to $\frac{1}{65}$ of a grain; the gram, to $\frac{1}{28}$ oz. avoirdupois; the kilo, to $2\frac{1}{5}$ pounds; and the metric ton, to $1\frac{1}{10}$ (one-half of $2\frac{1}{5}$) short tons. Unless precise equivalents are desired, the values here given are accurate enough for all practical purposes. It should be firmly kept in mind that the weight of 1 liter of water is 1 kilogram, and that 1 liter of water will fill a space of 1 cubic decimeter.

OPERATIONS WITH METRIC UNITS.

23. The rules for adding, subtracting, multiplying, and dividing metric numbers are the same as for the corresponding operations on decimals. Be sure, however, that all numbers are expressed in the same units.

Example.—What is the sum of 45.68 Dm., 63.4 Hm., and 6,845 cm.?

Solution.—Reducing all the numbers to the same unit, say meters, and adding as in decimals, the sum is 6,865.25 m.

$$\begin{array}{r} 456.8 \\ 6340. \\ 68.45 \\ \hline 6865.25 \end{array} \text{ m. Ans.}$$

Example.—From 5.462 kilos take 7 Hg. 4 g. 9 cg.

Solution.—Expressing both numbers in kilos, and subtracting, the result is 4.75791 Kg.

$$\begin{array}{r} 5.462 \\ .70409 \\ \hline 4.75791 \end{array} \text{ Kg. Ans.}$$

EXAMPLES FOR PRACTICE.

24. Solve the following:

(*a*) What is the difference between 8.5 Kg. and 976 grams? Ans. 7.524 Kg.

(*b*) How much silk is contained in $12\frac{1}{2}$ pieces, if each piece contains 48.75 m.? Ans. 609.375 m.

(*c*) If 735 kilos of flour are equally distributed among 35 persons, how many kilos will each person receive? Ans. 21 Kg.

(*d*) In the last example, how many pounds did each person receive, counting $2\frac{1}{5}$ pounds to a kilo? Ans. 46.2 lb.

(*e*) Add 74 Hl., 147.2 l., 5,006.3 cl., and 6.5421 Kl., expressing the result in liters. Ans. 14,139.363 l.

(*f*) What is the weight in pounds of 197,862 cl. of water at 39.2° Fahrenheit? Ans. 4,353 lb., nearly.

FORMULAS.

25. A **formula** is an abridged statement of a general rule, in which symbols are used.

The symbols used are the letters of the alphabet, which represent numbers, and the signs $+$, $-$, $\times$, $\div$, $\sqrt{}$, etc., which have the same meaning as in arithmetic.

To illustrate, let the following example be taken: If a person exchanges 10 books, worth $3 per volume, for cloth at $2 per yard, how many yards will he obtain? A rule for solving this example, and all others like it, may be stated as follows: Multiply the number of books by the price per volume, and divide the product by the price of the cloth. The result will be the number of yards of cloth.

A more concise way of stating the rule is by using letters. Thus,

Let A = number of books;
B = price per volume;
C = price of the cloth;
D = number of yards of cloth.

Then, according to the rule,

$$\frac{\text{number of books} \times \text{price per volume}}{\text{price of cloth}} = \text{number of yards of cloth, or}$$

$$\frac{A \times B}{C} = D.$$

26. This last expression is a formula; the letters A, B, C, and D stand for the numbers given in the particular example to which it is applied; and the sign of multiplication ($\times$), and the horizontal dividing line of the fraction, which indicates division, show what operations must be performed upon the numbers to produce the answer D. In the example in question, $A = 10$, the number of books; $B = 3$, the price per volume; $C = 2$, the price of the cloth. Hence, writing for A, B, and C their values, 10, 3, and 2; D, the number of yards $= \frac{10 \times 3}{2} = 15$. Ans.

In modern technical works the rules for solving examples are commonly given by formulas, and it is important to understand how to use them. Having become accustomed to them, they will be found more convenient than rules written out in words.

27. *The multiplication sign,* $\times$, *is generally omitted in formulas, multiplication being indicated by simply writing the letters or expressions together.* Thus, the formula $\frac{A \times B}{C} = D$, given above, would ordinarily be written $\frac{A\,B}{C} = D$. The expression $4ab$ means the same as $4 \times a \times b$. Evidently, the sign cannot be omitted between *two figures*, as addition, instead of multiplication, would be indicated. Thus, 32 means $30 + 2$, not 3×2.

28. Formulas are usually written with the letter whose value is to be obtained standing alone at the left of the sign of equality. *To apply a formula, therefore, we have simply to substitute the given values for the letters on the right of the sign of equality and then perform the operations indicated by the signs.*

Example.—What is the value of v, in $v = \frac{a + b\,c}{d}$ when $a = 5$, $b = 10$, $c = 4$, and $d = 20$?

Solution.—Writing for a, b, c, and d their values,

$$v = \frac{5 + 10 \times 4}{20} = \frac{45}{20} = 2\tfrac{1}{4}. \quad \text{Ans}$$

Example.—What is the value of D, if $D = \frac{P\,c}{6\,a}$, and $P = 5$, $c = 300$, and $a = 10$?

SOLUTION.—Substituting the values of the letters,

$$D = \frac{5 \times 300}{6 \times 10} = 25. \quad \text{Ans.}$$

EXAMPLE.—The letters having the same values as before, what does x equal in the formula $x = \dfrac{c}{2\,P\,a}$?

SOLUTION.—Substituting, $x = \dfrac{300}{2 \times 5 \times 10} = \dfrac{300}{100} = 3.$ Ans.

EXAMPLE.—When $A = 10$, $B = 8$, $C = 5$, and $D = 4$, what is the value of E in the following:

$$(a)\ E = \frac{B\,C\,D}{A\left(2 + \frac{D}{C}\right)}? \qquad (b)\ E = \frac{A - \frac{3}{4}\,D + \frac{4\,B}{A + C}}{A - \frac{2\,B}{A + 22}}?$$

SOLUTION.—(*a*) Substituting,

$$E = \frac{8 \times 5 \times 4}{10\left(2 + \frac{4}{5}\right)}.$$

To simplify the denominator, notice that $2 + \frac{4}{5}$ is equivalent to the mixed number $2\frac{4}{5}$; hence, $10\left(2 + \frac{4}{5}\right) = 10 \times 2\frac{4}{5} = 10 \times \frac{14}{5} = \frac{140}{5}$.

Therefore, $$E = \frac{8 \times 5 \times 4}{\frac{140}{5}} = \frac{160 \times 5}{140} = 5\tfrac{5}{7}. \quad \text{Ans.}$$

(*b*) Substituting the values of the letters,

$$E = \frac{10 - \frac{3}{4} \times 4 + \frac{4 \times 8}{10 + 5}}{10 - \frac{2 \times 8}{10 + 22}} = \frac{10 - 3 + \frac{32}{15}}{10 - \frac{16}{32}} = \frac{9\frac{2}{15}}{9\frac{1}{2}} = \frac{\frac{137}{15}}{\frac{19}{2}} = \frac{274}{285}. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

29. Find the numerical values of x in the following formulas, when $a = 9$, $b = 8$, $c = 2$, $d = 10$, and $e = 3$:

(*a*) $x = \dfrac{d + c\,e}{b\,d - 40}$. Ans. $x = \frac{2}{5}$.

(*b*) $x = \dfrac{\frac{2}{3}(a + e)}{c\,e}$. Ans. $x = 1\frac{1}{3}$.

(*c*) $x = \dfrac{a\,d}{2\,c} + a\,b\,c$. Ans. $x = 166.5$.

(*d*) $x = \dfrac{a\,e\,d}{b\,c} + 4\frac{1}{8}$. Ans. $x = 21$.

INVOLUTION.

30. A **power** of a number is the result obtained by taking the number any number of times as a factor.

Thus, 4 is the *second* power of 2, since $2 \times 2 = 4$.

27 is the *third* power of 3, since $3 \times 3 \times 3 = 27$.

The *second* power is called the *square*, and the *third* power the *cube*.

The number itself is called the *first* power of the number.

31. **Involution** is the process of finding any required power of a number.

32. An **exponent** is a small figure placed to the right and a little above a number to denote how many times the number is to be used as a factor.

Thus, $4^3 = 4 \times 4 \times 4 = 64$; $2^4 = 2 \times 2 \times 2 \times 2 = 16$.

An exponent is sometimes called the index of the power.

33. The student should memorize the *squares* of numbers from 1 to 25, and their *cubes* as far at least as to 12.

34. **To find any power of a number:**

EXAMPLE.—Find the cube or third power of 35.

SOLUTION.— $35 \times 35 = 1{,}225 = 35^2$.
$1{,}225 \times 35 = 42{,}875 = 35^3$. Ans.

EXAMPLE.—What is the fourth power of 15?

SOLUTION.— $15 \times 15 = 225 = 15^2$.
$225 \times 15 = 3{,}375 = 15^3$.
$3{,}375 \times 15 = 50{,}625 = 15^4$. Ans.

The same result can be obtained by multiplying the second power by itself.

Thus, $225 \times 225 = 50{,}625 = 15^2 \times 15^2$.

EXAMPLE.—What is the value of 1.2^3?

SOLUTION.— $1.2 \times 1.2 \times 1.2 = 1.728$. Ans.

EXPLANATION.—The operation is performed as in ordinary multiplication, disregarding the decimal point, and then the result is pointed off in accordance with the rule for multiplication of decimals.

35. Example.—What is the cube of $\frac{3}{8}$?

Solution.— $\left(\frac{3}{8}\right)^3 = \frac{3^3}{8^3} = \frac{3 \times 3 \times 3}{8 \times 8 \times 8} = \frac{27}{512}$. Ans.

Explanation.—To raise a fraction to any power, it is necessary to raise the numerator and the denominator separately to the power desired.

Example.—Find the fourth power of $1\frac{7}{8}$.

Solution.—

$$\left(1\frac{7}{8}\right)^4 = \left(\frac{15}{8}\right)^4 = \frac{15^4}{8^4} = \frac{15 \times 15 \times 15 \times 15}{8 \times 8 \times 8 \times 8} = \frac{50{,}625}{4{,}096}. \text{ Ans.}$$

36. Rule.—I. *To raise a whole number or a decimal to any power, use it as a factor as many times as there are units in the exponent.*

II. *To raise a fraction to any power, raise both the numerator and denominator to the power indicated by the exponent.*

EXAMPLES FOR PRACTICE.

37. Raise the following to the powers indicated:

(*a*) 85^2.	Ans.	(*a*)	7,225.
(*b*) $(\frac{12}{13})^2$.		(*b*)	$\frac{144}{169}$.
(*c*) 6.5^2.		(*c*)	42.25.
(*d*) 14^4.		(*d*)	38,416.
(*e*) $(\frac{3}{4})^3$.		(*e*)	$\frac{27}{64}$.
(*f*) $(\frac{5}{6})^3$.		(*f*)	$\frac{125}{216}$.
(*g*) $(\frac{7}{2})^3$.		(*g*)	$\frac{343}{8}$.
(*h*) 1.4^5.		(*h*)	5.37824.

EVOLUTION.

38. The process of finding the root of a number is called **evolution,** and consists of finding one of the equal factors which, when multiplied together, will give a product equal to the given number—it is the reverse of involution.

39. When the given number is resolved into *two* equal factors, one of them is called the **square root;** when resolved into *three* equal factors, one of them is called the

cube root; when resolved into *four* equal factors, one of them is called the **fourth root**; etc. For example, the number 36 is equal to the product of the two equal factors 6 and 6; hence, 6 is the square root of 36. Again, the number 343 is equal to the product of the three equal factors 7, 7, and 7; hence, 7 is the cube root of 343. Also, 32 is equal to the product of the five equal factors 2, 2, 2, 2, and 2; hence, 2 is the fifth root of 32.

40. To indicate that some root of a number is to be extracted, a sign ($\sqrt{}$), called the **radical sign,** is placed before the number; and to indicate what root is to be extracted, a small figure, called the **index,** is placed within the sign. Thus, the square root of 36, the cube root of 243, and the fifth root of 32 would be, respectively, indicated by $\sqrt[2]{}36$, $\sqrt[3]{}243$, and $\sqrt[5]{}32$. It is usual, however, to combine the vinculum with the radical sign, in which case the above expressions would become $\sqrt[2]{36}$, $\sqrt[3]{343}$, and $\sqrt[5]{32}$, respectively. Square root is always indicated by simply placing the radical sign before the number and omitting the index; thus, $\sqrt{36}$.

41. The roots most commonly required to be found are the square root and cube root, and, occasionally, the fifth root. The fourth root is very seldom required. Hence, we shall describe only the methods to be pursued in extracting square and cube roots.

SQUARE ROOT.

42. The greatest number that can be written with *one* figure is 9, and $9^2 = 81$; the greatest number that can be written with *two* figures is 99, and $99^2 = 9{,}801$; with *three* figures 999, and $999^2 = 998{,}001$; with four figures 9,999, and $9{,}999^2 = 99{,}980{,}001$; etc.

In each of the above it will be noticed that the square of the number contains just twice as many figures as the given number. Hence, in order to find the square root of a number, first find how many figures there will be in the root;

this is done by pointing off the number into *periods* of *two* figures each, *beginning with the unit figure.* The number of periods will indicate the number of figures in the root.

Thus, the square root of 83,740,801 must contain four figures, since, pointing off the periods, we get 83′74′08′01, or four periods; consequently, there must be four figures in the root. In like manner, the square root of 50,625 must contain three figures, since there are three (5′06′25) periods.

43. The square of any number wholly decimal always contains twice as many figures as the number. For example, $.1^2 = .01$; $.13^2 = .0169$; $.751^2 = .564001$; etc. Hence, in pointing off a decimal, begin at the decimal point and proceed to the right. If the last period contains but one figure, complete it by annexing a cipher; thus, .43′42′94′40.

44. Hereafter, when referring to any digit of a number, the digit occupying the highest order in the number will be called the first digit, or figure; that occupying the next highest order will be called the second digit, or figure; etc. So, also, when a number is divided into periods, the extreme left-hand period will be called the first period; the one next to it, the second period; etc.

45. The process of extracting the square root of a number is best illustrated by means of an example.

Example.—Extract the square root of 517.92.

Solution.—First point off the number into periods of two figures each, beginning at the decimal point; this gives 5′17′.92. Now find the whole number whose square most nearly equals the first period. In the present case this is 2, since $2^2 = 4$, and $3^2 = 9$, and the difference between 5 and 4 is less than the difference between 5 and 9. Call this number (2 in the present case) the *trial root.* Divide the first period (which call the *trial dividend*) by twice the trial root, and to the quotient add one-half the trial root. Thus, $\frac{5}{2 \times 2} + \frac{2}{2} = 1.25 + 1 = 2.25$. Call this result the new trial root; retain two figures of it, increasing the second by 1 if the third figure is 5 or greater, thus obtaining for the above result 2.3 or 23, not regarding the decimal point for the present.

Repeat the process, using the first and second periods of the given number for a trial dividend, and 23 for a trial root.

Thus, $\frac{5'17}{2\times 23}+\frac{23}{2}=22.74$, or 22.7 to three figures.

Repeat the process once more, using 227 for the trial root, and the first three periods of the given number for the trial dividend.

Thus, $\frac{5'17'92}{2\times 227}+\frac{227}{2}=227.58$, using five figures.

To determine the correct position of the decimal point, apply the principle that *there must be as many places in the whole-number part of the root as there are periods in the whole-number part of the given number*. In 517.92 there are two periods to the left of the decimal point; hence, $\sqrt{517.92}=22.758$. Ans.

46. The method as outlined in the above example will always give the first five figures of the root, and will generally give the first six figures. These are all that are usually required. If, however, a greater number of figures is required, repeat the process once more, using the answer as obtained above for the trial root, and using all the figures of the given number for the trial dividend. The result then obtained should be correct to from 10 to 13 figures.

47. Rule.—I. *Begin at the decimal point and point off the number into periods of two figures each. Having done this, neglect the decimal point until required to decide its position in the root.*

II. *Find the number whose square is nearest in value to the first period; call this number the trial root. Using the first period of the given number as a trial dividend, divide it by twice the trial root; add to the quotient one-half the trial root, and retain two figures of the result, increasing the second figure by 1 if the third figure is 5 or greater.*

III. *Use the first two periods for a trial dividend (the second period being a cipher period if the number contains but one period) and the result obtained in* II *for a trial root. Repeat the process, retaining three figures of the result, which use as a trial root when three periods are used for a trial dividend. The first five figures of this last result will be the first five figures of the root. Increase the fifth figure by 1 if the sixth figure is 5 or greater, and then locate the decimal point.*

IV. *If the number contains but one or two periods, additional periods may be obtained by annexing ciphers.*

48. The following formula will assist the student in remembering this rule:

$$\text{Root} = \frac{D}{2R} + \frac{R}{2},$$

in which D = the trial dividend, and R = the trial root.

EXAMPLE.—Extract the square root of 3; that is, find the value of $\sqrt{3}$.

SOLUTION.—The number whose square is nearest 3 is 2; hence, we use 2 for the first trial root.

Then, $$\frac{3}{2 \times 2} + \frac{2}{2} = 1.75, \text{ say } 1.8.$$

Substituting 18 for R,

$$\frac{3'00}{2 \times 18} + \frac{18}{2} = 17.3.$$

Substituting 173 for R,

$$\frac{3'00'00}{2 \times 173} + \frac{173}{2} = 173.205.$$

Hence, $\sqrt{3} = 1.73205$, which is correct to six figures. Ans.

EXAMPLE.— $\sqrt{.00217} = ?$

SOLUTION.—Pointing off the number into periods of two figures each, it becomes .00'21'70, annexing a cipher to complete the last period. Treating 21 as the first period, the number whose square is nearest to 21 is 5, since $4^2 = 16$ and $5^2 = 25$, and there is a greater difference between 16 and 21 than there is between 21 and 25.

Hence, $$\frac{21}{2 \times 5} + \frac{5}{2} = 4.6.$$

Substituting 46, $$\frac{21'70}{2 \times 46} + \frac{46}{2} = 46.6.$$

Substituting 466, $$\frac{21'70'00}{2 \times 466} + \frac{466}{2} = 465.83.$$

We locate the decimal point as follows: Since the number is wholly decimal, the root must also be wholly decimal; and since there are two ciphers following the decimal point, there must be *one cipher after the decimal point in the root*, for if any number have *one* cipher following the decimal point, its square will have two ciphers following the decimal point. Hence, $\sqrt{.00217} = .046583$. Ans.

EXAMPLE.— $\sqrt{.3} = ?$

SOLUTION.—Annexing a cipher to complete the period (since the number is wholly decimal), we obtain $\sqrt{.30} = ?$ The number whose square is nearest 30 is 5.

Substituting 5, $\frac{30}{2 \times 5} + \frac{5}{2} = 5.5.$

Substituting 55, $\frac{30'00}{2 \times 55} + \frac{55}{2} = 54.8.$

Substituting 548, $\frac{30'00'00}{2 \times 548} + \frac{548}{2} = 547.72.$

Since the number is wholly decimal, the root is wholly decimal, and $\sqrt{.3} = .54772$. Ans.

When there are ciphers between the decimal point and the first digit of a number that is wholly decimal, and it is required to ascertain the position of the decimal point in the root, count the ciphers between the decimal point and the first digit and divide the result by 2. The quotient in each case, neglecting any remainder that may be obtained, will denote the number of ciphers between the decimal point and the first digit of the root. Thus, the square root of .00000027 will have $6 \div 2 = 3$ ciphers between the decimal point and the first digit of the root; likewise, the square root of .0000027 will have $5 \div 2 = 2$ ciphers between the decimal point and the first figure of the root. Another way to ascertain the position of the decimal point is to point off the given number into periods, and write one cipher in the root for each cipher period between the decimal point and the first digit of the root. Thus, the square root of .00′00′00′25 = .0005; and the square root of .00′04 = .02.

49. If the first period is a perfect square, the result obtained by the first approximation will be the same as the trial root. In such cases, we must proceed as follows:

Use the first two periods of the given number for the trial dividend; annex a cipher to the trial root and proceed as before. Or, find the number composed of two figures whose square is nearest in value to the number represented by the first two periods of the given number, and use it for the trial root.

For example, to extract the square root of 190.72, it is

seen that the first period, 1, is a perfect square; hence, use the first two periods. By the first method,

$$\frac{1'90}{2\times 10}+\frac{10}{2} = 14.5;\ \frac{1'90'72}{2\times 145}+\frac{145}{2} = 138.3;$$

$$\frac{1'90'72'00}{2\times 1,383}+\frac{1,383}{2} = 1,381.01,$$

or 13.8101 after locating the decimal point. By the second method, we take 14 for the trial root and obtain:

$$\frac{1'90}{2\times 14}+\frac{14}{2} = 13.8;\ \frac{1'90'72}{2\times 138}+\frac{138}{2} = 138.101,$$

or 13.8101 after locating the decimal point.

Although the first method is a little longer, it saves the labor of trying to find the number whose square is nearest to the number represented by the first two periods. When using the first method, the substituting of three figures for R in the trial root will not generally give the first five figures of the root in the result. Hence, it is necessary to substitute four figures of the result for R, and repeat the process. The result thus obtained will usually be correct to six figures.

50. When the given number whose root it is desired to find is a perfect square, the process just described does not give the exact root; but the exact root may always be found as shown in the following examples:

Example.— $\sqrt{15,625} = ?$

Solution.—Pointing off into periods, we get 1'56'25. Since the first period is a perfect square, it is necessary to employ one of the methods described in Art. **49.** Using the first method,

$$\frac{1'56}{2\times 10}+\frac{10}{2} = 12.8.$$

Substituting 128 for R, $$\frac{1'56'25}{2\times 128}+\frac{128}{2} = 125.035.$$

Substituting 1,250 for R, $$\frac{1'56'25'00}{2\times 1,250}+\frac{1,250}{2} = 1,250.$$

Now, since this last result is the same as the trial root, we know that the last trial dividend is a perfect square. Hence, locating the decimal point, $\sqrt{15,625} = 125$. Ans.

EXAMPLE.— $\sqrt{64,128,064} = ?$

SOLUTION.—Pointing off into periods, we get 64′12′80′64. If we assume that $R = 8$, the first result will be 8, since 64 is a perfect square. Thus, $\frac{64}{2 \times 8} + \frac{8}{2} = 8$. Hence, we use the first two periods for a trial dividend, and 80 for R, getting,

$$\frac{64'12}{2 \times 80} + \frac{80}{2} = 80.075, \text{ say } 80.1.$$

Substituting 801 for R, $$\frac{64'12'80}{2 \times 801} + \frac{801}{2} = 800.8.$$

Substituting 8,008 for R, $$\frac{64'12'80'64}{2 \times 8{,}008} + \frac{8{,}008}{2} = 8{,}008.$$

Hence, $\sqrt{64,128,064} = 8,008$. Ans.

EXAMPLE.— $\sqrt{5,688,327,241} = ?$

SOLUTION.—Pointing off into periods, we get 56′88′32′72′41.

Substituting 7 for R, $$\frac{56}{2 \times 7} + \frac{7}{2} = 7.5.$$

Substituting 75 for R, $$\frac{56'88}{2 \times 75} + \frac{75}{2} = 75.42.$$

Substituting 754 for R, $$\frac{56'88'32}{2 \times 754} + \frac{754}{2} = 754.21.$$

Since the first four figures of this result are the same as those of the preceding operation, we may be sure that the fifth figure of this result is also correct; and, since the given number contains five periods, and the square of the fifth figure is the same as the last figure of the fifth period of the given number, the given number is probably a perfect square. Hence, substituting 75,421 for R, and using all five periods,

$$\frac{56'88'32'72'41}{2 \times 75{,}421} + \frac{75{,}421}{2} = 75{,}421.$$

Therefore, $\sqrt{5,688,327,241} = 75,421$. Ans.

EXAMPLES FOR PRACTICE.

51. In the following examples, — after the answer indicates that the last figure was increased by 1 in consequence of the next figure being 5 or a greater digit; + after the answer indicates that the next figure was less than 5. When the answer is followed by neither + nor —, the root is exact.

(*a*)	What is the square root of 90?	(*a*)	9.4868+.
(*b*)	What is the square root of 54.3?	(*b*)	7.3689−.
(*c*)	What is the square root of 2,796?	(*c*)	52.877+.
(*d*)	What is the square root of 3,510.76?	Ans. (*d*)	59.252−.
(*e*)	What is the square root of 1,485,961?	(*e*)	1,219.
(*f*)	What is the square root of 2,164.1104?	(*f*)	46.52.
(*g*)	What is the square root of 47.80754449?	(*g*)	6.9143.

CUBE ROOT.

52. The first step in finding the cube root of a number is to find how many figures there will be in the root. Proceeding as in Art. **42,** we have $9^3 = 729$; $99^3 = 970,299$; $999^3 = 997,002,999$. It appears that in each of the above cases, the cube of the number contains three times as many figures as the number. Therefore, if a number is pointed off into periods of *three* figures each, beginning at the right, the cube root of the number will contain as many figures as there are periods. Thus, the cube root of 1,767,172,329 must contain four figures, since the number contains four periods.

53. The cube of any number wholly decimal contains three times as many figures as the number cubed. Thus, $.2^3 = .008$, $.02^3 = .000008$, $.007^3 = .000000343$, $.317^3 = .031855013$. The first step in finding the cube root of a decimal is to separate it into periods of three figures each, beginning at the decimal point. The location of the decimal point in the root is readily determined from the number of ciphers between the decimal point and the first digit; the whole number obtained by dividing this number by 3 will denote the number of ciphers immediately following the decimal point in the root. The cube root of .000076, neglecting the decimal point, is 42358. To locate the decimal point, we observe that there are four ciphers following the decimal point in the given number; $\frac{4}{3} = 1$ and 1 remainder. Therefore, there will be one cipher following the decimal point in the root; or $\sqrt{.000076} = .042358$. In dividing the number of ciphers by 3, no attention is paid to the remainder, if there be one.

54. The process of extracting the cube root of a number may be shown by an example.

EXAMPLE.—Extract the cube root of 19,397.82.

SOLUTION.—The number is first divided into periods of three figures, beginning at the decimal point; this gives 19′397.820. The first period is 19, and the nearest perfect cube is 27; therefore, $\sqrt[3]{27} = 3$ is taken as the first trial root. The first period is divided by *three times the square* of the trial root, and to the quotient is added two-thirds of the trial root. Thus, in the present case, $\frac{19}{3 \times 3^2} + \frac{2}{3} \times 3 = .703 + 2 = 2.703$. Two figures of this quotient are retained, the decimal point being neglected as in the process of extracting square root; this gives 27 as the second trial root. Using the trial root and the first two periods as a trial dividend, the process is repeated Thus, $\frac{19'397}{3 \times 27^2} + \frac{2}{3} \times 27 = 8.86 + 18 = 26.86$. Retaining three figures, the third trial root is 269. The process is repeated, using three periods. Thus, $\frac{19'397'820}{3 \times 269^2} + \frac{2}{3} \times 269 = 89.35669 + 179.33333 = 268.69002$. Since there are two periods in the whole-number part of the given number, there must be two places in the whole-number part of the root. Therefore, the required root is 26.869. The result thus obtained is correct to six figures, since the first seven figures of the root are 26.86897.

55. Instead of always using the cube root of the perfect cube that is nearest in value to the first period, for the first trial root, a better way is to proceed as follows: Find the two perfect cubes between which the first period lies, and divide their difference by the index of the root (in this case 3); then add the result (neglecting any remainder) to the smaller perfect cube. If the first period is less than the result thus obtained, use the cube root of the *smaller* perfect cube for the first trial root; but if the first period is equal to or greater than this result, use the cube root of the greater perfect cube.

For example, suppose that the first period is 267; it lies between $6^3 = 216$ and $7^3 = 343$. To determine whether to use 6 or 7 for the trial root, proceed as above directed. $343 - 216 = 127$; $127 \div 3 = 42$; and $216 + 42 = 258$. Hence, 7 would be used for the trial root.

Again, suppose the first period was 35. $3^3 = 27$; $4^3 = 64$;

$64-27 = 37$; $37 \div 3 = 12$; $27+12 = 39$. Hence, 3 would be used for the first trial root.

In the same manner, if the first period were 4, the trial root used would be 2, for $1^3 = 1$; $2^3 = 8$; $(8-1) \div 3 = 2$; and $1+2 = 3$, which is less than 4.

If the trial root is determined as explained in this article, the third approximation (when three figures are used for the trial root) will usually give six figures of the root exactly; if four figures are used for the trial root, the answer will be correct to eight figures.

56. Rule.—I. *Beginning at the decimal point, separate the number into periods of three figures each. Having done this, neglect the decimal point until it is required to determine its position in the root.*

II. *Find the first trial root as described in Art.* **55.** *Using the first period as a trial dividend, divide it by three times the square of the trial root, and add to the quotient two-thirds of the trial root. Retain two figures of the result, increasing the second figure by 1 if the third figure is 5 or greater.*

III. *Use the first two periods for a trial dividend and the result obtained in* **II** *for a trial root, and repeat the process described in* **II.** *Retain three figures of the result as a new trial root, and, using three periods as a trial dividend, repeat the process once more, retaining five or six figures of the result, and locate the decimal point.*

IV. *If the number contains but one or two periods, the additional periods may be obtained by annexing ciphers.*

57. The above process of extracting cube root is expressed by the following formula:

$$\text{Root} = \frac{D}{3R^2} + \frac{2R}{3},$$

in which D = the trial dividend, and R = the trial root.

EXAMPLE.—Extract the cube root of 75.

SOLUTION.— $4^3 = 64$; $5^3 = 125$; $(125-64) \div 3 + 64 = 84$. Hence, 4 will be used for the first trial root.

First approximation, $\frac{75}{3 \times 4^2} + \frac{2 \times 4}{3} = 4.23$, or 4.2.

Second approximation, $\frac{75'000}{3 \times 42^2} + \frac{2 \times 42}{3} = 42.17$, or 42.2.

Third approximation, $\frac{75'000'000}{3 \times 422^2} + \frac{2 \times 422}{3} = 421.71652$.

Locating the decimal point, $\sqrt[3]{75} = 4.2172$ to five figures. Ans. The exact root to eight figures is 4.2171633.

58. If the first period is a perfect cube, proceed in a manner similar to that described in Art. **49.** Also, if the given number is a perfect cube, the exact root may be found by a method similar to that described in Art. **50.**

Example.—Extract the cube root of 1,728.

Solution.—Pointing off into periods of three figures each, it is seen that the first period, 1, is a perfect cube; hence, use 10 for the trial root in the first approximation.

First approximation, $\frac{1'728}{3 \times 10^2} + \frac{2 \times 10}{3} = 12.43$, or 12.4.

Second approximation, $\frac{1'728'000}{3 \times 124^2} + \frac{2 \times 124}{3} = 120.13$, or 120.1.

Third approximation, $\frac{1'728'000'000}{3 \times 1{,}201^2} + \frac{2 \times 1{,}201}{3} = 1{,}200.0008$,

or 12 after fixing the decimal point.

Substituting 12 for R,

$\frac{1{,}728}{3 \times 12^2} + \frac{2 \times 12}{3} = 12$; hence, 12 is the cube root of 1,728. Ans.

Example.—Extract the cube root of 59'273.381'699.

Solution.—The trial root is evidently 4.

First approximation, $\frac{59}{3 \times 4^2} + \frac{2 \times 4}{3} = 3.9$.

Second approximation, $\frac{59'273}{3 \times 39^2} + \frac{2 \times 39}{3} = 38.98$, or 39.0.

Third approximation, $\frac{59'273'381}{3 \times 390^2} + \frac{2 \times 390}{3} = 389.90002$, or 389.9

Fourth approximation, $\frac{59'273'381'699}{3 \times 3{,}899^2} + \frac{2 \times 3{,}899}{3} = 3{,}899$.

Hence, $\sqrt[3]{59{,}273.381699} = 38.99$. Ans.

EXAMPLES FOR PRACTICE.

59. In the following examples, — after the answer indicates that the last figure was increased by 1 in consequence of the next figure being 5 or a greater digit; + after the answer indicates that the next figure was less than 5. When the number is followed by neither + nor —, the root is exact.

(*a*) What is the cube root of 80?	Ans.	(*a*) 4.3089–.
(*b*) What is the cube root of 8.451?		(*b*) 2.0369–.
(*c*) What is the cube root of 201,539,270.647?		(*c*) 586.3.
(*d*) What is the cube root of 609,210?		(*d*) 84.7726–.
(*e*) What is the cube root of 5,000.211?		(*e*) 17.1.

---

60. To assist the student in determining the first trial root for square and cube roots, we have prepared the following table. The table is not a necessity, it is merely a help.

SQUARE ROOT.

If the first period is less than 3, use 1 for the trial root.
If the first period is less than 7, use 2 for the trial root.
If the first period is less than 13, use 3 for the trial root.
If the first period is less than 21, use 4 for the trial root.
If the first period is less than 31, use 5 for the trial root.
If the first period is less than 43, use 6 for the trial root.
If the first period is less than 57, use 7 for the trial root.
If the first period is less than 73, use 8 for the trial root.
If the first period is less than 91, use 9 for the trial root.
If the first period is between 91 and 100, use 10 for the trial root.

CUBE ROOT.

If the first period is less than 3, use 1 for the trial root.
If the first period is less than 14, use 2 for the trial root.
If the first period is less than 39, use 3 for the trial root.
If the first period is less than 84, use 4 for the trial root.
If the first period is less than 155, use 5 for the trial root.
If the first period is less than 258, use 6 for the trial root.
If the first period is less than 399, use 7 for the trial root.
If the first period is less than 584, use 8 for the trial root.
If the first period is less than 819, use 9 for the trial root.
If the first period is between 819 and 1,000, use 10 for the trial root.

EXAMPLE.—Extract the cube root of 990.

SOLUTION.—Referring to the table, we see that 10 should be used for the trial root.

First approximation,

$$\frac{990}{3 \times 10^2} + \frac{2 \times 10}{3} = 9.967.$$

We must use 99.7 for the next trial root, since if we increase the number expressed by the first two figures by 1 in consequence of the third figure being 6, the result is 10.0, the same as the first trial root. This indicates that the first approximation is correct to at least three figures.

Second approximation,

$$\frac{990'000}{3 \times 99.7^2} + \frac{2 \times 99.7}{3} = 99.6655.$$

Hence, $\sqrt[3]{990} = 9.96655$. Ans.

The exact root to eight figures is 9.9665549.

61. To extract the square root or the cube root of a fraction, reduce the fraction to a decimal and extract the required root of the decimal.

Example.—Extract the cube root of $\frac{7}{24}$.

Solution.—Reducing $\frac{7}{24}$ to a decimal, it becomes $.291\dot{6}$.

The first period is 291 (the others are all 6's, since the 6 repeats); hence, referring to the table, Art. **60**, 7 should be used for the trial root.

First approximation, $\frac{291}{3 \times 7^2} + \frac{2 \times 7}{3} = 6.64+.$

Second approximation, $\frac{291'666}{3 \times 66^2} + \frac{2 \times 66}{3} = 66.31+.$

Third approximation, $\frac{291'666'666}{3 \times 663^2} + \frac{2 \times 663}{3} = 663.176+.$

Therefore, $\sqrt[3]{\frac{7}{24}} = \sqrt[3]{.291\dot{6}} = .663176$. Ans.

ARITHMETIC.

MENSURATION.

1. Mensuration treats of the measurement of lines, angles, surfaces, and solids.

LINES AND ANGLES.

2. A **straight line** is one that does not change its direction throughout its whole length—it is the shortest distance between two points. To distinguish one straight line from another, two of its points are designated by letters. The line shown in Fig. 1 would be called the line *A B*.

FIG. 1.

3. A **curved line** changes its direction at every point. (Fig. 2.)

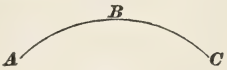

FIG. 2.

4. Parallel lines are equally distant from each other at all points. (Fig. 3.)

FIG. 3.

5. A line is **perpendicular** to another when it meets that line so as not to incline towards it on either side. (Fig. 4.)

FIG. 4.

6. A **horizontal line** is a line parallel to the horizon or water level. (Fig. 5.)

7. A **vertical line** is a line perpendicular to a horizontal line; consequently, it has the direction of a plumb-line. (Fig. 5.)

FIG. 5.

8. An **angle** is the amount of divergence between two lines that intersect, or meet; the point of meeting is called the **vertex** of the angle. Thus, in Fig. 6, the two lines form an angle whose vertex is at B. Angles are distinguished by naming the vertex and a point on each line. Thus, in Fig. 6, the angle formed by the lines $A B$ and $C B$ is called the angle $A B C$, or the angle $C B A$; the letter at the vertex is always placed at the middle. When an angle stands alone so that it cannot be mistaken for any other angle, only the vertex letter need be used. Thus, the angle referred to might be designated simply as the angle B.

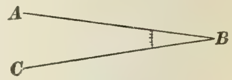

FIG. 6.

9. A **right angle** is one of the angles formed by the intersection of two lines which are perpendicular to each other. In Fig. 7, the line $A B$ is perpendicular to the line $C D$; therefore, the angles $A B C$ and $A B D$ are right angles.

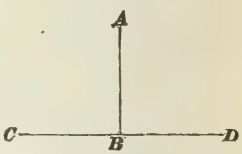

FIG. 7.

10. An **acute angle** is less than a right angle. The angle $A B C$, Fig. 8, is an acute angle.

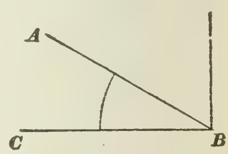

FIG. 8.

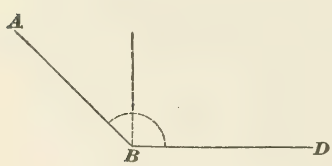

FIG. 9.

11. An **obtuse angle** is greater than a right angle. The angle $A B D$, Fig. 9, is an obtuse angle.

QUADRILATERALS.

12. A **plane figure** is any part of a plane, or flat, surface, bounded by straight or curved lines.

13. A **quadrilateral** is a plane figure bounded by four straight lines.

14. A **parallelogram** is a quadrilateral whose opposite sides are parallel.

There are four kinds of parallelograms: the **rectangle,** the **square,** the **rhomboid,** and the **rhombus.**

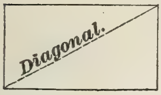

FIG. 10.

15. A **rectangle** is a parallelogram whose angles are all right angles. (Fig. 10.)

FIG. 11.

16. A **square** is a rectangle whose sides are all of the same length. (Fig. 11.)

FIG. 12.

17. A **rhomboid** is a parallelogram whose opposite sides are equal, and whose angles are not right angles. (Fig. 12.)

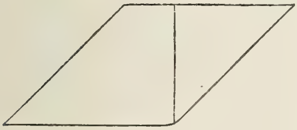

FIG. 13.

18. A **rhombus** is a rhomboid having equal sides. (Fig. 13.)

FIG. 14.

19. A **trapezoid** is a quadrilateral having only two of its sides parallel. (Fig. 14.)

20. The **altitude** of a parallelogram or trapezoid is the perpendicular distance between the parallel sides. The length of the dotted lines in Figs. 12, 13, and 14 is the altitude.

21. The **base** of a quadrilateral is the side on which it is supposed to stand. Any side may be taken as the base.

22. The **area** of a plane figure is the number of square units contained in its surface. The square unit may be a square inch, square foot, square yard, square meter, etc., as is most convenient.

23. The area of a parallelogram is equal to the product of the base and the altitude. This can be shown readily in

the case of the rectangle. Suppose, for example, the leaf of a book is 6 inches wide and 9 inches long (Fig. 15). It is a rectangle with a base of 9 inches and an altitude of 6 inches. Suppose the base to be divided into 9 equal parts, each 1 inch in length, and assume lines to be drawn through each point of division, parallel to the short sides of the rectangle. In a similar manner, suppose the altitude, or short side, to be divided into 6 equal parts, each 1 inch long, and through these points of division let lines be drawn parallel to the base. The rectangle is divided by these two sets of lines into little squares, as shown in Fig. 15. The area of one of the small squares is 1 square inch, since each of its sides is 1 inch in length. There are 9 of the squares in each horizontal row, and there are 6 rows. Hence, the total number of the little squares is $6 \times 9 = 54$, and the area of the surface is 54 square inches.

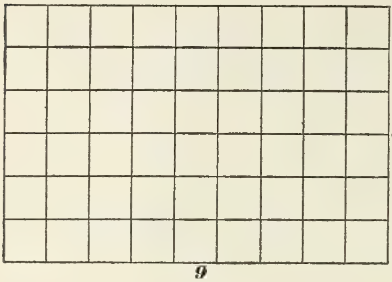

FIG. 15.

24. Rule.—*To find the area of a rectangle, multiply the base by the altitude.*

25. In ordinary language, the base and altitude of a rectangular surface are spoken of as length and breadth; the area of the surface is obtained by multiplying together the length and breadth. In applying the above rule, care must be taken that the base and altitude, or length and breadth, are reduced to the same kind of units. For example, if the base is given in feet and the altitude in inches, they cannot be multiplied together unless both are feet or both inches. This principle is of great importance, and holds good throughout the subject of Mensuration.

It must not be understood from the foregoing that *feet can be multiplied by feet* or *inches by inches.* In multiplication the multiplier is *always abstract.* In Fig. 15 there are 9 square inches in 1 row, and 6 times as many in 6 rows.

The operation in reality is 9 sq. in. $\times 6 = 54$ sq. in., or 6 sq. in. $\times 9 = 54$ sq. in.

26. EXAMPLE.—What is the area of a floor 16 feet long and $13\frac{1}{2}$ feet wide ?

SOLUTION.—The base is 16 feet and the altitude is $13\frac{1}{2}$ feet.

Area = base $\times$ altitude $= 16 \times 13\frac{1}{2} = 216$ sq. ft. Ans.

27. The area of any parallelogram is equivalent to the area of a rectangle of the same base and altitude. In Fig. 16, the plane figure *A B D C* is a rhomboid. Suppose the corner *A C E* is cut off, as shown, and placed at the other end in the position *B D F*. If the cutting line *A E* is perpendicular to the base *C D*, the new figure *A B F E* is a rectangle. It is plain that the base and altitude of the rectangle are the same as the base and altitude of the rhomboid, and that the areas of the two figures are the same.

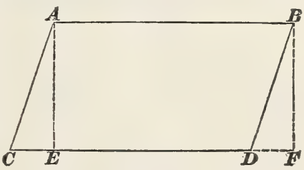

FIG. 16.

28. *To find the area of any parallelogram, multiply the base by the altitude.*

EXAMPLE.—A piece of cloth 1 yard wide is "cut on the bias," that is, it has the shape shown in Fig. 16. If the length of the strip is 8 feet, what is its area ?

SOLUTION.—The altitude is 1 yd. = 3 ft., and the base is 8 ft. Hence,

Area = base $\times$ altitude $= 8 \times 3 = 24$ sq. ft. Ans.

29. **Rule.**—*To find the area of a trapezoid, multiply one-half the sum of the parallel sides by the altitude.*

30. The reason for this rule will appear from an examination of Fig. 17. If *E* and *F* be the middle points of the sides that are not parallel, and if *A E 3* and *B F 2* be cut off below by *4–3* and *1–2*, perpendicular to *A B*, and placed above, as shown, we have a rectangle whose area is equal to *E F* $\times$ *H I*. But *E F* is as much less than *A B* as it is greater than *D C*. In other words,

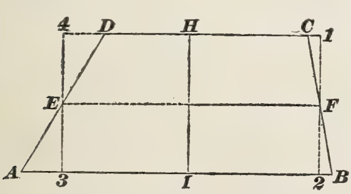

FIG. 17.

EF = half the sum of the parallel sides of the trapezoid.
Hence, area of trapezoid $= \frac{DC + AB}{2} \times HI.$

EXAMPLE.—A piece of land has the form of a trapezoid. The parallel sides are, respectively, 40 rd. and 56 rd. long, and the perpendicular distance between them is 35 rods. How many acres are contained in the piece?

SOLUTION.—One-half the sum of the parallel sides $= \frac{40 + 56}{2} = 48$ rd.

$$\text{Area} = 48 \times 35 = 1{,}680 \text{ sq. rd.} = \frac{1{,}680}{160} = 10.5 \text{ acres. Ans.}$$

31. The **perimeter** of a quadrilateral is the sum of the lengths of its four sides.

EXAMPLE.—A room is 23 ft. long and 18 ft. wide. What is its perimeter?

SOLUTION.—Perimeter $= 23 + 23 + 18 + 18 = 23 \times 2 + 18 \times 2 = 82$ ft. Ans.

PLASTERING, PAINTING, AND KALSOMINING.

32. Plastering, painting, and kalsomining are usually estimated by the square yard. Allowances for doors, windows, etc. are not regulated by any established usage. Sometimes no deduction is made for them, sometimes one-half their extent is deducted; but this is a matter usually specified in the contract.

33. EXAMPLE.—At 22 cents per square yard, what will it cost to plaster a room 65 ft. long, 22 ft. wide, and 15 ft. high; deducting in full for 8 doors 4 ft. 6 in. wide and 11 ft. 6 in. high; 10 windows 3 ft. 6 in. wide and 8 ft. high; and a baseboard $6\frac{1}{2}$ in. high extending around the room?

SOLUTION.—Perimeter of the room $= 65 \times 2 + 22 \times 2 = 174$ ft.
Area of walls .... $= 174 \times 15 = 2{,}610$ sq. ft.
Area of ceiling .... $= 65 \times 22 = 1{,}430$ sq. ft.
Total .... $= 4{,}040$ sq. ft.
Area of doors .... $= 4\frac{1}{2} \times 11\frac{1}{2} \times 8 = 414$ sq. ft.
Area of windows .... $= 3\frac{1}{2} \times 8 \times 10 = 280$ sq. ft.
Area of baseboard = (perimeter less the width of 8 doors) $\times \frac{6\frac{1}{2}}{12} = (174 - 4\frac{1}{2} \times 8) \times \frac{6\frac{1}{2}}{12} = 74\frac{3}{4}$ sq. ft.
Total, after reduction .... $= 3{,}271\frac{1}{4}$ sq. ft.
Area in square yards .... $= 3{,}271\frac{1}{4} \div 9 = 363\frac{17}{36}$ sq. yd.
Cost .... $= \$.22 \times 363\frac{17}{36} = \$79.96$. Ans.

34. Rule.—*Multiply the perimeter of the room by the height of the ceiling for the area of the walls. To this add the area of the ceiling, and from the sum make such deductions as are specified. Reduce the results to square yards, and multiply the price per square yard by the number denoting the area in square yards.*

EXAMPLES FOR PRACTICE.

35. Solve the following examples:

1. What will it cost to plaster a room 24 ft. by 30 ft., the ceiling being 9 ft. 6 in. high, at 25 cents a square yard, if no deductions are made for openings? Ans. $48.50.

2. At 12 cents per square yard, what will it cost to paint the walls and ceiling of a hall 60 ft. long, 45 ft. wide, and 15 ft. high, deducting one-half for 4 doors, 11 ft. high and 8 ft. wide, and 8 windows, 9 ft. high and 4 ft. wide? Ans. $73.73.

3. What must be paid, at 5 cents per square yard, for kalsomining 3 rooms, each having a ceiling 8 ft. 9 in. high, and the following dimensions, respectively: 18 ft. by 20 ft., 21 ft. by 27 ft., and 24 ft. by 30 ft., there being no deductions? Ans. $22.76.

PAPERING.

36. Wall paper as made in the United States is 18 inches ($\frac{1}{2}$ yard) wide, and is sold in *single rolls* and *double rolls;* a single roll is 8 yards long, and a double roll is 16 yards long. When cutting the paper, paper hangers divide the rolls into strips of sufficient length to reach from the baseboard to a short distance (say 6 inches) above the lower edge of the border. There is always considerable waste in cutting, owing to the matching of the figures forming the design, and the fact that there is a part of a strip left over after cutting up the roll. The parts of strips thus left over are used for the surface above doors, and above and below windows, and other irregular places. Although double rolls are usually counted as two single rolls, there is a choice between them in certain cases. Thus, suppose the strips were required to be 9 feet (3 yards) long; only 2 strips could be cut from a single roll, or 4 strips from 2 single rolls, while 5 strips could be cut from a double roll. The length of a roll of border is the same as the length of a roll of paper.

37. On account of the waste in cutting, the varying sizes and shapes of rooms, the number of windows, doors, etc., it is difficult to estimate exactly the number of rolls required. We give herewith two rules, both of which are used in practice:

Rule I.—*From the perimeter of the room subtract the widths of openings* (*windows and doors*), *and reduce the result to half-yards; the number of half-yards so obtained will be the total number of strips required. Find the number of strips that can be cut from a roll and divide the first result by the second; the quotient will be the number of rolls required.*

Rule II.—*Divide the number of half-yards in the perimeter of the room by the number of strips that can be cut from a roll; the quotient will be the number of rolls required.*

38. If computed by the first rule, the number of rolls obtained may be too small, and if computed by the second rule, too large. But, since paper dealers will usually take back all rolls that are intact, the second rule will generally give the best results, as it will prevent the loss of time required to send to the dealer for extra rolls, in case they are required.

Example.—Find how much paper will be needed to cover the walls and ceiling of a room 15 ft. by 20 ft., the border for both walls and ceiling to be 18 inches wide. The baseboard is 8 inches high, and the height of walls from floor to ceiling is 9 feet.

Solution.—Since the widths of the openings are not specified, it will be necessary to use rule II.

Perimeter of room $= 2 \times 15 + 2 \times 20 = 70$ ft. $= 23\frac{1}{3}$ yd. $= 46\frac{2}{3}$ half-yards, or 47 strips. Assuming that the strips extend the height of the baseboard above the bottom edge of the border, the length of a strip is (since 18 in. $= 1\frac{1}{2}$ ft.) $9 - 1\frac{1}{2} = 7\frac{1}{2}$ ft. $= 2\frac{1}{2}$ yd. Hence, the number of strips in a single roll is $8 \div 2\frac{1}{2} = 3$ strips, and the number of rolls required is $47 \div 3 = 15\frac{2}{3}$, or 16 rolls.

In papering the ceiling, the direction in which the strips are to run must be considered. If the strips run lengthwise of the room, the distance between the edges of the border is $20 - 2 \times 1\frac{1}{2} = 17$ ft., and the length of the strips must be at least 18 ft., or 6 yd. long; hence, but one strip can be cut from a single roll, and but two from a double roll. The width of the room in half-yards is $(15 \div 3) \times 2 = 10$; hence, allowing for the border, 9 strips, or 9 single rolls will be required.

If the strips run crosswise of the room, the length of a strip between the edges of the border will be $15 - 2 \times 1\frac{1}{2} = 12$ ft., and the length of

a strip must be at least 13 ft., or $4\frac{1}{3}$ yd.; hence, 1 strip may be obtained from a single roll, or $16 \div 4\frac{1}{3} = 3$ strips from a double roll. The length of the room in half-yards is $(20 \div 3) \times 2 = 13\frac{1}{3}$; hence, allowing the paper to extend 6 in. beyond the inner edge of the border at both ends of the room, 12 strips will be required. The number of double rolls required will be $12 \div 3 = 4$ double rolls. Consequently, there is less waste, in this case, when the paper runs crosswise than when it runs lengthwise.

Since the perimeter of the room is 70 ft., or $23\frac{1}{3}$ yd., $23\frac{1}{3} \div 8 = 3$ single rolls of border for the walls, and the same amount for the ceiling will be required. Therefore, 16 single rolls of paper are required for the walls, 4 double rolls for the ceiling, 3 single rolls of border for the walls, and 3 single rolls for the ceiling. Ans.

CARPETING.

39. Carpet is made of various widths. Ingrain carpet is usually 36 inches, or 1 yard wide; Brussels carpet is 27 inches, or $\frac{3}{4}$ yard wide. Carpet borders are $22\frac{1}{2}$ inches, or $\frac{5}{8}$ yard wide. A linear yard of ingrain carpet contains a square yard, and a linear yard of Brussels carpet contains $\frac{3}{4}$ of a square yard. If no allowance is made for cutting and matching the strips of carpet, the number of linear yards of carpet required for a room is found by dividing the area of the room in square yards by the area of a linear yard of the carpet.

Example.—How many yards of Brussels carpet are required to cover a floor 36 ft. long and 21 ft. wide, making no allowance for cutting and matching?

Solution.—Area of floor $= 36 \times 21 = 756$ sq. ft. $= \frac{756}{9} = 84$ sq. yd. A linear yard of Brussels carpet has an area of $\frac{3}{4}$ sq. yd. Hence, the number of linear yards required is $84 \div \frac{3}{4} = 112$ yd. Ans.

40. In practice, there is usually considerable loss due to cutting and matching. To find the number of yards required for a room, when allowance is made for loss, the width of the room is divided by the width of a single strip. The quotient is the number of strips required, supposing them to run lengthwise of the room. The number of strips multiplied by the length in yards of a single strip, making allowance for the loss required for matching, is the number of linear yards required.

Example.—How many yards of Brussels carpet are required to cover a room 23 ft. long and 15 ft. wide, making an allowance of 1 ft. on each strip for matching? The carpet is supposed to run lengthwise.

Solution.—Width of room $= 15$ ft. $= 180$ in. Width of carpet $= 27$ in. Number of strips $= 180 \div 27 = 6\frac{2}{3}$. Hence, 7 strips must be used, the excess, 9 in., being cut off or turned under. Allowing 1 foot for matching, length of strip $= 23 + 1 = 24$ ft. $= 8$ yd. Number of linear yards required $= 7 \times 8 = 56$ yd. Ans.

41. The number of linear yards of carpet border required for a room is equal to the perimeter of the room in yards.

Example.—How many yards of border are required in carpeting a room 42 ft. long and $26\frac{1}{2}$ ft. wide?

Solution.—Perimeter of room $= 42 \times 2 + 26\frac{1}{2} \times 2 = 137$ ft. $= \frac{137}{3}$ $= 45\frac{2}{3}$ yd. Ans.

BOARD MEASURE.

42. In measuring lumber, the unit is the **board foot,** which is a board 1 foot long, 1 foot wide, and 1 inch (or less) thick. One board foot is equal to $\frac{1}{12}$ of a cubic foot. Hence, to find the number of board feet in any piece of lumber:

Rule.—*Multiply the length in feet by the breadth in feet, and this product by the thickness in inches, if it be more than one inch; or, otherwise, multiply the length in feet by the breadth in inches, and this product by the thickness in inches, and then divide by 12.*

Example.—How many board feet are contained in a joist 18 feet long, 14 inches wide, and 12 inches thick?

Solution.— $\frac{18 \times 14 \times 12}{12} = 252$ board feet. Ans.

43. Lumber is sold by the thousand (M) feet, the term foot being always used instead of the longer term, board foot. Hence, to find the cost, divide the number of feet by 1,000, and multiply by the cost per M.

Example.—What will be the cost of 19 boards, 14 feet long, 15 inches wide, and $1\frac{1}{2}$ inches thick, at $23.50 per M?

Solution.—Number of thousand feet $= \frac{19 \times 14 \times 15 \times 1\frac{1}{2}}{12 \times 1,000} = .498\frac{3}{4}$. Hence, $.498\frac{3}{4} \times 23.50 = \$11.72$. Ans.

44. When expressing the size of anything that is rectangular, it is customary to write the dimensions and connect them by the sign of multiplication. Thus, to express the size of a room that is 12 feet long and 10 feet wide, it would be written $12' \times 10'$, and read *12 feet by 10 feet.* In such cases the abbreviations (′) and (″) are generally used instead of feet and inches. If three dimensions are to be expressed, all three are connected by the cross (read *by*), the length being written first, then the breadth, and, lastly, the thickness or height. Thus, a room 18 feet long, 14 feet wide, and 10 feet high would be expressed as a room $18' \times 14' \times 10'$. Hence, the joist in the example, Art. **42,** would be expressed as $18' \times 14'' \times 12''$.

45. Shingles are sold in bundles of 250 ($\frac{1}{4}$ M). The lengths of all shingles in bundle are the same (usually 12″, 14″, or 16″), but the width varies. The *average* width, however, is generally 4″, the width of all bundles being alike. When laying shingles, 4″ are usually exposed to the weather, the remaining portions being concealed by the other shingles. Hence, to find the number of shingles required to cover a roof:

46. Rule.—*Compute the total area of the roof in square inches, and divide this area by the product of the average width of the shingle and the length that is exposed to the weather.*

Example.—What would it cost to shingle a roof, each side measuring $40' \times 16'$, if the shingles cost \$4.50 per M?

Solution.—Since the size of the exposed portion is not stated, it will be assumed as $4'' \times 4''$. Then, for one side, $\frac{40 \times 16 \times 144}{4 \times 4} = 5{,}760$ shingles will be required, and for both sides, $5{,}760 \times 2 = 11{,}520$ shingles. Therefore, the cost will be $11.52 \times 4.50 =$ \$51.84. Ans.

We multiply by 144 in order to reduce the square feet (40×16) to square inches. Allowance should also be made for waste.

47. If the exposed portion is $4'' \times 4''$, it will take 9 shingles for each square foot; hence, in such cases it is only necessary to find the total area in square feet and multiply by

9 to find the number of shingles. Thus, in the last example, the total area in square feet is $40 \times 16 \times 2 = 1,280$ sq. ft., and $1,280 \times 9 = 11,520$ shingles, the same result as before.

EXAMPLES FOR PRACTICE.

48. Solve the following:

1. How many shingles are required for a roof which measures $45' \times 17'$ on one side and $45' \times 24'$ on the other side, the exposed portion of the shingles being $4'' \times 5''$? Ans. 13,284 shingles.

2. (*a*) How many thousand feet of lumber are contained in a pile having 42 layers of boards 16 feet long, the width of the layers being 11 feet, and the thickness of the boards, 1 inch? (*b*) What would be its cost at $18.75 per M. Ans. $\begin{cases} (a) & 7.392 \text{ M.} \\ (b) & \$138.60. \end{cases}$

3. What is the area in square feet of a parallelogram whose base is $58\frac{1}{4}''$ and altitude is $23\frac{5}{8}''$? Ans. 9.5566+ sq. ft.

4. How many square yards of oilcloth will cover a floor $15' \times 13\frac{1}{2}'$? Ans. $22\frac{1}{2}$ sq. yd.

5. If Brussels carpet costs 95 cents per yard, what will be the cost of carpeting a room $13\frac{1}{2}' \times 18'$, allowing 1 ft. on each strip for waste in matching? Ans. $36.10.

6. How many sheets of tin $20'' \times 14''$ are required to cover a roof $56' \times 30'$? Ans. 864 sheets.

7. At 18 cents per square yard, what will be the cost of plastering the ceiling and walls of a room 23 ft. long, 16 ft. wide, and 12 ft. high, making allowance for 3 doors, 3 ft. 6 in. wide by 7 ft. 6 in. high, 5 windows, 3 ft. 6 in. wide by 5 ft. 4 in. high, and a baseboard 8 in. high? Ans. $21.74.

8. At $2.50 per square yard, what is the cost of paving a street $\frac{1}{2}$ mile long and 60 feet wide? Ans. $44,000.

9. How many double rolls of paper and border are required to cover the walls of the room of example 7, assuming that the border, which is 18 in. wide, extends the height of the baseboard over the paper? Use rule I, Art. **37.** Ans. $\begin{cases} \text{9 rolls for walls.} \\ \text{2 rolls for border.} \end{cases}$

10. How many board feet in a stick of timber $27' \times 9'' \times 8''$? Ans. 162 ft.

11. How many single rolls of paper would be required to paper the ceiling of the room of example 7, assuming that there is no border, and that the paper overlaps on the walls at least 2 in.? Ans. 11 rolls.

THE TRIANGLE.

49. A **triangle** is a plane figure having three sides.

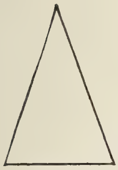
Fig. 18.

50. An **isosceles** triangle is one having two of its sides equal, as in Fig. 18.

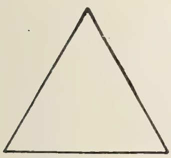
Fig. 19.

51. An **equilateral** triangle is one having all of its sides equal. (Fig. 19.)

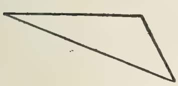
Fig. 20.

52. A **scalene** triangle is one having no two of its sides equal. (Fig. 20.)

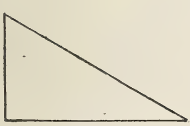
Fig. 21.

53. A **right-angled** triangle is any triangle having one right angle. The side opposite the right angle is called the **hypotenuse.** (Fig. 21.) A right-angled triangle may be isosceles or scalene.

54. The **altitude** of any triangle is a line drawn from the vertex of the angle opposite the base perpendicular to the base, or to the base extended. In Figs. 22 and 23 the vertical dotted line *A B* is the altitude of the triangle. The **perimeter** of a triangle is the sum of the lengths of the three sides.

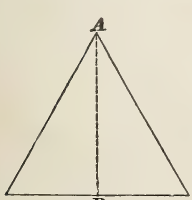

Fig. 22.

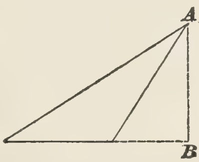

Fig. 23.

55. If in any parallelogram a straight line, called the **diagonal,** is drawn, connecting two opposite corners, the parallelogram is divided into two equal triangles, as DAB and DCB, Fig. 24. The area of each triangle, therefore, is equal to one-half the area of the parallelogram, or to one-half the product of the base and the altitude. Any side of a triangle may be taken as the base.

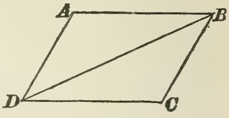

FIG. 24.

56. Rule.—*To find the area of a triangle, multiply the base by the altitude and divide the product by 2.*

EXAMPLE.—The base of a triangle is 36 in. long and its altitude is $20\frac{1}{2}$ in. What is the area of the triangle?

SOLUTION.—Area $= \frac{\text{base} \times \text{altitude}}{2} = \frac{36 \times 20\frac{1}{2}}{2} = 369$ sq. in. Ans.

57. Rule.—*To find the altitude or the base of a triangle, having given the area and the base or the altitude, multiply the area by 2, and divide by the given dimension.*

EXAMPLE.—What must be the altitude of a triangular piece of sheet metal to contain 100 sq. in., if the base is 10 in. long?

SOLUTION.— $100 \times 2 = 200$; $200 \div 10 = 20$ in. Ans.

58. The following relations between the sides of a right-angled triangle are frequently useful:

1. *The square of the hypotenuse is equal to the sum of the squares of the two short sides.*

2. *The square of one of the short sides is equal to the difference between the square of the hypotenuse and the square of the other short side.*

59. These relations may be expressed by formulas.

Let a = one short side of the triangle;
b = other short side of the triangle;
c = hypotenuse of the triangle.

Then,
$$c^2 = a^2 + b^2;$$
$$b^2 = c^2 - a^2;$$
$$a^2 = c^2 - b^2.$$

Since the square of the hypotenuse is equal to the sum of the squares of the short sides, the hypotenuse itself must be equal to the *square root* of the sum of the squares of the short sides.

60. Rule.—*To find the hypotenuse of a right-angled triangle, square each of the short sides; add the squares together and extract the square root of the sum.*

Denoting the sides by the letters a, b, and c, as above, the rule may be expressed by the formula:

$$c = \sqrt{a^2 + b^2}.$$

EXAMPLE.—The short sides of a right-angled triangle are 6 ft. and 8 ft. long, respectively; what is the length of the hypotenuse?

SOLUTION.—In this case, $a = 6$ ft., and $b = 8$ ft. Using the formula,

hypotenuse $= c = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ ft. Ans.

61. Rule.—*To find one of the short sides of a right-angled triangle, subtract from the square of the hypotenuse the square of the given short side, and extract the square root of the difference.*

The rule may be expressed by the formula:

$$b = \sqrt{c^2 - a^2}.$$

EXAMPLE.—A ladder 40 feet long is so placed that its top just reaches the top of a house 32 feet from the ground. What is the distance of the foot of the ladder from the house?

SOLUTION.—The ladder, the side of the house, and the ground form three sides of a right-angled triangle, of which the ladder is the hypotenuse c, and the height of the house is the known short side a. Using the formula, the other short side is,

$$b = \sqrt{40^2 - 32^2} = \sqrt{1{,}600 - 1{,}024} = \sqrt{576} = 24 \text{ ft.}$$

Therefore, the foot of the ladder is 24 feet from the house. Ans.

EXAMPLES FOR PRACTICE.

62. Solve the following examples:

1. Find the number of acres in a triangular field whose base is 184 rd. long, and altitude is 69 rd. Ans. 39.68– A.

2. The area of a triangle is 19.5 sq. in., and its base is 8 in. long. What is the altitude? Ans. $4\frac{7}{8}$ in.

3. The hypotenuse of a right-angled triangle is 13 in. long, and one of the short sides is 5 in. long. (*a*) Find the length of the other short side, and (*b*) the area of the triangle.

Ans. $\begin{cases} (a) & 12 \text{ in.} \\ (b) & 30 \text{ sq. in.} \end{cases}$

4. The lengths of the parallel sides of a trapezoid are 22 ft. and 19 ft., respectively; the distance between them is 47 ft. What is the area of the trapezoid?

Ans. $963\frac{1}{2}$ sq. ft.

THE CIRCLE.

63. A **circle** is a figure bounded by a curved line, called the **circumference,** every point of which is equally distant from a point within, called the **center.** (Fig. 25.) The circumference of a circle is also called its **periphery.**

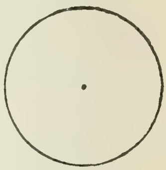

FIG. 25.

64. NOTE.—When a surface is bounded by straight lines, the length of the bounding line is called the *perimeter;* when the bounding line is a curve, the length of the curve is called the *periphery.* Thus, we speak of the perimeter of a polygon, and the periphery of a circle.

65. The **diameter** of a circle is a straight line passing through the center and terminated at both ends by the circumference. (See *A B*, Fig. 26.)

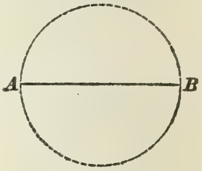

FIG. 26.

66. The **radius** of a circle is a straight line drawn from the center to the circumference. It is equal in length to one-half the diameter. The plural of radius is **radii,** and we say that all radii of a circle are equal. (*O A*, Fig. 27, is a radius.)

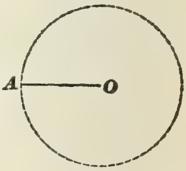

FIG. 27.

67. If a circle is divided by a diameter, each half is called a **semicircle,** and each half-circumference is called a **semi-circumference.**

68. It has been found that the length of the circumference of any circle divided by the length of the diameter

gives a constant number. This number is very nearly 3.1416; it is generally denoted by the Greek letter π (pronounced *pi*).

69. Rule.—*To find the circumference of a circle, multiply the diameter by 3.1416.*

Let
C = circumference of circle;
D = diameter of circle;
R = radius of circle;
π = 3.1416.

The above rule may be expressed by the formula,

$$C = \pi D = 3.1416\ D.$$

EXAMPLE.—If a car wheel is 36 in. in diameter, what is its circumference?

SOLUTION.— $C = 3.1416\ D = 3.1416 \times 36 = 113.0976$ in. Ans.

70. Rule.—*To find the diameter of a circle, divide the circumference by 3.1416.*

Formula:
$$D = \frac{C}{\pi} = \frac{C}{3.1416}.$$

EXAMPLE.—The circumference of a tree is 10 ft. 4 in.; what is the diameter?

SOLUTION.— 10 ft. 4 in. = 124 in. Using the formula,

$$D = \frac{124}{3.1416} = 39.47 \text{ in. Ans.}$$

71. Rule.—*To find the area of a circle, multiply the square of the radius by 3.1416, or multiply the square of the diameter by .7854.*

Formulas:
$$A = \pi R^2 = 3.1416\ R^2,$$
$$A = \tfrac{1}{4}\pi D^2 = .7854\ D^2,$$

in which A denotes the area of the circle.

EXAMPLE.—If the diameter of a circular piston is 14 in., what is its area?

SOLUTION.—The radius is one-half the diameter (Art. 66), or 7 in.

Hence, $A = 3.1416 \times 7^2 = 3.1416 \times 49 = 153.9384$ sq. in.
or, $A = .7854 \times 14^2 = .7854 \times 196 = 153.9384$ sq. in. Ans.

72. Rule.—*To find the diameter of the circle, the area being given, divide the area by .7854 and extract the square root of the quotient.*

Formula: $$D = \sqrt{\frac{A}{.7854}}.$$

Example.—What is the diameter of a circular field whose area is 1,296.64 sq. yd.?

Solution.—Substituting the known area in the formula,

$$D = \sqrt{\frac{1,296.64}{.7854}} = \sqrt{1,650.9} = 40.63 \text{ yd. Ans.}$$

EXAMPLES FOR PRACTICE.

73. Solve the following examples:

1. Find (*a*) the circumference and (*b*) area of a circle 34 ft. in diameter.
Ans. $\begin{cases} (a) & 106.814 \text{ ft.} \\ (b) & 907.92 \text{ sq. ft.} \end{cases}$

2. What is the area of a circle 4 ft. $6\frac{1}{2}$ in. in diameter?
Ans. 2,332.834 sq. in.

3. If the area of a circular sheet of metal is 130 square inches, what is (*a*) the diameter? (*b*) the circumference?
Ans. $\begin{cases} (a) & 12.866 \text{ in.} \\ (b) & 40.42 \text{ in.} \end{cases}$

4. (*a*) What must be the diameter in rods of a circular race track 1 mile in length? (*b*) What is the area of the field enclosed?
Ans. $\begin{cases} (a) & 101.859 \text{ rd.} \\ (b) & 50.93 \text{ A.} \end{cases}$

5. Find (*a*) the circumference and (*b*) the area of a locomotive driving wheel, the diameter of which is 5 ft. $6\frac{1}{2}$ in.
Ans. $\begin{cases} (a) & 208.916 \text{ in.} \\ (b) & 3,473.235 \text{ sq. in.} \end{cases}$

THE PRISM AND CYLINDER.

74. A **solid**, or body, has three dimensions: length, breadth, and thickness. The sides that enclose it are called its **faces**, and the intersections of the sides are called the **edges**.

75. A **prism** is a solid whose ends are equal and parallel plane figures, and whose sides are parallelograms. Prisms

take their names from the form of their bases. Thus, a triangular prism is one having a triangle for its base.

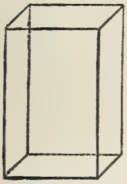

FIG. 28.

76. A **parallelopipedon** is a prism whose bases (ends) are parallelograms. (Fig. 28.)

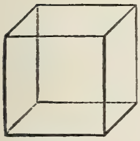

FIG. 29.

77. A **cube** is a prism whose faces are equal squares. (Fig. 29.) All the faces of a cube are equal. A cube is also a parallelopipedon.

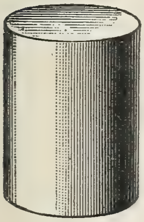

FIG. 30.

78. A **cylinder** is a body of uniform diameter throughout its entire length, whose ends are equal parallel circles. (Fig. 30.)

79. The **altitude** of a prism, or of a cylinder, is the perpendicular distance between its bases.

80. A **right prism** is one whose sides are perpendicular to the bases.

81. A **right cylinder** is one in which the line joining the centers of the two circular bases is perpendicular to those bases.

82. In the case of plane figures, we have had to do with perimeters and areas. In the case of solids, we have to do with the areas of their outside surfaces, and with their contents or volumes.

83. The **entire surface** of any solid is the area of the whole outside of the solid.

84. The **convex surface** of a solid is the same as the entire surface, except that in the case of prisms and cylinders the areas of the ends are not included.

85. Rule.—*To find the convex surface of a prism or cylinder, multiply the perimeter of the base by the altitude.*

EXAMPLE.—A block of marble is 24 in. long and its ends are 9 in. square. What is the area of its convex surface?

SOLUTION.— $9 \times 4 = 36$ in. = the perimeter of the base; $36 \times 24 = 864$ sq. in., the convex area. Ans.

86. To find the entire area of the outside surface, add the areas of the two ends to the convex area. Thus, in the last example, the area of the two ends $= 9 \times 9 \times 2 = 162$ sq. in.; $864 + 162 = 1{,}026$ sq. in.

EXAMPLE.—(*a*) What is the convex surface of a cylindrical tank with flat ends 23 ft. long and 4 ft. 6 in. in diameter? (*b*) What is the entire surface?

SOLUTION.—Perimeter of end $= 4\frac{1}{2} \times 3.1416 = 14.137$ ft.

(*a*) Convex surface $= 14.137 \times 23 = 325.151$ sq. ft. Ans.

Area of one end $= .7854 \times (4\frac{1}{2})^2 = 15.904$ sq. ft.

(*b*) Entire surface $= 325.151 + 2 \times 15.904 = 356.959$ sq. ft. Ans.

87. The **volume** of a solid is the quantity of space it occupies. As shown in Art. **14**, § 4, the measuring unit is a cube whose edges are equal in length to a linear unit; it may be a cubic inch, cubic foot, cubic yard, or cubic meter. Fig. 31 represents a rectangular prism 4 ft. long, 3 ft. wide, and 2 ft. thick. Dividing the prism by lines, as shown, it is seen that there are four equal slices, each of which is made up of $2 \times 3 = 6$ cubes. In all, there are $4 \times 2 \times 3 = 24$ cubes, each containing 1 cubic foot; that is, the volume of the prism is 24 cubic feet. It is seen that the number of cubes in each horizontal layer is just equal to the

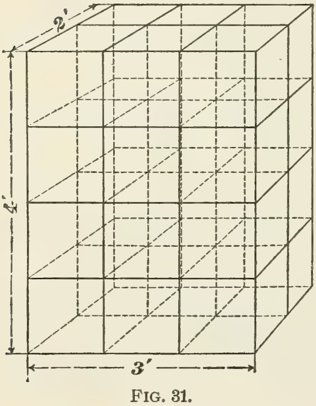

FIG. 31.

number of square feet in the base; and the number of layers is equal to the number of feet in the altitude. The same reasoning holds true for a prism with triangular base, or for a cylinder.

88. Rule.—*To find the volume of a prism or cylinder, multiply the area of the base by the altitude.*

In applying this rule, all dimensions must have the same unit.

EXAMPLE.—A packing box is $4\frac{1}{2}$ ft. long, 4 ft. wide, and $3\frac{1}{4}$ ft. deep. What is its volume?

SOLUTION.—Area of base $= 4\frac{1}{2} \times 4 = 18$ sq. ft. Altitude $= 3\frac{1}{4}$ ft. Volume, or cubical contents $= 18 \times 3\frac{1}{4} = 58\frac{1}{2}$ cu. ft. Ans.

EXAMPLE.—(*a*) How many cubic feet of water can be run into a circular cistern 8 ft. in diameter and 10 ft. deep? (*b*) how many gallons?

SOLUTION.—(*a*) The problem is to find the volume of a cylinder whose altitude is 10 ft., and whose bases are 8 ft. in diameter.

$$\text{Area of base} = .7854 \times 8^2 = 50.265 \text{ sq. ft.}$$
$$\text{Volume} = 50.265 \times 10 = 502.65 \text{ cu. ft. Ans.}$$

(*b*) According to Art. **21,** § 4, 1 gallon contains 231 cu. in. Hence, the cistern can hold $\frac{502.65 \times 1{,}728}{231} = 3{,}760$ gal., very nearly. Ans.

89. The dimensions of a rectangular solid are spoken of as length, breadth, and thickness. According to Art. **87,** the volume of the solid is the product of these three dimensions.

EXAMPLE.—A brick is 8 in. long, 4 in. wide, and 2 in. thick; what is its volume?

SOLUTION.—Volume $=$ length $\times$ breadth $\times$ thickness $= 8 \times 4 \times 2 = 64$ cu. in. Ans.

MASONRY.

90. In estimating the cubical contents of stone walls, the perch of $24\frac{3}{4}$ cubic feet is used. As stated in Art. **16,** § 4, the perch is often assumed to be 25 cubic feet.

91. Rule.—*To find the number of perches of masonry in a wall, divide the volume of the wall in cubic feet by* $24\frac{3}{4}$.

EXAMPLE.—How many perches in a wall 8 rd. long, $4\frac{1}{2}$ ft. high, and 2 ft. thick?

SOLUTION.—Length of wall $= 8 \times 16\frac{1}{2} = 132$ ft. Cubical contents of wall $= 132 \times 4\frac{1}{2} \times 2 = 1{,}188$ cu. ft. Number of perches $= 1{,}188 \div 24\frac{3}{4} = 48$. Ans.

92. In estimating the contents of stone foundations for buildings, the length of the wall is measured on the outside, thus counting each corner twice. This is illustrated in Fig. 32. If a wall 2 feet thick measures $12' \times 20'$ on the outside, and we assume that the corners are parts of the longer sides, we have two walls each 20 feet long, and two walls each 8 feet long. The actual length is, therefore, $2 \times 20 + 2 \times 8 = 56$ ft. The length estimated on the outside is $2 \times 20 + 2 \times 12 = 64$ ft. To find the actual length of such a wall, subtract four times the thickness of the wall from the length measured on the outside. Thus, in the above case, actual length $= 64 - 4 \times 2 = 56$ ft.

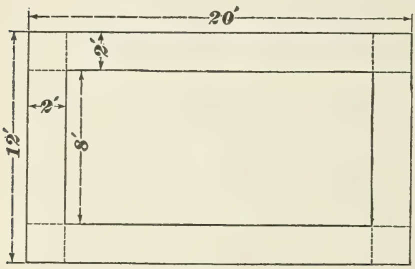

FIG. 32.

Usually, masons make no allowance for windows or doors in estimating their work. In estimating the quantity of stone required for the wall, such allowance should be made.

EXAMPLE.—(*a*) How many perches of stone are required to build the walls of a church 60 ft. long by 32 ft. wide, the walls being 24 ft. high and $2\frac{1}{4}$ ft. thick? There are 8 windows, each 5 ft. wide and 11 ft. high, and 2 doors, each 6 ft. wide and 9 ft. high. (*b*) What is the cost of laying the walls at $3.50 per perch?

SOLUTION.—

Length of wall (outside) $= 2 \times 60 + 2 \times 32 = 184$.
Actual length $= 184 - 4 \times 2\frac{1}{4} = 175$ feet.
Actual cubical contents $= 175 \times 24 \times 2\frac{1}{4} = 9{,}450$ cu. ft.
Allowance for windows $= 5 \times 11 \times 2\frac{1}{4} \times 8 = 990$ cu. ft.
Allowance for doors $= 6 \times 9 \times 2\frac{1}{4} \times 2 = 243$ cu. ft.
Net contents $= 9{,}450 - (990 + 243) = 8{,}217$ cu. ft.
(*a*) Perches required for wall $= 8{,}217 \div 24\frac{3}{4} = 332$. Ans.

(*b*) Since in estimating the cost of the work, no allowance is made for corners, doors, and windows,

Cubical contents $= 184 \times 24 \times 2\frac{1}{4} = 9{,}936$ cu. ft.
Perches of stonework $= 9{,}936 \div 24\frac{3}{4} = 401\frac{5}{11}$.
Cost of laying walls $= 401\frac{5}{11} \times \$3.50 = \$1{,}405.09$. Ans.

93. In **brickwork,** the unit of measurement is one thousand (M) bricks. The dimensions of an ordinary brick are $8'' \times 4'' \times 2''$. In some localities they are made smaller, in others, larger. To allow for mortar, $\frac{1}{4}$ inch is added to the length and to the thickness in making calculations. On this assumption, the ordinary brick with its mortar has a volume of $8\frac{1}{4} \times 4 \times 2\frac{1}{4} = 74\frac{1}{4}$ cubic inches. Since a cubic foot contains 1,728 cubic inches, it takes $1{,}728 \div 74\frac{1}{4} = 23\frac{3}{11}$ bricks to make a cubic foot of wall.

94. **Rule.**—*To find the number of ordinary bricks in a wall, multiply its volume in cubic feet by* $23\frac{3}{11}$.

EXAMPLE.—How many bricks are required in a wall 80 ft. long, $16\frac{1}{2}$ ft. high, and 4 ft. thick?

SOLUTION.—
Volume of wall $= 80 \times 16\frac{1}{2} \times 4 = 5{,}280$ cu. ft.
Number of bricks $= 5{,}280 \times 23\frac{3}{11} = 122{,}880$ or 122.88 M. Ans.

95. In estimating brickwork, it is customary in most localities to use the outside, or gross, length of the wall, and to allow for doors and windows. The practice, however, is not uniform, and in some cases no allowance is made for corners or openings.

EXAMPLE.—What will be the cost of erecting the walls of a building 64 ft. long and 40 ft. wide, the wall being 36 ft. high and 3 bricks (= 1 ft.) thick? Allowance is to be made for 40 windows, each 6 ft. $\times$ 2 ft. 9 in., and 8 doors, each 8 ft. $\times$ 3 ft. 6 in. The bricks cost \$5.75 per M, and the laying costs \$1.40 per M.

SOLUTION.—Outside length of wall $= 64 \times 2 + 40 \times 2 = 208$ ft.
Net length of wall $= 208 - 4 \times 1 = 204$ ft.
Contents of wall $= 204 \times 36 \times 1 = 7{,}344$ cu. ft.
Deduction for windows $= 6 \times 2\frac{3}{4} \times 1 \times 40 = 660$ cu. ft.
Deduction for doors $= 8 \times 3\frac{1}{2} \times 1 \times 8 = 224$ cu. ft.
Net contents of wall $= 7{,}344 - (660 + 224) = 6{,}460$ cu. ft.
Number of bricks $= 6{,}460 \times 23\frac{3}{11} = 150{,}342 = 150.342$ M.
Cost of bricks $= 150.342 \times \$5.75 = \$864.47$.
Cost of laying $= 150.342 \times \$1.40 = \$210.48$.
Total cost of erecting walls $= \$864.47 + \$210.48 = \$1{,}074.94$. Ans.

EXAMPLES FOR PRACTICE.

96. Solve the following examples:

1. Find the cost of building a stone wall to enclose a rectangular yard 160 ft. long and 108 ft. wide. The wall is 9 ft. high and 2 ft. 6 in. thick, and the price of laying is $2.25 per perch. Ans. $1,096.36.

2. How many thousand bricks are required for a house 18 ft. wide, 38 ft. long, and 32 ft. high, walls 3 bricks thick, making allowance for 3 doors, each 3 ft. 4 in. by 7 ft. 6 in., and 16 windows, each 3 ft. by 6 ft.? Ans. 71.983 M.

3. Philadelphia bricks are $8\frac{1}{4}'' \times 4\frac{1}{8}'' \times 2\frac{3}{8}''$. Allowing $\frac{1}{4}$ in. on length and thickness for mortar, how many of these bricks are required to make a cubic foot? Ans. $18\frac{3}{4}$, nearly.

BINS, CISTERNS, ETC.

97. It is frequently necessary to estimate the capacity of a bin, box, or vessel, in bushels, barrels, or gallons. The volume of the bin or vessel in cubic feet or cubic inches is divided by the number of cubic feet or cubic inches in a bushel, barrel, or gallon, as the case may be.

Example.—How many bushels of wheat can be put into a bin 35 ft. long, 6 ft. wide, and 8 ft. high?

Solution.—Cubical contents of the bin $= 35 \times 6 \times 8 = 1{,}680$ cu. ft. $= 1{,}680 \times 1{,}728 = 2{,}903{,}040$ cu. in. One bushel contains 2,150.42 cu. in. (Art. **24**, § 4). Number of bushels $= 2{,}903{,}040 \div 2{,}150.42 = 1{,}350$ bu., nearly. Ans.

98. For convenience of reference, the following table of capacities is given:

TABLE I.—DRY MEASURE.

1 heaped bushel	=	2,747.71 cu. in.	= 1.59 cu. ft., nearly.
1 stricken bushel	=	2,150.42 cu. in.	= 1.25 cu. ft., nearly.
1 peck..........	=	537.6 cu. in.	
1 quart..........	=	67.2 cu. in.	
1 pint...........	=	33.6 cu. in.	

LIQUID MEASURE.

1 hogshead	=	8.422 cu. ft.
1 barrel...	=	4.211 cu. ft.
1 gallon...	=	231 cu. in.
1 quart....	=	57.75 cu. in.
1 pint.....	=	28.875 cu. in.

99. Rule.—*To find the capacity of a bin or other vessel in dry measure or in liquid measure, divide the volume of the bin or vessel in cubic inches by the number of cubic inches in the unit of measure.*

Example.—How many liquid quarts are contained in a rectangular pail $8'' \times 5'' \times 4''$?

Solution.—Volume of pail $= 8 \times 5 \times 4 = 160$ cu. in. In one liquid quart there are 57.75 cu. in. Hence, the capacity of the pail is $160 \div 57.75 = 2.77$ quarts. Ans.

Example.—How many gallons in a milk can 16 inches in diameter and 30 inches high?

Solution.—Volume of can = area of base × altitude $= .7854 \times 16^2 \times 30 = 6{,}031.8$ cu. in. A gallon contains 231 cu. in. Number of gallons $= 6{,}031.8 \div 231 = 26.11$ gal. Ans.

100. The following table of *approximate* capacities is very convenient in rough calculations:

TABLE II.

1 cu. ft.	=	.63 of a heaped bushel.
1 cu. ft.	=	.8 of a stricken bushel.
1 cu. ft.	=	7.5 liquid gallons.
1 cu. ft.	=	$\frac{19}{80}$ of a barrel.

The following short rules are approximate, but the results are sufficiently accurate for all practical purposes.

101. Rule.—*To find the capacity of a bin in heaped bushels, multiply the volume in cubic feet by .63.*

102. Rule.—*To find the capacity of a bin in stricken bushels, multiply the volume in cubic feet by .8.*

Example.—(*a*) How many stricken bushels in a bin $18' \times 13' \times 7'$? (*b*) How many heaped bushels in the same bin?

Solution.—Volume $= 18 \times 13 \times 7 = 1{,}638$ cubic feet.

(*a*) Stricken bushels $= 1{,}638 \times .8 = 1{,}310.4$ bu. Ans.

(*b*) Heaped bushels $= 1{,}638 \times .63 = 1{,}031.94$ bu. Ans.

103. Rule.—*To find the number of gallons in a cistern or other vessel, multiply the volume in cubic feet by 7.5.*

104. Rule.—*To find the number of barrels in a cistern, multiply the volume in cubic feet by* $\frac{19}{80}$.

Example.—A rectangular cistern 9 ft. 6 in. long, 6 ft. wide, and 4 ft. deep contains (*a*) how many gallons? (*b*) how many barrels?

SOLUTION.—Volume of cistern $= 9\frac{1}{2} \times 6 \times 4 = 228$ cubic feet.

(*a*) $228 \times 7\frac{1}{2} = 1{,}710$ gal. Ans.

(*b*) $228 \times \frac{19}{80} = 54.15$ bbl. Ans.

105. Rule.—*To find the number of gallons in a cylindrical vessel, multiply the square of the diameter in inches by the height in inches, and that product by .0034.*

EXAMPLE.—An oil tank 7 ft. 6 in. long and 24 in. in diameter contains how many gallons?

SOLUTION.— 7 ft. 6 in. = 90 in. Capacity $= 24^2 \times 90 \times .0034 = 176\frac{1}{4}$ gal. Ans.

EXAMPLES FOR PRACTICE.

106. Solve the following examples by the exact methods:

1. A wagon body is 14 ft. long, 4 ft. wide, and 24 in. deep. How many bushels of shelled corn will it hold? Ans. 90 bu.

2. A rectangular can is $30'' \times 16'' \times 11\frac{1}{2}''$. How many more liquid quarts than dry quarts will it hold? Ans. 13.44 liquid quarts.

3. How many barrels are contained in a cylindrical cistern 9 ft. deep and 8 ft. 6 in. in diameter? Ans. 121.28 bbl.

4. A tin cup is 4 in. in diameter and $5\frac{1}{2}$ in. deep. (*a*) How many liquid pints will it hold? (*b*) how many dry pints?

Ans. $\begin{cases} (a) & 2.394 \text{ pt.} \\ (b) & 2.057 \text{ pt.} \end{cases}$

5. How many dry pecks can be put into a hogshead? Ans. 27.07 pk.

Solve the following examples by the approximate rules:

6. A box which holds exactly 14 stricken bushels will hold how many liquid gallons? Ans. 131.25 gal.

7. How many bushels of wheat in a bin $21' \times 6\frac{1}{2}' \times 4\frac{1}{2}'$? Ans. 491.4 bu.

8. How many barrels are contained in a cistern 11 ft. 4 in. deep and 7 ft. 6 in. in diameter? Ans. 118.91 bbl.

9. How many heaped bushels of potatoes are contained in a bin $30' \times 18' \times 7\frac{1}{2}'$? Ans. 2,551.5 bu.

10. How many gallons of water can be pumped into a cylindrical stand pipe 12 ft. in diameter and 80 ft. high? Ans. 67,859 gal.

11. A cubical box holds 100 bushels of wheat. What are its dimensions? Ans. $5' \times 5' \times 5'$.

COAL AND HAY.

107. A *ton* (2,000 lb.) of Lehigh coal, egg size, measures $34\frac{1}{2}$ cubic feet in the bin.

A ton of Schuylkill coal, egg size, measures 35 cubic feet.

A ton of pink gray and red ash coal, egg size, measures 36 cubic feet.

A ton of Wyoming coal, egg size, measures 31 cubic feet.

The bulk of a ton of hay is dependent upon the pressure to which it is subjected. Roughly speaking, a ton of hay lying unpressed measures 500 cubic feet; when in a small stack, 400 cubic feet, and in mows compressed with grain, or in well settled stacks, 300 cubic feet.

EXAMPLES FOR PRACTICE.

108. 1. How many tons of hay are contained in a well compressed mow $30' \times 18' \times 15'$? Ans. 27 tons.

2. How many tons of Lehigh coal will fill a bin 17 ft. long, 13 ft. wide, and 8 ft. high? Ans. 51.2 tons.

3. How many tons of Wyoming coal will fill a car 32 ft. long, $6\frac{1}{2}$ ft. wide, and 4 ft. deep? Ans. 26.84 tons.

THE PYRAMID AND CONE.

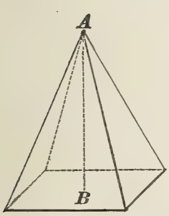

FIG. 33.

109. A **pyramid** is a solid whose base is a plane figure, and whose sides are triangles uniting at a common point, called the **vertex.** (Fig. 33.) If a straight line be drawn on one of the sides of a pyramid from the vertex so as to be perpendicular to one edge of the base, this line is called the **slant height.**

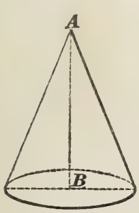

FIG. 34.

110. A **cone** is a solid whose base is a circle, and whose convex surface tapers uniformly to a point called the **vertex.** (Fig. 34.) If a straight line be drawn on the cone from the vertex to the edge of the base, this line is called the **slant height.**

111. The **altitude** of a pyramid or a cone is the perpendicular distance from the vertex to the base. (See $A\ B$, Figs. 33 and 34.)

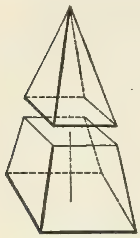

FIG. 35.

112. If a pyramid be cut by a plane parallel to the base, so as to form two parts, the lower part is called the **frustum** of the pyramid. (Fig. 35.)

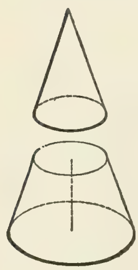

FIG. 36.

113. If a cone be cut in a similar manner, the lower part is called the **frustum** of the cone. (Fig. 36.)

114. The upper end of the frustum of a pyramid or a cylinder is called the **upper base,** and the lower end, the **lower base.** The **altitude** of a frustum is the perpendicular distance between the bases.

115. The **entire area** of a pyramid or cone is the area of the whole outside surface. The **convex area** is the *entire area* less the area of the *base.*

116. **Rule.—I.** *To find the convex area of a pyramid or cone, multiply the perimeter of the base by one-half the slant height.*

II. *The entire area is equal to the convex area plus the area of the base.*

EXAMPLE.—What is (*a*) the convex area and (*b*) the entire area of a cone whose altitude is 18 in., and whose base is 14 in. in diameter?

SOLUTION.—(*a*) The perimeter of the base is $\pi\ d = 3.1416 \times 14 = 43.9824$ in. The slant height is evidently equal to the hypotenuse

of a right-angled triangle whose altitude is the same as the altitude of the cone (18 in.) and whose base is equal to the radius of the base of the cone ($\frac{14}{2} = 7$ in.). Hence, slant height $= \sqrt{18^2 + 7^2} = 19.3132$ in. Therefore, applying rule, convex area $= 43.9824 \times \frac{19.3132}{2} = 424.72$ sq. in. Ans.

(*b*) Area of base $= 14^2 \times .7854 = 153.94$ sq. in. Hence, entire area $= 424.72 + 153.94 = 578.66$ sq. in. Ans.

117. Rule.—*To find the volume of a pyramid or a cone, multiply the area of the base by one-third the altitude.*

Example.—What is the volume of a cone whose altitude is 18 in., and whose base is 14 in. in diameter.

Solution.—Area of the base $= 14^2 \times .7854 = 153.94$ sq. in. Hence, the volume $= 153.94 \times \frac{18}{3} = 923.64$ cu. in. Ans.

Example.—Find the volume of a pyramid whose base is a square, measuring 15 in. on a side, and whose altitude is $16\frac{1}{2}$ in.

Solution.—Area of base $= 15^2 = 225$ sq. in. Ans.

$$\text{Volume} = 225 \times \frac{16\frac{1}{2}}{3} = 1{,}237\tfrac{1}{2} \text{ cu. in. Ans.}$$

118. Rule.—I. *The convex area of a frustum of a pyramid or cone is equal to the sum of the perimeters of the bases multiplied by one-half the slant height.*

II. *The entire area equals the convex area plus the areas of the bases.*

Example.—What is (*a*) the convex and (*b*) the entire area of a frustum of a square pyramid whose slant height is 22 in., one of the edges of the upper base being 6 in. long and of the lower base 14 in. long ?

Solution.—(*a*) If one of the edges of the upper base is $6 \times 4 = 24$ in., and the perimeter of the lower base is $14 \times 4 = 56$ in., perimeter of upper base is $6 \times 4 = 24$ in., and of the lower base, $14 \times 4 = 56$ in. Applying rule, $(24 + 56) \times \frac{22}{2} = 880$ sq. in. Ans.

(*b*) Area of upper base is $6 \times 6 = 36$ sq. in., and of the lower base, $14 \times 14 = 196$ sq. in. Since the entire area equals the convex area plus the area of bases, the entire area is $880 + 36 + 196 = 1{,}112$ sq. in. Ans.

119. Rule.—*To find the volume of the frustum of a pyramid or a cone, add together the areas of the upper and*

lower bases, and the square root of the product of the two areas; multiply this sum by one-third the altitude.

EXAMPLE.—Given a frustum of a square pyramid; each edge of the lower base measures 12 in., each edge of the upper base measures 5 in., and its altitude is 16 in.; what is its volume?

SOLUTION.—Area of upper base $= 5 \times 5 = 25$ sq. in.; area of lower base $= 12 \times 12 = 144$ sq. in.; the square root of the product of the areas of the two bases $= \sqrt{25 \times 144} = 60$. Adding these three results, and multiplying by one-third the altitude, $25 + 144 + 60 = 229$; $229 \times \frac{16}{3} = 1{,}221\frac{1}{3}$ cu. in. = the volume. Ans.

EXAMPLE.—How many gallons of water will a circular tank hold that is 4 ft. in diameter at the top, 5 ft. in diameter at the bottom, and is 8 ft. deep?

SOLUTION.—There are 231 cu. in. in a gallon, and the volume of the tank should be found in cubic inches. The tank is in the shape of a frustum of a cone. The diameter of the upper base $= 4 \times 12 = 48$ in.; the diameter of the lower base $= 5 \times 12 = 60$ in., and the depth $= 8 \times 12 = 96$ in. Area of upper base $= 48^2 \times .7854 = 1{,}809.56$ sq. in.; area of lower base $= 60^2 \times .7854 = 2{,}827.44$ sq. in.;

$$\sqrt{1{,}809.56 \times 2{,}827.44} = 2{,}261.95.$$

Whence, $1{,}809.56 + 2{,}827.44 + 2{,}261.95 = 6{,}898.95$;

$6{,}898.95 \times \frac{96}{3} = 220{,}766.4$ cu. in. = contents. Now, since there are 231 cu. in. in 1 gallon, the tank will hold $220{,}766.4 \div 231 = 955.7$ gallons, nearly. Ans.

THE SPHERE.

FIG. 37.

120. A **sphere** is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within, called the center. (Fig. 37.)

The word **ball** is often used instead of sphere.

121. The **diameter** of a sphere is a straight line passing through its center, the ends of which terminate at the surface.

122. The **radius** of a sphere is a line drawn from the center to the surface.

123. **Rule.**—*To find the area of the surface of a sphere, square the diameter and multiply the result by 3.1416.*

Let A = area of surface of sphere;
V = volume of sphere;
D = diameter of sphere;
Then, $A = \pi D^2 = 3.1416\,D^2$.

EXAMPLE.—What is the area of the surface of a sphere whose diameter is 14 in.?

SOLUTION.—Area $= 3.1416\,D^2 = 3.1416 \times 14^2 = 615.75$ sq. in. Ans.

From this it will be seen that the surface of a sphere equals the circumference of a great circle multiplied by the diameter, a rule often used.

124. **Rule.**—*To find the volume of a sphere, cube the diameter and multiply the result by .5236.*

Formula, $V = \frac{1}{6}\pi D^3 = .5236\,D^3$.

EXAMPLE.—What is the weight of a lead ball 12 in. in diameter, a cubic inch of lead weighing .41 lb.?

SOLUTION.—Volume $= .5236\,D^3 = .5236 \times 12^3 = 904.78$ cu. in.
Weight of ball $= 904.78 \times .41 = 370.96$ lb. Ans.

EXAMPLES FOR PRACTICE.

125. Solve the following:

1. What is the volume of a ball 6 in. in diameter? (*b*) What is the surface of the ball?
Ans. { (*a*) 113.098 cu. in. (*b*) 113.098 sq. in.

2. Find the volume of a cone whose altitude is 12 in., and the circumference of whose base is 31.416 in. Ans. 314.16 cu. in.

NOTE.—Find the diameter of the base, and then its area.

3. A conical vessel is 22 in. deep and 3 ft. in diameter. How many gallons are required to fill it? Ans. 32.31 gal.

4. Find the volume of a log of wood 18 ft. long and 32 in. in diameter. Ans. 100.53 cu. ft.

5. The diameter of the earth is 7,918 mi. Assuming it to be a true sphere, what is its volume in cubic miles?
Ans. 259,923,849,377.3152 cu. mi.

6. The frustum of a triangular pyramid has an altitude of 15 ft. The lower base is a right-angled triangle whose short sides are 8 ft. and 15 ft.; the upper base is another right-angled triangle whose short sides are 6 ft. and 11 ft. 3 in. Find the volume of the frustum.

Ans. 693.75 cu. ft.

7. A cubic inch of cast iron weighs .26 of a pound. What is the weight of a cast-iron cannon ball 8 in. in diameter? Ans. 69.7 lb.

8. How many square feet of copper are required to cover a ball 3 ft. 4 in. in diameter? Ans. 34.9 sq. ft.

9. (*a*) How many cubic inches in a pan 10 in. across the top, 8 in. across the bottom, and 4 in. deep? (*b*) How many liquid quarts will the pan hold?

Ans. $\begin{cases} (a) & 255.52 \text{ cu. in.} \\ (b) & 4.424 \text{ qt.} \end{cases}$

NOTE.—The pan is a frustum of a cone.

GAUGING OF CASKS.

126. A **cask** resembles two frustums of cones with their larger bases placed together.

The **bung diameter** of a cask is the diameter measured half way between the two ends; it is usually the greatest diameter.

The **mean diameter** of a cask is the mean between the bung diameter and head diameter. To find the mean diameter, add together the head diameter and bung diameter and divide the sum by 2.

127. Rule.—*To find the number of gallons in a cask, multiply the square of the mean diameter in inches by the length in inches, and that product by .0034.*

To find the number of liters in the cask, multiply by .0129 instead of .0034. If the cask is partly filled, stand it on end, find the mean diameter of the part filled, multiply its square by the height, and that product by .0034.

EXAMPLE.—The diameter of a cask is 27 in. at the head, 33 in. at the bung, and the cask is 3 ft. long; how many gallons will it hold?

SOLUTION.—Mean diameter $= \frac{27 + 33}{2} = 30$ in. Length = 3 ft. = 36 in. Capacity $= 30^2 \times 36 \times .0034 = 110.16$ gal. Ans.

EXAMPLES FOR PRACTICE.

128. Solve the following:

1. (*a*) What is the capacity in gallons of a cask whose length is 32 in., bung diameter 25 in., and head diameter 19 in.? (*b*) What is the capacity in liters?

Ans. $\begin{cases} (a) & 52.66 \text{ gal.} \\ (b) & 199.8 \text{ liters.} \end{cases}$

2. The height of the liquid in a cask is 14 in.; the head diameter is 20 in., and the diameter at the level of the liquid is 24 in. How many gallons are in the cask? Ans. 23.04 gal.

3. A cask has the following dimensions: head diameter, 16 in.; bung diameter, 21 in.; length, 34 in. If the cask contains 23 gal., how many more gallons may be put into it? Ans. 16.564 gal.

ARITHMETIC.

RATIO.

1. Suppose that it is desired to compare two numbers, say 20 and 4. If we wish to know how many times larger 20 is than 4, we divide 20 by 4 and obtain 5 for the quotient; thus, $20 \div 4 = 5$. Hence, we say that 20 is 5 times as large as 4, i. e., 20 contains 5 times as many units as 4. Again, suppose we desire to know what part of 20 is 4. We then divide 4 by 20 and obtain $\frac{1}{5}$; thus, $4 \div 20 = \frac{1}{5}$, or .2. Hence, 4 is $\frac{1}{5}$, or .2, of 20. This operation of comparing two numbers is termed *finding the ratio* of the two numbers. Ratio, then, is a comparison. It is evident that the two numbers to be compared must be expressed in the same unit; in other words, the two numbers must both be abstract numbers or concrete numbers of the same kind. For example, it would be absurd to compare 20 horses with 4 birds, or 20 horses with 4. Hence, **ratio** may be defined as a comparison between two numbers of the same kind.

2. A ratio may be *expressed* in three ways; thus, if it is desired to compare 20 and 4, and express this comparison as a ratio, it may be done as follows: $20 \div 4$, $20 : 4$, or $\frac{20}{4}$. All three are read *the ratio of 20 to 4*. The ratio of 4 to 20 would be expressed thus: $4 \div 20$, $4 : 20$, or $\frac{4}{20}$. The first method of expressing a ratio, although correct, is seldom or never used; the second form is the one most often met with, while the third is rapidly growing in favor, and is likely to supersede

the second. The third form, called the fractional form, is preferred by modern mathematicians, and possesses great advantages to students of algebra and of higher mathematical subjects. The second form seems to be better adapted to arithmetical subjects, and is the one we shall ordinarily adopt. There is still another way of expressing a ratio, though seldom or never used in the case of a simple ratio like that given above. Instead of the colon, a straight vertical line is used; thus, 20 | 4.

3. The **terms** of a ratio are the two numbers to be compared; thus, in the above ratio, 20 and 4 are the terms. When both terms are considered together, they are called a **couplet**; when considered separately, the first term is called the **antecedent,** and the second term, the **consequent.** Thus, in the ratio 20 : 4, 20 and 4 form a couplet, and 20 is the antecedent and 4 the consequent.

When a ratio is expressed in the fractional form, the antecedent becomes the numerator and the consequent the denominator. Thus, the ratio of \$35 to \$7 is written \$35 : \$7, or $\frac{\$35}{\$7}$, \$35 and \$7 being in both cases the antecedent and consequent, respectively.

4. A ratio may be **direct** or **inverse.** The *direct ratio* of 20 to 4 is 20 : 4, while the *inverse ratio* of 20 to 4 is 4 : 20. The direct ratio of 4 to 20 is 4 : 20, and the inverse ratio is 20 : 4. An inverse ratio is sometimes called a **reciprocal ratio.** The **reciprocal** of a number is 1 divided by the number. Thus, the reciprocal of 17 is $\frac{1}{17}$; of $\frac{3}{8}$ is $1 \div \frac{3}{8} = \frac{8}{3}$; i. e., the reciprocal of a fraction is the fraction inverted. The inverse ratio of 20 to 4 may be expressed as 4 : 20, or as $\frac{1}{20} : \frac{1}{4}$. The two ratios have equal values; for,

$$4 \div 20 = \tfrac{1}{5}, \text{ and } \frac{1}{20} \div \frac{1}{4} = \frac{1}{20} \times \frac{4}{1} = \frac{1}{5}.$$

5. The term **vary** implies a ratio. When we say that two numbers vary as some other two numbers, we mean that

the relation between the first two numbers is the same as the relation between the other two numbers.

6. The **value** of a ratio is the result obtained by performing the division indicated. Thus, the value of the ratio 20 : 4 is 5; it is the quotient obtained by dividing the antecedent by the consequent.

7. By expressing the ratio in the fractional form, for example, the ratio of 20 to 4 as $\frac{20}{4}$, it is easy to see, from the laws of fractions, that if both terms be multiplied or both divided by the same number, it will not alter the value of the ratio. Thus,

$$\frac{20}{4} = \frac{20 \times 5}{4 \times 5} = \frac{100}{20}; \text{ and } \frac{20}{4} = \frac{20 \div 4}{4 \div 4} = \frac{5}{1}.$$

8. It is also evident, from the laws of fractions, that multiplying the antecedent or dividing the consequent multiplies the ratio, and dividing the antecedent or multiplying the consequent divides the ratio.

9. When a ratio is expressed in words, as the ratio of 20 to 4, the first number named is always regarded as the antecedent and the second as the consequent, without regard to whether the ratio itself is direct or inverse. *When not otherwise specified, all ratios are understood to be direct.* To express an inverse ratio, the simplest way of doing it is to express it as if it were a direct ratio, with the first number named as the antecedent, and then transpose the antecedent to the place occupied by the consequent and the consequent to the place occupied by the antecedent; or, if the ratio is expressed in the fractional form, invert the fraction. Thus, to express the inverse ratio of 20 to 4, first write it 20 : 4, and then transpose the terms, as 4 : 20; or as $\frac{20}{4}$, and then invert, as $\frac{4}{20}$. Or, the reciprocals of the numbers may be taken, as explained above. To **invert** a ratio is to transpose its terms.

EXAMPLES FOR PRACTICE.

10. What is the value of the following:

		Ans.	
(*a*)	The ratio of 98 to 49?	(*a*)	2.
(*b*)	The ratio of \$45 to \$9?	(*b*)	5.
(*c*)	The ratio of $6\frac{1}{4}$ to $\frac{2}{4}$?	(*c*)	$12\frac{1}{2}$.
(*d*)	The ratio of 3.5 to 4.5?	(*d*)	$.77\frac{7}{9}$.
(*e*)	The inverse ratio of 76 to 19?	(*e*)	$\frac{1}{4}$.
(*f*)	The inverse ratio of 49 to 98?	(*f*)	2.
(*g*)	The inverse ratio of 18 to 24?	(*g*)	$1\frac{1}{3}$.
(*h*)	The inverse ratio of 9 to 15?	(*h*)	$1\frac{2}{3}$.
(*i*)	The ratio of 10 to 3, multiplied by 3?	(*i*)	10.
(*j*)	The ratio of 35 to 49, multiplied by 7?	(*j*)	5.
(*k*)	The ratio of 18 to 64, divided by 9?	(*k*)	$\frac{1}{32}$.
(*l*)	The ratio of 14 to 28, divided by 5?	(*l*)	$\frac{1}{10}$.
(*m*)	111 gal. : 37 gal. = ?	(*m*)	3.

11. Instead of expressing the value of a ratio by a single number as above, it is customary to express it by means of another ratio in which the consequent is 1. Thus, suppose that it is desired to find the ratio of the weights of two pieces of iron, one weighing 45 pounds and the other weighing 30 pounds. The ratio of the heavier to the lighter is then 45 : 30, an inconvenient expression. Using the fractional form, we have $\frac{45}{30}$. Dividing both terms by 30, the consequent, we obtain $\frac{1\frac{1}{2}}{1}$, or $1\frac{1}{2} : 1$. This is the same result as obtained above, for $1\frac{1}{2} \div 1 = 1\frac{1}{2}$, and $45 \div 30 = 1\frac{1}{2}$.

12. A ratio may be squared, cubed, or raised to any power, or any root of it may be taken. Thus, if the ratio of two numbers is 105 : 63, and it is desired to cube this ratio, the cube may be expressed as $105^3 : 63^3$. That this is correct is readily seen; for, expressing the ratio in the fractional form, it becomes $\frac{105}{63}$, and the cube is $\left(\frac{105}{63}\right)^3 = \frac{105^3}{63^3} = 105^3 : 63^3$. Also, if it is desired to extract the cube root of the ratio $105^3 : 63^3$ it may be done by simply dividing the exponents

by 3, obtaining 105 : 63. This may be proved in the same way as in the case of cubing the ratio. Thus, $105^3 : 63^3 = \left(\frac{105}{63}\right)^3$, and $\sqrt[3]{\left(\frac{105}{63}\right)^3} = \frac{105}{63} = 105 : 63$.

NOTE.—The root of a fraction may be found in two ways; either by reducing the fraction to a decimal and then extracting the root, or by extracting the root of both the numerator and the denominator. Thus, $\sqrt{\frac{49}{64}} = \sqrt{.765625} = .875$; also, $\sqrt{\frac{49}{64}} = \frac{\sqrt{49}}{\sqrt{64}} = \frac{7}{8} = .875$.

13. Since $\left(\frac{105}{63}\right)^3 = \left(\frac{5}{3}\right)^3$, it follows that $105^3 : 63^3 = 5^3 : 3^3$ (this expression is read: the ratio of 105 cubed to 63 cubed equals the ratio of 5 cubed to 3 cubed), it follows that the antecedent and consequent may always be multiplied or divided by the same number, irrespective of any indicated powers or roots, without altering the value of the ratio. Thus, $24^2 : 18^2 = 4^2 : 3^2$. For, performing the operations indicated by the exponents, $24^2 = 576$ and $18^2 = 324$. Hence, $576 : 324 = 1\frac{7}{9}$, or $1\frac{7}{9} : 1$. Also, $4^2 = 16$ and $3^2 = 9$; hence, $16 : 9 = 1\frac{7}{9}$ or $1\frac{7}{9} : 1$, the same result as before. Also, $24^2 : 18^2 = \frac{24^2}{18^2} = \left(\frac{24}{18}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{4^2}{3^2} = 4^2 : 3^2$.

The case of equal roots of a ratio may be proved for roots in a similar manner. Thus, $\sqrt[3]{24^3} : \sqrt[3]{18^3} = \sqrt[3]{4^3} : \sqrt[3]{3^3}$. For the $\sqrt[3]{24^3} = 24$, and $\sqrt[3]{18^3} = 18$; and $24 : 18 = 1\frac{1}{3}$, or $1\frac{1}{3} : 1$. Also, $\sqrt[3]{4^3} = 4$, and $\sqrt[3]{3^3} = 3$; $4 : 3 = 1\frac{1}{3}$, or $1\frac{1}{3} : 1$.

NOTE.—If the numbers composing the antecedent and consequent have different exponents, or if different roots of those numbers are indicated, the operations described in Art. **13** cannot be performed. This is evident; for, consider the ratio $4^2 : 8^3$. When expressed in the fractional form it becomes $\frac{4^2}{8^3}$, which cannot be expressed either as $\left(\frac{4}{8}\right)^2$ or as $\left(\frac{4}{8}\right)^3$, and, hence, cannot be reduced as described above.

14. Since ratios are merely abstract numbers, they may be compared. For example, which is the greater, 7 : 5 or 18 : 12? Performing the divisions, 7 : 5 = 1.4 and 18 : 12

= 1.5; therefore, the latter is the greater. A convenient way of comparing ratios is to express them as fractions and reduce the fractions to a common denominator. For example, it is desired to find which is the greater, the ratio of 8 to 9 or the ratio of 9 to 10. Ratio $8 : 9 = \frac{8}{9}$ and $9 : 10 = \frac{9}{10}$. Reducing to a common denominator, $\frac{8}{9} = \frac{80}{90}$ and $\frac{9}{10} = \frac{81}{90}$. Since 81 is greater than 80, the ratio of 9 to 10 is the greater.

EXAMPLES FOR PRACTICE.

15. Which is the greater:

(*a*) 10 : 7 or 16 : 11?
(*b*) 2 : 3 or 7 : 10?
(*c*) 22 : 7 or 355 : 113?

Ans. (*a*) 16 : 11. (*b*) 7 : 10. (*c*) 22 : 7.

1. What is the cube root of the ratio of 135 to 5? Ans. 3.

2. What is the square root of the ratio of 9 to 576? Ans. .125.

3. Which is the greater, the square of the ratio of 5 to 9, or the cube root of the ratio of 125 to 4,096? Ans. The latter.

4. If the antecedent is 5 hours and the consequent is 50 minutes, what is the ratio? Ans. 6.

5. What is the ratio of $\frac{3}{8}$ to $\frac{11}{16}$? Ans. $\frac{6}{11}$.

6. What is the square of the inverse ratio of $\frac{1}{2}$ to 3? Ans. 36.

7. If in the ratio 40 : 12, the antecedent is multiplied by 3, what is the value of the new ratio? Ans. 10.

PROPORTION.

16. Proportion is an equality of ratios, the equality being indicated by the double colon (::) or by the sign of equality (=). Thus, to write in the form of a proportion the two equal ratios, 8 : 4 and 6 : 3, we may employ one of the three following forms:

$$8 : 4 :: 6 : 3 \qquad (1)$$
$$8 : 4 = 6 : 3 \qquad (2)$$
$$\frac{8}{4} = \frac{6}{3} \qquad (3)$$

17. The first form is the one most extensively used, by reason of its having been exclusively employed in all the older works on mathematics. The second and third forms are being adopted by all modern writers on mathematical subjects, and, in time, will probably supersede the first form. In this paper, we shall adopt the second form, unless some statement can be made clearer by using the third form.

18. A proportion may be *read* in two ways. The old way to read the above proportion is—*8 is to 4 as 6 is to 3;* the new way is—*the ratio of 8 to 4 equals the ratio of 6 to 3.* The student may read it either way, but we recommend the latter.

19. Each ratio of a proportion is termed a **couplet.** In the above proportion, 8 : 4 is a couplet; so, also, is 6 : 3.

20. The numbers forming a proportion are called **terms;** and they are numbered consecutively from left to right, thus:

first second third fourth
$$8 : 4 = 6 : 3$$

Hence, in any proportion the ratio of the first term to the second term equals the ratio of the third term to the fourth term.

21. The first and fourth terms of a proportion are called the **extremes,** and the second and third terms, the **means.** Thus, in the foregoing proportion, 8 and 3 are the extremes and 4 and 6 are the means.

22. A **direct proportion** is one in which both couplets are direct ratios.

23. An **inverse proportion** is one which requires one of the couplets to be expressed as an inverse ratio. Thus, 8 is to 4 inversely as 3 is to 6, must be written 8 : 4 = 6 : 3; i. e., the second ratio (couplet) must be inverted.

24. Proportion forms one of the most useful sections of arithmetic. In the arithmetics of our grandfathers it was called "The Rule of Three."

25. The test of a proportion is the following principle: *In any proportion, the product of the extremes is equal to the product of the means.*

The truth of this principle may be shown by an example. The proportion $9 : 3 = 51 : 17$ is evidently true, since the ratio of each couplet is three. Expressed in the fractional form, the proportion is $\frac{9}{3} = \frac{51}{17}$. To reduce these fractions to a common denominator, multiply both terms of the first by 17 and both terms of the second by 3; thus, $\frac{9 \times 17}{3 \times 17} = \frac{51 \times 3}{17 \times 3}$. The fractions are still equal, because their values have not been changed by multiplying the numerator and denominator by the same number; and since the denominators are equal, each being the product 3×17, the numerators 9×17 and 51×3 must be equal. But the numerator 9×17 is the product of the extremes, and the numerator 51×3 is the product of the means. Hence, in every case, these products are equal if the four numbers form a proportion.

EXAMPLE.—Can the four numbers, 16, 125, 12, and 94 form a proportion?

SOLUTION.—Taking 16 and 94 for the extremes and 125 and 12 for the means, the product $16 \times 94 = 1{,}504$ and $125 \times 12 = 1{,}500$ are not equal, and the numbers cannot form a proportion. Ans.

26. The problem that most frequently occurs in proportion is to find one of the terms when the other three terms are given. Suppose it is required to find a number that will form a proportion with the numbers 6, 13, and 30; that is, $6 : 13 = 30$: what number? Placing the product of the extremes equal to the product of the means,

$$6 \times \text{what number} = 13 \times 30?$$

Since 6 times the required number is 13×30, the number must be $\frac{1}{6}$ of 13×30; or,

$$\text{Number} = \frac{13 \times 30}{6} = 65.$$

The unknown extreme is therefore equal to the product of the means divided by the known extreme.

27. Rule.—I. *To find an unknown extreme, divide the product of the means by the given extreme.*

II. *To find an unknown mean, divide the product of the extremes by the given mean.*

Example.—What is the third term in the proportion $17 : 51 = x : 42$?

Solution.—In this case, a mean is unknown. The extremes are 17 and 42, and the other mean is 51. Hence,

$$\text{Third term} = \frac{17 \times 42}{51} = 14. \text{ Ans.}$$

Example.—What is the first term in the proportion $: 4 = 21 : 6$?

Solution.—Applying the rule,

$$\text{Unknown term} = 4 \times 21 \div 6 = 14. \text{ Ans.}$$

28. In the statement of a problem, represent the unknown term by the letter x. Thus, in the last example, the proportion would be written $x : 4 = 21 : 6$, and the solution would be written $x = \frac{4 \times 21}{6} = 14$.

29. Proportion is useful in solving a class of problems containing three quantities, of which two are of the same kind and have a certain ratio, and it is required to find a fourth quantity which shall bear the same ratio to the remaining quantity. The following is an example of a problem of this kind:

Example.—If 4 horses can be bought for $250 how much must be paid for 11 horses at the same price?

Solution.—Here the 4 horses and the 11 horses are the two numbers of the same kind having a definite ratio, and we desire to find a fourth number to which $250 shall bear the same relation that 4 horses do to 11 horses.

In solving a problem of this kind the student first asks himself: "What is it I wish to find?" In the present case it is dollars. The 4 horses are bought for a known amount of money, and the 11 horses, at the same price, cost an amount as yet unknown, but to be found. Since the horses are all bought at the same price, it is evident that the

cost of 11 horses bears the same relation to \$250, the cost of 4 horses, that 11 horses do to 4 horses. In other words, the ratio of the cost of 11 horses to the cost of 4 horses is equal to the ratio of 11 horses to 4 horses.

These two equal ratios may be put in the form of a proportion; thus,

Cost of 11 horses : \$250 = 11 horses : 4 horses.

The unknown term is an extreme, and is found by dividing the product of the means by the known extreme.

$$\text{Cost of 11 horses} = \frac{250 \times 11}{4} = \$687.50. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

30. Find the value of x in each of the following:

(*a*)	\$16 : \$64 :: x : 4.		(*a*)	$x = 1$.
(*b*)	x : 85 :: 10 : 17.		(*b*)	$x = 50$.
(*c*)	24 : x :: 15 : 40.		(*c*)	$x = 64$.
(*d*)	18 : 94 :: 2 : x.	Ans.	(*d*)	$x = 10\frac{4}{9}$.
(*e*)	\$75 : \$100 = x : 100.		(*e*)	$x = 75$.
(*f*)	15 pwt. : x = 21 : 10.		(*f*)	$x = 7\frac{1}{7}$ pwt.
(*g*)	x : 75 yd. = \$15 : \$5.		(*g*)	$x = 225$ yd.

1. If 75 pounds of lead cost \$2.10, what would 125 pounds cost at the same rate? Ans. \$3.50.

2. If A does a piece of work in 4 days and B does it in 7 days, how long will it take A to do what B does in 63 days? Ans. 36 days.

3. The circumferences of any two circles are to each other as their diameters. If the circumference of a circle 7 inches in diameter is 22 inches, what is the circumference of a circle 31 inches in diameter? Ans. $97\frac{3}{7}$ inches.

INVERSE PROPORTION.

31. In the example given in Art. **29,** the proportion was direct. The *greater* the number of horses bought, the *greater* must be the amount paid for them; that is, the cost of the horses varies *directly* as the number of horses. In some cases, the nature of the problem requires an inverse proportion. Such a case may be best shown by an example.

Example.—If 6 men can do a piece of work in 8 days, in what time can 18 men do it, working at the same rate?

Solution.—It is clear that by employing 18 men instead of 6, the work will be done in *less* time; that is, the *greater* the number of men

the *less* the time required to do the work. The time varies *inversely* as the number of men, and the statement of the problem requires an inverse proportion. The ratio of the number of days required by 18 men to the 8 days required by 6 men is equal to the *inverse* ratio of 18 men to 6 men. Denoting the unknown number of days by x and writing the proportion as a direct proportion, the result is

$$6 : 18 = 8 : x.$$

But since the proportion is an inverse one, one of the couplets must be inverted. If the first couplet is inverted, the result is $18 : 6 = 8 : x$; if the second, $6 : 18 = x : 8$. In either case,

$$x = \frac{8 \times 6}{18} = 2\tfrac{2}{3} \text{ days. Ans.}$$

32. Sometimes the word *inverse* occurs in the statement of the example; in such cases the proportion can be written at once, merely inverting one of the couplets. But it frequently happens that only by carefully studying the conditions of the example, can it be ascertained whether the proportion is direct or inverse. When in doubt, the student can always satisfy himself as to whether the proportion is direct or inverse by first ascertaining what is required, and stating the proportion as a direct proportion. Then, in order that the proportion may be true, if the first term is smaller than the second term, the third term must be smaller than the fourth; or if the first term is larger than the second term, the third term must be larger than the fourth term. Keeping this in mind, the student can always tell whether the required term will be larger or smaller than the other term of the couplet to which the required term belongs. Having determined this, the student then refers to the example, and ascertains from its conditions whether the required term is to be larger or smaller than the other term of the same kind. If the two determinations agree, the proportion is direct, otherwise it is inverse, and one of the couplets must be inverted.

33. Example.—A carriage wheel, 16 ft. in circumference, makes 200 turns in passing over a certain distance. How many turns will a wheel 10 ft. in circumference make in going the same distance?

Solution.—The quantities considered are the circumferences of the wheels and the number of turns made. Forming a direct proportion:

$$16 : 10 = 200 : x.$$

An examination of this proportion shows that x (the number of turns of the smaller wheel) must be smaller than 200. But x must evidently be larger than 200, since a wheel 10 feet in diameter will make more turns in going a given distance than a wheel 16 feet in diameter. Hence, the proportion is an inverse one. Inverting the second couplet,

$$16 : 10 = x : 200,$$

from which $x = \frac{16 \times 200}{10} = 320$ turns. Ans.

EXAMPLE.—If A's *rate* of doing work is to B's as 5 : 7, and A does a piece of work in 42 days, in what time will B do it?

SOLUTION.—The required term is the number of days it will take B to do the work. Hence, stating as a direct proportion,

$$5 : 7 = 42 : x.$$

Now, since 7 is greater than 5, x will be greater than 42. But, referring to the statement of the example, it is easy to see that B works faster than A; hence it will take B a less number of days to do the work than A. Therefore, the proportion is an inverse one, and should be stated,

$$5 : 7 = x : 42,$$

from which $x = \frac{5 \times 42}{7} = 30$ days. Ans.

Had the example been stated thus: The time that A requires to do a piece of work is to the time that B requires as 5 : 7; A can do it in 42 days, in what time can B do it? it is evident that it would take B a longer time to do the work than it would A; hence, x would be greater than 42, and the proportion would be direct, the value of x being $\frac{7 \times 42}{5} = 58.8$ days.

34. The unknown term x may occupy the place of any term in a proportion without destroying the proportion. Thus, consider the example: If 4 men earn \$25 in one week, how much will 12 men earn in the same time? The proportion may be formed in any of the four following ways:

$$x : \$25 = 12 \text{ men} : 4 \text{ men};$$
$$\$25 : x = 4 \text{ men} : 12 \text{ men};$$
$$12 \text{ men} : 4 \text{ men} = x : \$25;$$
$$4 \text{ men} : 12 \text{ men} = \$25 : x.$$

From any one of the above proportions,

$$x = \frac{\$25 \times 12}{4} = \$75.$$

EXAMPLES FOR PRACTICE.

35. Solve the following:

1. If a pump which discharges 4 gal. of water per min. can fill a tank in 20 hr., how long will it take a pump discharging 12 gal. per min. to fill it? Ans. $6\frac{2}{3}$ hr.

2. If a pump discharges 90 gal. of water in 20 hr., in what time will it discharge 144 gal.? Ans. 32 hr.

3. The weight of any gas (the volume and pressure remaining the same) varies inversely as the absolute temperature. If a certain quantity of some gas weighs 2.927 lb. when the absolute temperature is 525°, what will the same volume of gas weigh when the absolute temperature is 600°, the pressure remaining the same? Ans. 2.561+ lb.

4. If 50 cu. ft. of air weigh 4.2 pounds when the absolute temperature is 562°, what will be the absolute temperature when the same volume weighs 5.8 pounds, the pressure being the same in both cases? Ans. 407°, very nearly.

5. If a man can make a journey in 15 days traveling 10 hours per day, how long will it take him if he travels 8 hours per day? Ans. $18\frac{3}{4}$ da.

6. A merchant sells $4,500 worth of goods and gains $1,300. At the same rate, what would he gain if he sold $8,000 worth? Ans. $2,311$\frac{1}{9}$.

7. If 14 men can dig a cellar in 12 days, how long will it take 8 men to dig it? Ans. 21 da.

PROPERTIES OF PROPORTION.

36. If the same operations (addition and subtraction excepted) be performed upon *all* the terms of a proportion, the proportion is not thereby destroyed. In other words, if all the terms of a proportion be (1) multiplied or (2) divided by the same number; (3) if all the terms be raised to the same power; if (4) the same root of all the terms be taken, or (5) if both couplets be inverted, the proportion still holds. We will prove these statements by a numerical example, and the student can satisfy himself by other similar ones. The fractional form will be used, as it is better suited to the purpose. Consider the proportion $8 : 4 = 6 : 3$. Expressing it in the fractional form, it becomes $\frac{8}{4} = \frac{6}{3}$. What we are to

prove is that if any of the five operations enumerated above be performed upon all the terms of this proportion, the first fraction will still equal the second fraction.

1. Multiplying all the terms by any number, say 7, $\frac{8\times7}{4\times7} = \frac{6\times7}{3\times7}$; or $\frac{56}{28} = \frac{42}{21}$. Now $\frac{56}{28}$ evidently equals $\frac{42}{21}$, since the value of either ratio is 2, and the same is true of the original proportion.

2. Dividing all the terms by any number, say 7, $\frac{8\div7}{4\div7} = \frac{6\div7}{3\div7}$; or $\frac{\frac{8}{7}}{\frac{4}{7}} = \frac{\frac{6}{7}}{\frac{3}{7}}$. But $\frac{8}{7}\div\frac{4}{7} = 2$, and $\frac{6}{7}\div\frac{3}{7} = 2$ also, the same as in the original proportion.

3. Raising all the terms to the same power, say the cube, $\frac{8^3}{4^3} = \frac{6^3}{3^3}$. This is evidently true, since $\frac{8^3}{4^3} = \left(\frac{8}{4}\right)^3 = 2^3 = 8$, and $\frac{6^3}{3^3} = \left(\frac{6}{3}\right)^3 = 2^3 = 8$ also.

4. Extracting the same root of all the terms, say the cube root, $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \frac{\sqrt[3]{6}}{\sqrt[3]{3}}$. It is evident that this is likewise true, since $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \sqrt[3]{\frac{8}{4}} = \sqrt[3]{2}$, and $\frac{\sqrt[3]{6}}{\sqrt[3]{3}} = \sqrt[3]{\frac{6}{3}} = \sqrt[3]{2}$ also.

5. Inverting both couplets, $\frac{4}{8} = \frac{3}{6}$, which is true, since each equals $\frac{1}{2}$.

37. If both terms of either couplet be multiplied or both be divided by the same number, the proportion is not destroyed. This should be evident from the preceding article, and also from Art. **7.** Hence, in any proportion, equal factors may be canceled from the terms of a couplet, before applying the rule of Art. **27.** Thus, in the proportion $45 : 9 = x : 7.1$, we may divide both terms of the first couplet by 9 (that is, cancel 9 from both terms), obtaining $5 : 1 = x : 7.1$; whence, $x = 7.1\times5\div1 = 35.5$. (See note in Art. **13.**)

POWERS AND ROOTS IN PROPORTION.

38. It was stated in Art. **12** that a ratio may be raised to any power, or any root of it may be taken. A proportion is frequently stated in such a manner that one of the couplets must be raised to some power or some root of it must be taken. In all such cases, both terms of the couplet so affected *must be raised to the same power, or the same root of both terms must be taken.*

39. EXAMPLE.—Knowing that the weight of a sphere varies as the cube of its diameter, what is the weight of a sphere 6 inches in diameter if a sphere 8 inches in diameter of the same material weighs 180 pounds?

SOLUTION.—This is evidently a direct proportion. Hence, we write

$$6^3 : 8^3 = x : 180.$$

Dividing both terms of the first couplet by 2 (See Art. **13**),

$$3^3 : 4^3 = x : 180, \text{ or } 27 : 64 = x : 180;$$

whence, $x = \dfrac{27 \times 180}{64} = 75\tfrac{15}{16}$ pounds. Ans.

EXAMPLE.—A sphere 8 inches in diameter weighs 180 pounds; what is the diameter of another sphere of the same material which weighs $75\tfrac{15}{16}$ pounds?

SOLUTION.—Since the weights of any two spheres are to each other as the cubes of their diameters, we have the proportion

$$180 : 75\tfrac{15}{16} = 8^3 : x^3$$

x, the required term must be cubed, because the other term of the couplet is cubed (see Art. **38**). But, $8^3 = 512$; hence,

$$180 : 75\tfrac{15}{16} = 512 : x^3, \text{ or } x^3 = \frac{75\tfrac{15}{16} \times 512}{180} = 216;$$

whence, $x = \sqrt[3]{216} = 6$ inches. Ans.

40. Since taking the same root of all the terms of a proportion does not change its value (Art. **36**), the above example might have been solved by extracting the cube root of all of the numbers, thus obtaining $\sqrt[3]{180} : \sqrt[3]{75\tfrac{15}{16}} = 8 : x$; whence, $x = \dfrac{8 \times \sqrt[3]{75\tfrac{15}{16}}}{\sqrt[3]{180}} = 8 \times \sqrt[3]{\dfrac{75\tfrac{15}{16}}{180}} = 8\sqrt[3]{\dfrac{1,215}{2,880}} = 8\sqrt[3]{\dfrac{27}{64}}$ $= 8 \times \tfrac{3}{4} = 6$ inches. The process, however, is longer and is not so direct, and the first method is to be preferred.

41. If two cylinders have *equal* volumes, but different diameters, the diameters are to each other inversely as the square roots of their lengths. Hence, if it is desired to find the diameter of a cylinder that is to be 15 inches long, and which shall have the same volume as one that is 9 inches in diameter and 12 inches long, we write the proportion

$$9 : x = \sqrt{15} : \sqrt{12}.$$

Since neither 12 nor 15 is a perfect square, we square all of the terms (Arts. **40** and **36**) and obtain

$$81 : x^2 = 15 : 12; \text{ whence, } x^2 = \frac{81 \times 12}{15} = 64.8,$$

and $x = \sqrt{64.8} = 8.05$ inches = diameter of 15-in. cylinder.

EXAMPLES FOR PRACTICE.

42. Solve the following examples:

1. The intensity of light varies inversely as the square of the distance from the source of light. If a gas jet illuminates an object 30 feet away with a certain distinctness, how much brighter will the object be at a distance of 20 feet? Ans. $2\frac{1}{4}$ times as bright.

2. In the last example, suppose that the object had been 40 feet from the gas jet; how bright would it have been compared with its brightness at 30 feet from the gas jet? Ans. $\frac{9}{16}$ as bright.

3. When comparing one light with another, the intensities of their illuminating powers vary as the squares of their distances from the objects they illuminate. If a man can just distinguish the time indicated by his watch, 50 feet from a certain light, at what distance could he just distinguish the time by a light 3 times as powerful. Ans. 86.6+ feet.

4. The quantity of air flowing through a mine varies directly as the square root of the pressure. If 60,000 cubic feet of air flow per minute when the pressure is 2.8 pounds per square foot, how much will flow when the pressure is 3.6 pounds per square foot? Ans. 68,034 cu. ft. per min., nearly.

5. In the last example, suppose that 70,000 cubic feet per minute had been required; what would be the pressure necessary for this quantity? Ans. 3.81+ lb. per sq. ft.

CAUSE AND EFFECT.

43. Many examples in proportion may be more easily solved by using the principle of *cause and effect.* That which may be regarded as producing a change or alteration in something, or as accomplishing something, may be called the **cause,** and the change or alteration, or thing accomplished, the **effect.**

44. *Like causes produce like effects.* Hence, when two causes of the same kind produce two effects of the same kind, the ratio of the causes equals the ratio of the effects; in other words, the ratio of the first cause to the second cause equals the ratio of the first effect to the second effect. Thus, in the question, if 3 men can lift 1,400 pounds, how many pounds can 7 men lift? we call 3 men and 7 men the *causes* (since they accomplish something, viz., the lifting of the weight), the number of pounds lifted, viz., 1,400 pounds and x pounds, are the effects. If we call 3 men the first cause, 1,400 pounds is the first effect; 7 men is the second cause and x pounds is the second effect. Hence, we may write

$$\begin{array}{ccccccc} \textit{1st cause} & & \textit{2d cause} & & \textit{1st effect} & & \textit{2d effect} \\ 3 & : & 7 & = & 1{,}400 & : & x \end{array}$$

whence $x = \dfrac{7 \times 1{,}400}{3} = 3{,}266\frac{2}{3}$ pounds.

45. The principle of cause and effect is extremely useful in the solution of examples in compound proportion, as we shall now show.

COMPOUND PROPORTION.

46. All the cases of proportion so far considered have been cases of **simple proportion;** i. e., each term has been composed of but one number. There are many cases, however, in which two or all of the terms have more than one number in them; all such cases belong to **compound proportion.** In all examples in compound proportion, both causes or both effects, or all four, consist of more than two numbers. We will illustrate this by an example.

EXAMPLE.—If 40 men earn \$1,280 in 16 days how much will 36 men earn in 31 days?

SOLUTION.—Since 40 men earn something, 40 men is a cause, and since they take 16 days in which to earn that something, 16 days is an element of the cause. For the same reason, 36 men working for 31 days is also a cause. The effects, that which is earned, are 1,280 dollars and x dollars. Then, 40 men and 16 days make up the first cause, and 36 men and 31 days make up the second cause. \$1,280 is the first effect and $\$x$ is the second effect. Hence, we write

$$\begin{array}{ccccccc} \textit{1st cause} & & \textit{2d cause} & & \textit{1st effect} & & \textit{2d effect} \\ \begin{array}{c}40\\16\end{array} & : & \begin{array}{c}36\\31\end{array} & = & 1{,}280 & : & x \end{array}$$

Now, instead of using the colon to express the ratio, we shall use the vertical line (see Art. 2), and the above becomes

$$\begin{array}{c}40\\16\end{array} \left| \begin{array}{c}36\\31\end{array} = 1{,}280 \right| x.$$

In the last expression, the product of all of the numbers included between the vertical lines must equal the product of all the numbers without them; i. e., $36 \times 31 \times 1{,}280 = 40 \times 16 \times x$.

$$\text{Or, } x = \frac{36 \times 31 \times \overset{\overset{2}{\cancel{80}}}{\cancel{1280}}}{\cancel{40} \times \cancel{16}} = \$2{,}232. \quad \text{Ans.}$$

47. The above might have been solved by canceling factors of the numbers in the original proportion. For, if any number within the lines has a factor common to any number without the lines, that factor may be canceled from both numbers. Thus, 16 is contained in

$$\begin{array}{c}\cancel{40}\\\cancel{16}\end{array} \left| \begin{array}{c}36\\31\end{array} = \begin{array}{r}2\\\cancel{80}\\\cancel{1280}\end{array} \right| x,$$

1,280, 80 times. Cancel 16 and 1,280, and write 80 above 1,280. 40 is contained in 80, 2 times. Cancel 40 and 80, and write 2 above 80. Now, since there are no more numbers that can be canceled, $x = 36 \times 31 \times 2 = \$2{,}232$, the same result as was obtained in the last article.

48. Rule.—*Write all the numbers forming the first cause in a vertical column, on the left of a vertical line; on the other side of this line write in a vertical column all the numbers forming the second cause. Write the sign of equality to*

the right of the second column, and on the right of this form a third column of the numbers composing the first effect, drawing a vertical line to the right; on the other side of this line, write for the fourth column the numbers composing the second effect. There must be as many numbers in the second cause as in the first cause, and in the second effect as in the first effect; hence, if any term is wanting, write x in its place. Multiply together all of the numbers within the vertical lines, and also all those without the lines (canceling previously, if possible), and divide the product of those numbers which do not contain x by the product of the others in which x occurs, and the result will be the value of x.

49. Example.—If 40 men can dig a ditch 720 feet long, 5 feet wide, and 4 feet deep in a certain time, how long a ditch 6 feet deep and 3 feet wide can 24 men dig in the same time?

Solution.—Here 40 men and 24 men are the causes and the two ditches are the effects. Hence,

$$\begin{array}{c|rr|l} & & \cancel{3} & \\ & & \cancel{18} & \\ & & \cancel{720} & x \\ \cancel{40} & 24 = & 5 & \cancel{3} \\ & & 4 & \cancel{6} \end{array} \quad \text{whence, } x = 24 \times 5 \times 4 = 480 \text{ feet. Ans.}$$

50. Example.—The volume of a cylinder varies directly as its length and directly as the square of its diameter. If the volume of a cylinder 10 inches in diameter and 20 inches long is 1,570.8 cubic inches, what is the volume of another cylinder 16 inches in diameter and 24 inches long?

Solution.—In this example, either the dimensions or the volumes may be considered the causes; say we take the dimensions for the causes. Then, squaring the diameters,

$$\begin{array}{c|cl|c} 10^2 & 16^2 & = 1{,}570.8 & x, \text{ or } \\ 20 & 24 & & \end{array} \quad \begin{array}{r|cl|c} 100 & 256 & = 1{,}570.8 & x; \\ \cancel{20} & \cancel{24} & & \\ 5 & 6 & & \end{array}$$

$$\text{whence, } x = \frac{256 \times 6 \times 1{,}570.8}{5 \times 100} = 4{,}825.4976 \text{ cubic inches. Ans.}$$

51. Example.—If a block of granite 8 ft. long, 5 ft. wide, and 3 ft. thick weighs 7,200 lb., what is the weight of a block of granite 12 ft. long, 8 ft. wide, and 5 ft. thick?

Solution.—Taking the weights as the effects, we have

$$\begin{array}{c|cl|l} & 4 & & \\ \cancel{8} & \cancel{12} & & \\ \cancel{5} & \cancel{8} & = 7{,}200 & x, \text{ or } x = 4 \times 7{,}200 = 28{,}800 \text{ pounds. Ans.} \\ \cancel{3} & \cancel{5} & & \end{array}$$

52. Example.—If 12 compositors in 30 days of 10 hours each set up 25 sheets of 16 pages each, 32 lines to the page, in how many days 8 hours long can 18 compositors set up, in the same type, 64 sheets of 12 pages each, 40 lines to the page?

Solution.—Here compositors, days, and hours compose the causes, and sheets, pages, and lines, the effects. Hence,

$$\begin{array}{c|cc|l}
3 & \cancel{3} & \cancel{5} & 2 \\
\cancel{12} & \cancel{18} & \cancel{25} & \cancel{64} \\
 & & \cancel{4} & \\
\cancel{30} & x & = \cancel{16} & \cancel{12}, \text{ or } x = 3 \times 10 \times 2 = 60 \text{ days. Ans.} \\
\cancel{6} & & & \cancel{4} \\
10 & \cancel{8} & \cancel{32} & \cancel{40} \\
 & & & \cancel{5}
\end{array}$$

53. In examples stated like that in Art. **50,** should an inverse proportion occur, write the various numbers as in the preceding examples, and then transpose those numbers which are said to vary inversely, from one side of the vertical line to the other side.

Example.—The centrifugal force of a revolving body varies directly as its weight, as the square of its velocity, and inversely as the radius of the circle described by the center of the body. If the centrifugal force of a body weighing 15 pounds is 187 pounds when the body revolves in a circle having a radius of 12 inches, with a velocity of 20 feet per second, what will be the centrifugal force of the same body when the radius is increased to 18 inches and the speed is increased to 24 feet per second?

Solution.—Calling the centrifugal force the effect, we have,

$$\begin{array}{c|l|l}
15 & 15 & \\
20^2 & 24^2 = 187 & x. \\
12 & 18 &
\end{array}$$

Transposing 12 and 18 (since the radii are to vary inversely) and squaring 20 and 24,

$$\begin{array}{c|l|l}
\cancel{15} & \cancel{15} & \\
 & 2 & \\
25 & \cancel{36} = 187 & x, \text{ or } x = \dfrac{12 \times 2 \times 187}{25} = 179.52 \text{ pounds. Ans.} \\
\cancel{400} & \cancel{576} & \\
\cancel{18} & 12 &
\end{array}$$

EXAMPLES FOR PRACTICE.

54. Solve the following by compound proportion:

1. If 12 men dig a trench 40 rods long in 24 days of 10 hours each, how many rods can 16 men dig in 18 days of 9 hours each? Ans. 36 rods.

2. If a piece of iron 7 feet long, 4 inches wide, and 6 inches thick weighs 600 pounds, how much will a piece of iron weigh that is 16 feet long, 8 inches wide, and 4 inches thick? Ans. $1{,}828\frac{4}{7}$ lb.

3. If 24 men can build a wall 72 rods long, 6 feet wide, and 5 feet high in 60 days of 10 hours each, how many days will it take 32 men to build a wall 96 rods long, 4 feet wide, and 8 feet high, working 8 hours a day? Ans. 80 days.

4. The horsepower of an engine varies as the mean effective pressure, as the piston speed, and as the square of the diameter of the cylinder. If an engine having a cylinder 14 inches in diameter develops 112 horsepower when the mean effective pressure is 48 pounds per square inch and the piston speed is 500 feet per minute, what horsepower will another engine develop, if the cylinder is 16 inches in diameter, piston speed is 600 feet per minute, and mean effective pressure is 56 pounds per square inch? Ans. 204.8 horsepower.

5. Referring to the example in Art. **50**, what will be the volume of a cylinder 20 inches in diameter and 24 inches long?

Ans. 7,539.84 cubic inches.

6. Knowing that the product of $3 \times 5 \times 7 \times 9$ is 945, what is the product of $6 \times 15 \times 14 \times 36$? Ans. 45,360.

7. If 60 yards of carpet $\frac{3}{4}$ of a yard wide will cover a room 15 feet wide and 27 feet long, how many yards of carpet $\frac{7}{8}$ of a yard wide will cover a room 18 feet wide and 35 feet long? Ans. 80 yards.

8. If it costs $72.50 to ship 9,000 pounds of freight a distance of 850 miles, how much will it cost to ship 22,000 pounds 675 miles at the same rate per mile? Ans. $140.74, nearly.

PROPORTIONAL PARTS.

55. Proportion may be used to divide a number into two or more parts having a given ratio to each other. For example, it is required to divide the number 40 into two parts which shall be in the ratio of 3 to 5. Adding these proportional parts, $3 + 5 = 8$; then the ratio of the proportional part 3 to the sum 8 is the same as the ratio of the smaller of the two numbers whose sum is 40 to 40.

That is, $3 : 8 = \text{smaller number} : 40$;

similarly, $5 : 8 = \text{larger number} : 40$.

From these proportions,

$$\text{smaller number} = \frac{3 \times 40}{8} = 15;$$

$$\text{larger number} = \frac{5 \times 40}{8} = 25.$$

56. Rule.—*Form a proportion of which one of the proportional parts is the first term, the sum of the proportional parts is the second term, the unknown part is the third term, and the number to be divided into parts is the fourth term. Form as many of these proportions as there are proportional parts, and solve. The results will be the parts required.*

EXAMPLE.—If \$6,900 is divided among four men, A, B, C, and D, in the proportion of 3, 4, 7, and 9, how much does each receive?

SOLUTION.—Sum of proportional parts $= 3+4+7+9 = 23$. The four proportions are:

$$3 : 23 = \text{A's share} : \$6{,}900$$
$$4 : 23 = \text{B's share} : \$6{,}900$$
$$7 : 23 = \text{C's share} : \$6{,}900$$
$$9 : 23 = \text{D's share} : \$6{,}900$$

Hence,

$$\text{A's share} = \frac{3 \times \$6{,}900}{23} = \$900$$
$$\text{B's share} = \frac{4 \times \$6{,}900}{23} = \$1{,}200$$
$$\text{C's share} = \frac{7 \times \$6{,}900}{23} = \$2{,}100$$
$$\text{D's share} = \frac{9 \times \$6{,}900}{23} = \$2{,}700. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

57. Solve the following:

1. Divide 1,350 into parts proportional to the numbers 4, 5, 6.
Ans. 360, 450, 540.

2. Divide 1,630 pounds into parts proportional to the fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$. Ans. 300 lb., 400 lb., 450 lb., and 480 lb.

SUGGESTION.—Reduce the fractions to least common denominator and divide the number into parts proportional to the numerators.

3. Gunpowder is composed of saltpeter, charcoal, and sulphur in the proportion of 75, $12\frac{1}{2}$, and $12\frac{1}{2}$. How many pounds of each of these substances in 1,473 pounds of powder?

Ans. $\begin{cases} \text{Saltpeter, } 1{,}104\frac{3}{4} \text{ lb.} \\ \text{Charcoal, } 184\frac{1}{8} \text{ lb.} \\ \text{Sulphur, } 184\frac{1}{8} \text{ lb.} \end{cases}$

4. A miller divides 1,980 bushels of wheat among three bins in the proportion of 2, 3, and 4. How many bushels does each bin contain?
Ans. 440 bu., 660 bu., 880 bu.

ARITHMETIC.

PERCENTAGE.

DEFINITIONS AND PRINCIPLES.

1. In certain operations pertaining to business, it is very convenient to regard the quantity on which we are to operate as being divided into 100 equal parts; thus, instead of using the ordinary fractions $\frac{1}{4}$, $\frac{3}{5}$, $\frac{2}{7}$, we use the equivalent fractions $\frac{25}{100}$, $\frac{60}{100}$, $\frac{28\frac{4}{7}}{100}$, or their equivalent decimals, .25, .60, .28$\frac{4}{7}$. This practice is a very convenient one in all computations involving United States money, because, since $1 equals 100 cents, it is easier to comprehend what part of the whole $\frac{35}{100}$ is than some other equivalent fraction, as $\frac{49}{140}$; it is also much easier to compute with fractions whose denominators are 100 than it is to compute with fractions whose denominators are composed of other figures.

2. **Percentage** is a term applied to those arithmetical operations in which the number or quantity to be operated upon is supposed to be divided into 100 equal parts.

3. The term **per cent.** means *by the hundred.* Thus, 8 per cent. of a number means 8 hundredths, i. e., $\frac{8}{100}$, or .08, of that number; 8 per cent. of 250 is $250 \times \frac{8}{100}$, or $250 \times .08 = 20$; 47 per cent. of 75 bushels is $75 \times \frac{47}{100} = 75 \times .47 = 35.25$ bushels. The statement that the population of a city has increased 22 per cent. in a given time, say from 1880 to 1890, is equivalent to saying that the increase is 22 in

every hundred; that is, for every 100 in 1880, there are 22 more, or 122, in 1890.

4. The **sign** of per cent. is %, and is read *per cent.* Thus, 6% is read six per cent.; $12\frac{1}{2}$% is read twelve and one-half per cent., etc.

5. When expressing the per cent. of a number to use in calculations, it is customary to express it decimally instead of fractionally. Thus, instead of expressing 6%, 25%, and 43% as $\frac{6}{100}$, $\frac{25}{100}$, and $\frac{43}{100}$, it is usual to express them as .06, .25, and .43.

6. The following table will show how many per cent. can be expressed either as a decimal or as a fraction:

Per Cent.	Decimal.	Fraction.	Per Cent.	Decimal.	Fraction.
1%	.01	$\frac{1}{100}$	$\frac{1}{4}$%	.0025	$\frac{\frac{1}{4}}{100}$ or $\frac{1}{400}$
2%	.02	$\frac{2}{100}$ or $\frac{1}{50}$	$\frac{1}{2}$%	.005	$\frac{\frac{1}{2}}{100}$ or $\frac{1}{200}$
5%	.05	$\frac{5}{100}$ or $\frac{1}{20}$	$1\frac{1}{2}$%	.015	$\frac{1\frac{1}{2}}{100}$ or $\frac{3}{200}$
10%	.10	$\frac{10}{100}$ or $\frac{1}{10}$	$6\frac{1}{4}$%	$.06\frac{1}{4}$	$\frac{6\frac{1}{4}}{100}$ or $\frac{1}{16}$
25%	.25	$\frac{25}{100}$ or $\frac{1}{4}$	$8\frac{1}{3}$%	$.08\frac{1}{3}$	$\frac{8\frac{1}{3}}{100}$ or $\frac{1}{12}$
50%	.50	$\frac{50}{100}$ or $\frac{1}{2}$	$12\frac{1}{2}$%	.125	$\frac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$
75%	.75	$\frac{75}{100}$ or $\frac{3}{4}$	$16\frac{2}{3}$%	$.16\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}$ or $\frac{1}{6}$
100%	1.00	$\frac{100}{100}$ or 1	$33\frac{1}{3}$%	$.33\frac{1}{3}$	$\frac{33\frac{1}{3}}{100}$ or $\frac{1}{3}$
125%	1.25	$\frac{125}{100}$ or $1\frac{1}{4}$	$37\frac{1}{2}$%	$.37\frac{1}{2}$	$\frac{37\frac{1}{2}}{100}$ or $\frac{3}{8}$
150%	1.50	$\frac{150}{100}$ or $1\frac{1}{2}$	$62\frac{1}{2}$%	.625	$\frac{62\frac{1}{2}}{100}$ or $\frac{5}{8}$
500%	5.00	$\frac{500}{100}$ or 5	$87\frac{1}{2}$%	.875	$\frac{87\frac{1}{2}}{100}$ or $\frac{7}{8}$

7. The names of the different terms used in percentage are: the *base*, the *rate* or *rate per cent.*, the *percentage*, the *amount*, and the *difference.*

8. The **base** is the number or quantity which is supposed to be divided into 100 equal parts.

9. The **rate per cent.** is that number of the 100 equal parts into which the base is supposed to be divided, that is taken or considered. The **rate** is the number of hundredths of the base, that is taken or considered. The distinction between the rate per cent. and the rate is this: the *rate per*

cent. is always 100 times the *rate.* Thus, 7% of 125 and .07 of 125 amount in the end to the same thing; the former, 7, is the *rate per cent.*—the *number* of hundredths of 125 intended; the latter, .07, is the *rate*, the *part* of 125 that is to be found; 7% is used in *speech*, .07 is the form used in *computation.* So, also, $12\frac{1}{2}\% = .125$, $\frac{1}{2}\% = .005$, $1\frac{3}{4}\% = .0175$.

10. The **percentage** is the result obtained by multiplying the base by the rate. Thus, 7% of $125 = 125 \times .07 = 8.75$, the percentage.

11. The **amount** is the sum of the base and the percentage.

12. The **difference** is the remainder obtained when the percentage is subtracted from the base.

13. The terms amount and difference are ordinarily used when there is an increase or a decrease in the base. For example, suppose the population of a village is 1,500 and it increases 25 per cent. This means that for every 100 of the original 1,500 there is an increase of 25, or a total increase of $15 \times 25 = 375$. This increase added to the original population gives the *amount*, or the population after the increase. If the population had decreased 375, the final population would have been $1{,}500 - 375 = 1{,}125$, and this would be the *difference.* The original population, 1,500, is the base on which the percentage is computed; the 25 is the rate per cent., and the increase or decrease, 375, is the percentage. If the base increases, the final value is the amount, and if it decreases, its final value is the difference.

BASE, RATE, AND PERCENTAGE.

14. **Rule.**—*To find the percentage, multiply the base by the rate.*

Example.—A farmer raised 650 bushels of wheat and sold 64% of it. How many bushels did he sell?

Solution.—The base is 650 bushels. Out of every 100 bushels raised 64 were sold; that is, the number of bushels sold was $\frac{64}{100}$ or .64 of the number raised.

$650 \times .64 = 416$ bushels, the percentage. Ans.

15. The rate per cent. also may be used in computation; it is then easy to see that it bears the same relation to 100 that the percentage does to the base. Thus, in the example just solved, the ratio 64% : 100% is equal to the ratio 416 : 650; that is,

$$64 : 100 = 416 : 650.$$

The rule of Art. **14** may be expressed by the proportion,

$$\text{rate per cent.} : 100 = \text{percentage} : \text{base}.$$

EXAMPLE.—A merchant gains 18% on goods that cost him $3,500. How many dollars does he gain?

SOLUTION.—Base = $3,500, rate per cent. = 18.

Let x represent the percentage or gain. Then $18 : 100 = x : \$3{,}500$,

or,

$$x = \frac{3{,}500 \times 18}{100} = \$630. \quad \text{Ans.}$$

16. Let both terms of the first couplet of the second proportion in Art. **15** be divided by 100 (see Art. **37**, § 7); the resulting proportion is

$$\frac{\text{rate per cent.}}{100} : 1 = \text{percentage} : \text{base}.$$

Dividing the rate per cent. by 100 gives the rate. Thus, if the rate per cent. is 64, dividing by 100 gives $\frac{64}{100} = .64$, the rate. Hereafter the rate will generally be used instead of the rate per cent., in which case, the above proportion will be used in the form

$$\text{rate} : 1 = \text{percentage} : \text{base}.$$

This proportion is very important, since it is the foundation of all calculations in percentage.

17. Placing the product of the means equal to the product of the extremes,

$$\text{percentage} = \text{base} \times \text{rate}.$$

Each extreme is equal to the product of the means divided by the other extreme. Hence,

$$\text{rate} = \frac{\text{percentage}}{\text{base}}, \text{ or } R = \frac{P}{B},$$

and

$$\text{base} = \frac{\text{percentage}}{\text{rate}}, \text{ or } B = \frac{P}{R}.$$

18. Rule.—*To find the rate, divide the percentage by the base.*

Example.—Bought 300 bushels of apples and sold 228 bushels. What per cent. of the number of bushels bought was sold?

Solution.—Here 300 is the base and 228 is the percentage; hence, applying rule,

$$\text{rate} = 228 \div 300 = .76 = 76\%. \quad \text{Ans.}$$

Example.—What per cent. of 875 is 25?

Solution.—Here 875 is the base, and 25 is the percentage; hence, applying rule,

$$25 \div 875 = .02\tfrac{6}{7} = 2\tfrac{6}{7}\%. \quad \text{Ans.}$$

Proof.— $875 \times .02\frac{6}{7} = 25.$

19. Rule.—*To find the base when the percentage and rate are given, divide the percentage by the rate.*

Example.—Bought a certain number of bushels of apples and sold 76% of them. If I sold 228 bushels, how many bushels did I buy?

Solution.—Here 228 is the percentage, and .76 is the rate; hence, applying the rule,

$$228 \div .76 = 300 \text{ bushels}. \quad \text{Ans.}$$

Any problem coming under these three rules can be solved by the proportion given in Art. **16.** Thus, in the last example,

$$.76 : 1 = 228 : \text{base};$$

$$\text{base} = \frac{1 \times 228}{.76} = 300 \text{ bushels}. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

20. What is

(*a*) 36% of 1,762?	Ans.	(*a*) 634.32.
(*b*) 19% of $89?		(*b*) $16.91.
(*c*) 47% of 2,400 bushels?		(*c*) 1,128 bushels.
(*d*) 113% of $1,640?		(*d*) $1,853.20.
What per cent. of		
(*e*) 360 is 90?	Ans.	(*e*) 25%.
(*f*) $900 is $360?		(*f*) 40%.
(*g*) 125 is 25?		(*g*) 20%.
(*h*) 150 is 750?		(*h*) 500%.
(*i*) 280 horses is 112 horses?		(*i*) 40%.
(*j*) 400 is 200?		(*j*) 50%.
(*k*) 47 is 94?		(*k*) 200%.
(*l*) 500 days is 250 days?		(*l*) 50%.
(*m*) 42 is 6% of what number?	Ans.	(*m*) 700.
(*n*) 126 is $31\frac{1}{2}$% of what number?		(*n*) 400.
(*o*) 198 is 36% of what number?		(*o*) 550.

ALIQUOT PARTS IN PERCENTAGE.

21. When the rate per cent. is an aliquot part of 100 per cent., it is more convenient to use the equivalent fraction in computations. The principal aliquot parts are shown in the table, Art. **6.**

EXAMPLE.—What is $16\frac{2}{3}\%$ of 684?

SOLUTION.— $16\frac{2}{3}\% = \frac{1}{6}$. $684 \times \frac{1}{6} = 114$. Ans.

EXAMPLE.— 65 is 25% of what number?

SOLUTION.— $25\% = \frac{1}{4}$; 65 is $\frac{1}{4}$ of $65 \times 4 = 260$. Ans.

The student should become familiar with the most common aliquot parts of 100, and use the fractions wherever possible in percentage computations.

EXAMPLES FOR PRACTICE.

22. Solve the following by aliquot parts:

(*a*)	What is 50% of 1,964?	Ans.	(*a*)	982.
(*b*)	What is $33\frac{1}{3}\%$ of $630?		(*b*)	$210.
(*c*)	What is $6\frac{1}{4}\%$ of 1,760?		(*c*)	110.
(*d*)	85 is $12\frac{1}{2}\%$ of what number?		(*d*)	680.
(*e*)	625 is $8\frac{1}{3}\%$ of what number?		(*e*)	7,500.

AMOUNT AND DIFFERENCE.

23. To find the relations existing between the amount or difference, and the base and rate, let us consider an example.

EXAMPLE.—In a factory where 2,100 men are employed, the force is increased 8%. How many new men are employed, and how many men are at work after the increase?

SOLUTION.— 8% of $2,100 = 2,100 \times .08 = 168$ men = number of new men employed. Ans. The total number of men after the force is increased is $2,100 + 168 = 2,268$ men. Ans.

In this example, the original number, 2,100, is the base, and the final number, 2,268, is the amount. For every 100 men originally in the shop, there are 8 more men, or 108 men, after the force is increased. At this rate per cent., the

number of men increases from 2,100 to 2,268. Therefore, the ratio of 108 to 100 is the same as the ratio of 2,268 to 2,100, or,

$$108 : 100 = 2{,}268 : 2{,}100.$$

Dividing the terms of the first couplet by 100 (Art. **37**, § 7),

$$1.08 : 1 = 2{,}268 : 2{,}100.$$

Since .08 is the rate, we have the proportion

$$1 + \text{rate} : 1 = \text{amount} : \text{base}.$$

24. If, in the above example, the force had been decreased 8%, the number of men thrown out of work would have been 168, and those left at work, 2,100 − 168 = 1,932. As before, the base is 2,100, but the final number, 1,932, is the difference, since the base has decreased. Out of every 100 men formerly at work, 8 have been discharged, leaving 100 − 8 = 92 still at work. Therefore,

$$92 : 100 = 1{,}932 : 2{,}100,$$

or,

$$.92 : 1 = 1{,}932 : 2{,}100.$$

But .92 = 1 − .08 = 1 − rate; hence,

$$1 - \text{rate} : 1 = \text{difference} : \text{base}.$$

25. Placing the product of the means equal to the product of the extremes, the proportions of Arts. **23** and **24** give the following formulas:

$$\text{amount} = \text{base} \times (1 + \text{rate}), \text{ or } A = B \times (1 + R)$$

$$\text{difference} = \text{base} \times (1 - \text{rate}), \text{ or } D = B \times (1 - R).$$

26. Rule.—*To find the amount, the base and rate being given, multiply the base by 1 plus the rate.*

Example.—The population of a city in 1880 was 43,000, and it increased 65% in the next ten years. What was its population in 1890?

Solution.—In this case, 43,000 is the base, .65 is the rate, and the required population in 1890 is the amount. Applying the rule,

$$\text{amount} = 43{,}000 \times (1 + .65) = 43{,}000 \times 1.65 = 70{,}950. \quad \text{Ans.}$$

27. Rule.—*To find the difference, the base and rate being given, multiply the base by 1 minus the rate.*

Example.—A speculator invested $26,500 in a business enterprise, and lost 16% of his investment. How much had he left?

Solution.—The original capital, $26,500, is the base, and .16 is the

rate. Since the capital is diminished, the portion of it remaining is the difference. Applying the rule,

difference = $26,500 × (1 − .16) = $26,500 × .84 = $22,260. Ans.

28. In the proportions of Arts. **23** and **24,** the fourth term is equal to the product of the means divided by the other extreme. That is,

$$\text{base} = \text{amount} \div (1 + \text{rate}), \text{ or } B = \frac{A}{1+R}.$$

$$\text{base} = \text{difference} \div (1 - \text{rate}), \text{ or } B = \frac{D}{1-R}.$$

29. Rule.—*To find the base, the amount and rate being given, divide the amount by 1 plus the rate.*

Example.—In 1895 the population of a village was 4,130, which was 18% more than the population in 1890. What was the population in 1890?

Solution.—The unknown population in 1890 is the base, upon which the 18% is computed. The final population in 1895 is the amount, since it is an increase. Applying the rule,

base = 4,130 ÷ (1 + .18) = 4,130 ÷ 1.18 = 3,500. Ans.

30. Rule.—*To find the base, the difference and rate being given, divide the difference by 1 minus the rate.*

Example.—A speculator lost 34% of an investment in stocks and had $10,560 remaining. What was the original investment?

Solution.—The original investment, on which the 34% is computed, is the base. Since there has been a loss, or decrease, the remaining $10,560 is the difference. Applying the rule,

base = $10,560 ÷ (1 − .34) = $10,560 ÷ .66 = $16,000. Ans.

31. The difficulty that the student is most likely to experience in percentage is the identification of the terms or elements. When an example is given, he must first determine from it which is the base, the percentage, the amount, etc. The rate is always recognized by the words *per cent.* or by the sign %. The base is the most important element, and it can be identified by referring to the rate. The student asks himself: "Of what number do I wish to find the percentage?" The answer to this question is the base. The percentage is always a number of the same kind as the base. If, from the statement of the problem, it is seen that the base increases or decreases, the percentage is the increase or decrease, and the final value

obtained, when the base has been increased or decreased by the percentage, is the amount or the difference. To be able to recognize the elements immediately requires much practice. As an exercise, several examples are given.

32. Example.— 15 tons of iron are obtained from 282 tons of ore. What per cent. of the ore is iron?

Solution.—Here the statement of the example shows that the rate is what is required. From the phrase, "what per cent. of the ore," we see that the ore is the thing of which a per cent. is taken. Therefore, the 282 tons must be the base. 15 tons is a number of the same kind as the base, and is the part of the base corresponding to the rate. It is, therefore, the percentage.

Rate = percentage ÷ base = 15 ÷ 282 = .05319 = 5.319%. Ans.

Example.—Out of a cargo of oranges, 8% spoiled and 4,600 boxes remained. How many boxes in the cargo? How many boxes spoiled?

Solution.—The rate is .08; .08 of what? The number of boxes in the cargo. Therefore, the base is the original number of boxes and the less number of boxes must be the difference. The number of boxes spoiled is the percentage.

Base = difference ÷ (1 − rate)
= 4,600 ÷ (1 − .08) = 4,600 ÷ .92 = 5,000 boxes. Ans.

Percentage or number of boxes spoiled = 5,000 × .08 = 400 boxes. Ans.

Example.—A farmer lost 63 sheep, which was 18% of his flock. How many had he left?

Solution.—The rate is .18. The number of sheep in the flock must be the base, since it is the number on which the 18% is computed. The decrease, or number of sheep lost, is the percentage, and the number remaining is the difference.

Base = percentage ÷ rate
= 63 ÷ .18 = 350 sheep = number originally in the flock.

Difference = base − percentage = 350 − 63 = 287 sheep left. Ans.

EXAMPLES FOR PRACTICE.

33. Solve the following:

			Ans.
(*a*)	What is $12\frac{1}{2}$% of $900?	(*a*)	$112.50.
(*b*)	What is $\frac{4}{5}$% of 627?	(*b*)	5.016.
(*c*)	What is $33\frac{1}{3}$% of 54?	(*c*)	18.
(*d*)	101 is $68\frac{3}{4}$% of what number?	(*d*)	$146\frac{10}{11}$.
(*e*)	784 is $83\frac{1}{3}$% of what number?	(*e*)	940.8.
(*f*)	What per cent. of 960 is 160?	(*f*)	$16\frac{2}{3}$%.
(*g*)	What per cent. of $3,606 is $$450\frac{3}{4}$?	(*g*)	$12\frac{1}{2}$%.
(*h*)	What per cent. of 280 is 112?	(*h*)	40%.

1. A man's salary is $1,800 per year and he saves $225. (*a*) What per cent. of his salary does he save? (*b*) What per cent. of it does he spend?

Ans. $\begin{cases} (a) & 12\frac{1}{2}\%. \\ (b) & 87\frac{1}{2}\%. \end{cases}$

2. A man has 32% of his money invested in stocks, 18% in grain, and the remainder, which is $7,620, in real estate. What is the total value of his property? Ans. $15,240.

3. If wool loses 32% of its weight in washing, how many pounds of unwashed wool are required to produce 35,360 pounds of washed wool?

Ans. 52,000 pounds.

4. In 1890 the population of a city was 85,000, which was 36% more than the population in 1880. What was the population in 1880?

Ans. 62,500.

5. If gunpowder contains 75% of saltpeter, 10% of sulphur, 15% of charcoal, how much of each is there in a ton of powder?

Ans. $\begin{cases} \text{Saltpeter, } 1{,}500 \text{ lb.} \\ \text{Sulphur, } 200 \text{ lb.} \\ \text{Charcoal, } 300 \text{ lb.} \end{cases}$

6. A man bequeathed to a charity 32% of his estate. To another charity he gave $23,100 which was 23% less than the amount given to the first charity. (*a*) What was the value of the estate? (*b*) What per cent. of the estate was given to the second charity?

Ans. $\begin{cases} (a) & \$93{,}750. \\ (b) & 24.64\%. \end{cases}$

7. A man owning a ship worth $225,000, sells $\frac{1}{4}$ of it to A, 20% of the remainder to B, and 35% of what then remains, to C. How much each do A, B, and C pay for their shares?

Ans. $\begin{cases} \text{A, } \$56{,}250. \\ \text{B, } \$33{,}750. \\ \text{C, } \$47{,}250. \end{cases}$

PROFIT AND LOSS.

34. **Profit and loss** treats of the gains or losses arising in business transactions.

If the price for which merchandise is sold is greater than the cost of the merchandise, the difference is *profit* or *gain*. If the selling price is less than the cost, the difference is *loss*.

35. The **gross cost** of merchandise is its first cost plus the expenses of purchase, transportation, and storage. Such expenses are commission, freight, insurance, drayage, etc.

36. The **net selling price** is the gross selling price, less all discounts and expenses of sale.

37. Computations in profit and loss are made according to the rules of percentage. The gross cost of the merchandise is the *base*, upon which the *rate* of profit or loss is computed. The profit or loss is the *percentage*. If the merchandise is sold at a profit, the net selling price is the *amount*; if at a loss, the net selling price is the *difference*.

38. Rule.—*To find the profit or loss, multiply the gross cost by the rate of gain or loss.* (Art. **14.**)

Formula.— *Profit or loss = cost × rate.*

Example.—A house costing $3,000 is sold for 22% above cost. What is the profit?

Solution.—Profit = cost × rate = $3,000 × .22 = $660. Ans.

39. Rule.—*To find the rate of profit or loss, divide the difference between the selling price and gross cost by the gross cost; or divide the profit or loss by the gross cost.* (Art. **18.**)

Formula.— *Rate = profit or loss ÷ gross cost.*

Example.—A merchant sold for $768 a lot of dry goods for which he paid $900. What was the per cent. loss?

Solution.— Loss = $900 − $768 = $132.
Rate of loss = loss ÷ cost = $132 ÷ $900 = .14$\frac{2}{3}$ or 14$\frac{2}{3}$%. Ans.

40. Rule.—*To find the selling price, the cost and rate of gain or loss being given, multiply the cost by 1 plus the rate of gain, or by 1 minus the rate of loss.*

Formulas.—

$$\textit{Selling price} = \begin{cases} \textit{Cost} \times (1 + \textit{rate of gain}). & \text{(Art. 26.)} \\ \textit{Cost} \times (1 - \textit{rate of loss}). & \text{(Art. 27.)} \end{cases}$$

Example.—If hay is bought for $8 per ton, and if baling and shipping costs $5.50 per ton additional, at what price must it be sold to yield a profit of 16%?

Solution.— Gross cost = $8 + $5.50 = $13.50.
Selling price = cost × (1 + rate) = $13.50 × 1.16 = $15.66. Ans.

41. Rule.—*To find the cost, the selling price and rate of gain or loss being given, divide the selling price by 1 plus the rate of gain, or by 1 minus the rate of loss.*

Formulas.—

$$Cost = \begin{cases} Selling\ price \div (1 + rate\ of\ gain). & \text{(Art. \textbf{29.})} \\ Selling\ price \div (1 - rate\ of\ loss). & \text{(Art. \textbf{30.})} \end{cases}$$

Example.—Sold drugs for \$112 and gained 75%. What was the cost of the drugs, and what was the profit?

Solution.—

Cost = Selling price ÷ (1 + rate) = \$112 ÷ 1.75 = \$64. Ans.

Profit = \$112 − \$64 = \$48. Ans.

EXAMPLES FOR PRACTICE.

42. What is the profit or loss

(*a*) If the gross cost is \$85 and the rate of gain is 32%?
(*b*) If the gross cost is \$837.50 and the rate of loss is 12%?
(*c*) If the gross cost is \$240 and the rate of gain is $16\frac{2}{3}$%?

Ans. (*a*) \$27.20. (*b*) \$100.50. (*c*) \$40.00.

What is the rate of gain or loss

(*d*) If the gross cost is \$6.50 and selling price is \$9.10?
(*e*) If the gross cost is \$14.00 and selling price is \$12.50?
(*f*) If the gross cost is \$3,500 and profit is \$500?

Ans. (*d*) 40%. (*e*) $10\frac{5}{7}$%. (*f*) $14\frac{2}{7}$%.

What is the selling price

(*g*) If the cost is \$945 and the rate of gain is $33\frac{1}{3}$%?
(*h*) If the cost is \$3.50 and the rate of gain is $12\frac{1}{2}$%?
(*i*) If the cost is \$125 and the rate of loss is 18%?

Ans. (*g*) \$1,260. (*h*) \$3.94. (*i*) \$102.50.

What is the cost

(*j*) If the selling price is \$575 and the rate of gain is 15%?
(*k*) If the selling price is \$28 and the rate of loss is $12\frac{1}{2}$%?
(*l*) If the selling price is \$3.50 and the rate of gain is 26%?

Ans. (*j*) \$500. (*k*) \$32. (*l*) $\$2.77\frac{7}{9}$.

1. A house and lot which cost \$3,250, is sold at a profit of 12%. What is (*a*) the profit and (*b*) the selling price?

Ans. (*a*) \$390. (*b*) \$3,640.

2. What must be the selling price of a suit of clothes which cost $18 in order that the profit may be $33\frac{1}{3}\%$? Ans. $24.

3. A harvesting machine costs the hardware merchant $90 net, and $6 for freight and cartage. If sold for $108, what is the gain per cent.? Ans. $12\frac{1}{2}\%$.

4. A carload of cattle is sold for $875, which is at a loss of 16%. What was the cost of the cattle? Ans. $1,041.67.

5. A sells a steam tug to B, gaining 14%, and B sells it to C for $4,104 and gains 20%. How much did the tug cost A? Ans. $3,000.

6. How much must hay sell for per ton, to gain 25%, if when sold for $8.40 per ton, there is a gain of $16\frac{2}{3}\%$? Ans. $9.

7. Six horses were sold at $125 each; three of them at a profit of 25% and the others at a loss of 25%. What was the net gain or loss? Ans. $50 loss.

TRADE DISCOUNTS.

43. Trade discounts are reductions made by manufacturers, jobbers, or merchants from their list or catalogue prices.

In many branches of business, manufacturers and dealers list their goods at a fixed price for each article, and allow a rate of discount on orders of a certain amount, a second discount on orders of larger amount, and perhaps a third discount on still larger orders. If it becomes necessary to raise or lower the price of the goods, the rate of discount is decreased or increased, the list price remaining the same. The system of discounts thus saves the expense of publishing a new price list every time prices change.

44. Merchandise is frequently sold at *time prices;* that is, payment is to be made in 30, 60, or 90 days after date of sale, and a certain rate of discount is allowed if payment is made at an earlier date. Business houses usually make announcements such as the following upon their bill heads: "Terms: 4 mo., or 5% 60 days;" "Terms: 60 days net; 30 days, 3% off; 10 days, 5% off." Even when no discount is stated in the terms, sellers will usually deduct the legal interest for the time remaining, if the payment is made

before it becomes due. Thus, if a payment due in 3 months is made 1 month after the sale, the seller should deduct the interest for the remaining 2 months.

45. Trade discounts are computed by the rules of percentage, the list price of the goods being the base. When several discounts are allowed, the first discount is computed on the list price, the second is computed on the remainder after deducting the first discount, and so on, each remainder being regarded as a base for the computation of the next discount. The several discounts, if there are more than one, form a **discount series.**

46. Rule.—*To find the selling price, multiply the list price by the rate, and subtract the discount thus obtained from the list price. If there is a discount series, compute the second discount, using the first remainder as a base, and subtract the discount from the remainder. Repeat the process, using each successive remainder as a base for computing the next discount. The last remainder is the selling price.*

EXAMPLE.—The list price of an article is \$62.50 and a discount of 40% is allowed. What is the selling price?

SOLUTION.—*First Method.*—

$$\text{Discount} = \$62.50 \times .40 = \$25.00.$$

$$\text{Selling price} = \$62.50 - \$25.00 = \$37.50. \quad \text{Ans.}$$

Second Method.—Since the list price is the base, and the selling price is the base less the percentage, i. e., the difference, the selling price may be found by applying the rule of Art. **27.** Thus,

$$\text{Selling price} = \$62.50 \times (1 - .40) = \$37.50. \quad \text{Ans.}$$

EXAMPLE.—On a bill of goods amounting to \$720, discounts of 30%, 10%, and 5% are allowed. What is the selling price?

SOLUTION.—*First Method.*—

$$\begin{aligned}
\text{First discount} &= \$720 \times .30 = \$216. \\
\text{Remainder} &= \$720 - \$216 = \$504. \\
\text{Second discount} &= \$504 \times .10 = \$50.40. \\
\text{Remainder} &= \$504 - \$50.40 = \$453.60. \\
\text{Third discount} &= \$453.60 \times .05 = \$22.68. \\
\text{Selling price} &= \$453.60 - \$22.68 = \$430.92. \quad \text{Ans.}
\end{aligned}$$

Second Method.—Regarding \$720 as divided into 100 parts, the first discount of 30% leaves 100 − 30 = 70 parts. The second discount of 10% is computed on the remainder after the first discount has been deducted;

that is, the second discount is 10% of 70 parts. Hence, the remainder after the second discount has been deducted is $70 \times (1 - .10) = 63$ parts. Similarly, the remainder after the third discount has been deducted is $63 \times (1 - .05) = 59.85$ parts, or 59.85% of \$720. Therefore,

$$\text{Selling price} = \$720 \times .5985 = \$430.92. \quad \text{Ans.}$$

Ordinarily, when applying this method, the work would be as follows:

$$\$720 \times (1 - .30) \times (1 - .10) \times (1 - .05)$$
$$= \$720 \times .70 \times .90 \times .95 = \$430.92. \quad \text{Ans.}$$

47. The discounts usually allowed are aliquot parts of 100%, and the labor of computation may be shortened by using the fractions corresponding to the rates of discount.

Example.—The gross amount of a bill of hardware is \$640, and discounts of 25%, 10%, and 5% are allowed. What is the net amount of the bill?

Solution.—

$25\% = \frac{1}{4}$, $10\% = \frac{1}{10}$, $5\% = \frac{1}{20}$.

4)	\$640	
	\$160	1st discount.
10)	\$480	1st remainder.
	\$ 48	2d discount.
20)	\$432	2d remainder.
	\$ 21.60	3d discount.
	\$410.40	net amount. Ans.

The solution is arranged as shown. To multiply \$640 by 25% or $\frac{1}{4}$, we divide by 4; then the discount \$160 is subtracted and the remainder, \$480, is divided by 10. The second discount, \$48, is subtracted, and the remainder is divided by 20. The final remainder, \$410.40, is the net amount or selling price.

48. When a discount series is allowed, business men usually reduce the series to an equivalent single discount; if there is a large number of sales, much labor of computation is saved by using the equivalent discount rather than the series.

49. Rule.—*To reduce a discount series to an equivalent single discount, subtract each rate of discount from 1, and multiply the remainders together. Subtract the product from 1, and the remainder will be the single discount.* (See second example, second method, Art. **46.**)

Example.—What single discount on the gross price is equivalent to a discount series of 25%, 20%, and 10%?

Solution.— $1 - .25 = .75$; $1 - .20 = .80$; $1 - .10 = .90$.

$$.75 \times .80 \times .90 = .54.$$

$$1 - .54 = .46, \text{ or } 46\%. \quad \text{Ans.}$$

50. Example.—The cost of a line of goods is \$350. What must they be marked to give a profit of 20% and allow a discount of 30% on the selling price?

Solution.—*First Method.*—The profit is $\$350 \times .20 = \$70$; therefore, the actual selling price is $\$350 + \$70 = \$420$. This is what remains after deducting 30% from the marked price. Since the 30% discount is computed on the marked price, that price must be the base, and the less price, \$420, obtained by subtracting the discount, is the difference. According to the rule, Art. **30**, base = difference ÷ (1 − rate); hence,

$$\text{marked price} = \$420 \div (1 - .30) = \$420 \div .70 = \$600. \quad \text{Ans.}$$

Second Method.—Regarding the \$350 as divided into 100 parts, 20 parts must be added to this in order to gain 20%; that is, the selling price must be 120 parts, or 120% of \$350. Now, if a discount of 30% is to be allowed from the list price and leave a remainder of 120 parts, it is evident that 120 parts is the difference, and the list price is the base. According to rule, Art. **30**,

list price (base) = selling price (difference) ÷ (1 − rate), or

$$\text{list price} = 120 \div (1 - .30) = 171\tfrac{3}{7} \text{ parts, or } 171\tfrac{3}{7}\%.$$

Hence, $\text{list price} = \$350 \times 1.71\tfrac{3}{7} = \$600.$ Ans.

51. Rule.—*To find the price at which goods must be marked to insure a given profit after allowing a discount, or a discount series, add to the cost the profit required, and divide the sum by 1 minus the discount or equivalent single discount.*

Example.—The cost of manufacturing hats is \$36 per dozen. At what price per dozen must they be marked that the manufacturer may realize $16\frac{2}{3}\%$ profit after allowing the trade discounts of 20% and $12\frac{1}{2}\%$?

Solution.—*First Method.*— $16\frac{2}{3}\% = \frac{1}{6}$. Profit $= \$36 \times \frac{1}{6} = \$6$; selling price $= \$36 + \$6 = \$42$. $1 - .20 = .80$; $1 - .12\frac{1}{2} = .87\frac{1}{2}$; $.80 \times .87\frac{1}{2} = .70$. The equivalent single discount is $1 - .70 = .30$. Marked price $= \$42 \div (1 - .30) = \$60$ per dozen. Ans.

Second Method.—Selling price expressed as per cent. $= 100 + 16\frac{2}{3} = 116\frac{2}{3}$. Marked price expressed as per cent. $= 116\frac{2}{3} \div (1 - .20) \times (1 - .12\frac{1}{2}) = 116\frac{2}{3} \div .70 = 166\frac{2}{3}\%$. Hence, marked price $= \$36 \times 1.66\frac{2}{3} = \$60$ per dozen. Ans.

EXAMPLES FOR PRACTICE.

52. Reduce the following discount series to equivalent single discounts:

		Ans.
(*a*)	25% and 16%.	(*a*) 37%.
(*b*)	30%, 20%, and 5%.	(*b*) 46.8%.
(*c*)	60%, 10%, and 5%.	(*c*) 65.8%.
(*d*)	40%, 20%, $12\frac{1}{2}\%$, and 4%.	(*d*) 59.68%.

1. A musical instrument is listed at $122 and discounts of 60% and 5% are allowed. What is the selling price? Ans. $46.36.

2. A bill of hardware is sold at the following discounts: $452.60 at 30% and 10%; $216 at $33\frac{1}{3}$% and 5%; $137.50 at 20%; and $83.75 net. What is the total net amount of the bill? Ans. $615.69.

3. A bill of goods amounting to $836.72 was bought May 5, 1890, on the following terms: 4 months, or 5% off 30 days. How much would pay the bill June 4, 1890? Ans. $794.88.

4. A wholesale dealer sells books at discounts of 20% and 5%. What must he mark a set of books that cost him $24 in order to make $26\frac{2}{3}$% profit, after allowing discounts? Ans. $40.

5. Plows are bought at a discount of 40% from the list price. What per cent. is gained by selling them at the list price? Ans. $66\frac{2}{3}$%.

6. A wholesale dealer offers silks at $3.50 per yard, subject to a discount of 20%, $12\frac{1}{2}$%, and 10%. How many yards can be bought for $352.80? Ans. 160.

7. A suit of clothing is marked 50% off. By selling at this price the clothier loses $12\frac{1}{2}$% of the cost of the suit, which was $12. What was the marked price of the suit? Ans. $21.00.

COMMISSION AND BROKERAGE.

53. **Commission,** or **brokerage,** is the sum paid an agent for transacting business for another person; as, for buying or selling merchandise or property, for collecting or investing money, etc.

54. The agent or party who transacts the business is called a **commission merchant,** or **broker;** the party for whom the business is transacted is called the **principal.** The term broker is applied to one who sells and buys stocks, bonds, bills of exchange, and money securities.

55. A **consignment** is a shipment of goods from one party to another; the party that ships the goods is called the **consignor,** or **shipper,** and the party to whom they are shipped is called the **consignee.**

56. When goods are sold on credit, the agent charges an additional amount for guaranteeing the payment of the sale. This extra charge is termed **guaranty.**

57. The **gross proceeds** of a sale or collection is the total amount realized by the agent before deducting his commission and other expenses connected with the transaction. The **net proceeds** is the amount due the principal after the commission and all other charges have been deducted.

58. An **account sales** is a detailed statement made by the agent to his principal, showing the goods sold and the prices obtained, giving a list of the charges and expenses, and the net proceeds due the principal. The charges include freight, cartage, storage, insurance, inspection, advertising, commission, and guaranty.

59. The **prime cost** of a purchase is the sum paid by the agent for the goods or property. The **gross cost** is the prime cost plus the commission and expenses incident to the purchase.

60. An **account purchase** is a detailed statement made by the agent to his principal, showing the cost of goods or property bought, the expenses attending the purchase, and the gross cost.

61. The commission or brokerage is usually computed at a certain per cent. of the gross proceeds of a sale or the prime cost of a purchase. In some cases, however, it is computed at a certain price per unit of weight or measure; as, so much per ton, per bushel, or per barrel. Examples in commission are solved by the rules of percentage. Either the gross proceeds or prime cost is the *base;* the net proceeds is the *difference;* the gross cost is the *amount;* the commission is the *percentage*, and the rate of commission is the *rate per cent.* The remittance from the principal to the purchasing agent, including both the investment and the commission, is an *amount.* The following rules are derived directly from the principles of percentage:

62. **Rule.**—*To find the commission, multiply the prime cost or gross selling price by the rate of commission.*

Formula.—

Commission = *cost or selling price* × *rate of commission.*

Example.—A real estate agent sells a house and lot for $4,375 and receives 2% commission. What is the commission and what are the net proceeds?

Solution.—Commission = selling price × rate = $4,375 × .02 = $87.50. Ans.

Net proceeds = selling price − commission = $4,375 − $87.50 = $4,287.50. Ans.

63. Rule.—*To find the prime cost or gross proceeds, the commission being given, divide the commission by the rate of commission.*

Formula.—

Prime cost or gross proceeds = commission ÷ rate of commission.

Example.—An agent received $319.50 commission for selling apples. If the rate of commission charged was $1\frac{1}{2}$%, what was the selling price of the apples?

Solution.—Selling price or gross proceeds = $319.50 ÷ .015 = $21,300. Ans.

64. Rule.—*To find the prime cost and commission, the remittance from the principal being given, subtract from the remittance the expenses of the purchase, if any, and divide the remainder by 1 plus the rate of commission. The quotient is the prime cost. Subtract the prime cost from the remainder and the difference is the commission.*

Example.—A principal sends his agent $21,611 with orders to buy cotton after deducting his commission and other charges. The agent paid $124.30 for freight, $51.70 for cartage, $15.00 for insurance, and deducted his commission of 2%. (*a*) How much remained to invest in cotton? (*b*) What was his commission?

Solution.—The expenses are first deducted. $21,611 − ($124.30 + $51.70 + $15) = $21,420, which is the sum of the prime cost and commission. According to the rule,

Prime cost = $21,420 ÷ (1 + .02) = $21,420 ÷ 1.02 = $21,000. Ans.
Commission = $21,420 − $21,000 = $420. Ans.

Note.—When a charge is made for guaranty, add the per cent. of guaranty to 1 plus the rate of commission, and proceed as above.

EXAMPLES FOR PRACTICE.

65. What is the commission

(*a*) If the gross proceeds are $300 and the rate of commission is $3\frac{1}{2}$%?

(*b*) If the gross proceeds are $9,375 and the rate of commission is 2%?

(*c*) If the prime cost is $831.75 and the rate of commission is $1\frac{1}{4}\%$?

(*d*) If the prime cost is $960 and the rate of commission is $\frac{5}{8}\%$?

Ans. $\begin{cases} (a) & \$10.50. \\ (b) & \$187.50. \\ (c) & \$10.40. \\ (d) & \$6.00. \end{cases}$

What are the net proceeds

(*e*) If the gross proceeds are $340, rate of commission 3%, and other expenses $4.30?

(*f*) If the gross proceeds are $6,375, rate of commission 4%, and other expenses $32.50?

(*g*) If the gross proceeds are $195.40, rate of commission $1\frac{1}{2}\%$, and other expenses $7.45?

Ans. $\begin{cases} (e) & \$325.50. \\ (f) & \$6{,}087.50. \\ (g) & \$185.02. \end{cases}$

What is the prime cost if

(*h*) The gross cost is $520 and the rate of commission is 4%?

(*i*) The gross cost is $1,606 and the rate of commission is $\frac{3}{8}\%$?

(*j*) The gross cost is $843, the rate of commission is 2%, and the expenses of buying are $27?

Ans. $\begin{cases} (h) & \$500. \\ (i) & \$1{,}600. \\ (j) & \$800. \end{cases}$

1. An agent receives $13.00 commission for selling $650 worth of goods. What rate of commission does he charge? Ans. 2%.

2. A commission merchant sold a quantity of wool for $4,650. He charged $2\frac{1}{2}\%$ commission, 2% guaranty, and the transportation, storage, and other expenses amounted to $184. How much should he send his principal? Ans. $4,256.75.

3. An agent received $550.50 to buy potatoes after deducting all expenses. He paid $26.50 for drayage, $32 for barrels, and charged $2\frac{1}{2}\%$ commission for buying. How many bushels did he buy at 60 cents per bushel? Ans. 800 bushels.

4. A commission merchant has consigned to him 400 barrels of flour which he sells at $4.75 per barrel, and charges $2\frac{1}{2}\%$ commission. With the net proceeds he buys sugar at $6\frac{1}{4}$ cents a pound and charges $2\frac{1}{2}\%$ commission for buying. (*a*) How many pounds of sugar does he buy? (*b*) What is the amount of his commissions? Ans. $\begin{cases} (a) & 28{,}917\frac{1}{8} \text{ lb.} \\ (b) & \$92.68. \end{cases}$

5. A New York agent received $1,134 with which to purchase hats. If he charges 3% commission and 2% additional for guaranty of quality, how many dozen hats can he buy at $13.50 per dozen? Ans. 80 dozen.

6. A commission merchant sold 2,500 bushels of wheat at 64 cents a bushel and a quantity of corn at 23 cents per bushel. The rate of commission on each sale was $1\frac{3}{4}\%$, and the total commission was \$35.084. How many bushels of corn were sold? Ans. 1,760 bu.

INSURANCE.

66. **Insurance** is a contract by which one party, the **underwriter,** or **insurer,** agrees, for a consideration, to make good a loss sustained by another party.

67. Insurance is of two kinds, **property insurance** and **personal insurance.** Property insurance includes *fire insurance* (indemnity for loss or damage by fire); *marine insurance* (indemnity for losses at sea); *transit insurance* (indemnity for loss of, or damage to, merchandise during transportation); *stock insurance* (indemnity for loss of live stock); and *accident insurance* (indemnity for breakage of fragile materials, as plate glass, etc.).

Personal insurance includes *life insurance*, which secures the payment of a certain amount to a specified person at the death of the party insured, or after the lapse of a specified time; *accident insurance*, which secures the payment of a certain sum in case of accident to the insured; *health insurance*, which secures the payment of a weekly sum during sickness; and insurance against the dishonesty of employes.

68. The **policy** is the written contract between the insurance company and the party insured; it contains a description of the property insured, the conditions upon which the insurance is taken, and the amount to be paid in case of loss.

69. The **premium** is the amount paid to the insurer for assuming the risk of loss or damage. The premium is a certain per cent. of the amount of insurance, as $\frac{5}{8}\%$, $\frac{3}{4}\%$. The rate of premium depends upon the nature of the risk and upon the length of time the insurance has to run. It is customary to speak of the rate of premium as the cost per \$100 of insurance; as 60 cents per \$100, \$1.20 per \$100, etc.

70. In property insurance, all computations are based on the rules of percentage. The amount of insurance is the *base*, the premium is the *percentage*, and the rate of premium is the *rate*.

71. **Rule.**—*To find the premium, multiply the amount of insurance by the rate of premium.* (Art. **14.**)

Formula.—

$$\textit{Premium} = \textit{amount of insurance} \times \textit{rate}.$$

EXAMPLE.—A house and furniture are insured against fire for $3,250, the rate of premium being $\frac{3}{4}$%, or 75 cents per $100 per year. What is the yearly premium?

SOLUTION.— $\frac{3}{4}\% = .0075$. Using the rule,

$$\text{Premium} = \$3{,}250 \times .0075 = \$24.37\tfrac{1}{2}. \quad \text{Ans.}$$

72. **Rule.**—*To find the amount of insurance, divide the premium by the rate of premium. To find the rate of premium, divide the premium by the amount of insurance.*

Formulas.—

$$\textit{Amount of insurance} = \textit{premium} \div \textit{rate of premium}.$$

$$\textit{Rate of premium} = \textit{premium} \div \textit{amount of insurance}.$$

EXAMPLE.—What amount of insurance can be obtained for $137.50, the rate of premium being 55 cents per $100?

SOLUTION.— Rate $= .55 \div 100.00 = .0055$.
Amount of insurance = premium ÷ rate = $\$137.50 \div .0055 = \$25{,}000$. Ans.

EXAMPLE.—A warehouse and contents together valued at $13,500 is insured for $\frac{2}{3}$ of its value. If the premium is $78.75 what is the rate of premium?

SOLUTION.—Amount of insurance $= \$13{,}500 \times \frac{2}{3} = \$9{,}000$.
Rate of premium $= \$78.75 \div \$9{,}000 = .00875 = 87\frac{1}{2}$ cents per $100, or $\frac{7}{8}$%. Ans.

EXAMPLES FOR PRACTICE.

73. 1. A store and contents valued at $16,400 is insured for $\frac{3}{5}$ of its value at $1\frac{1}{8}$% premium. What is the cost per year of insurance?

Ans. $110.70.

2. A premium of \$162.50 is paid for the insurance of a steamer worth \$32,500, which is insured for $\frac{4}{5}$ of its value. What is the premium in cents per \$100? Ans. $62\frac{1}{2}$ cents per \$100.

3. A stock of goods is insured for $\frac{2}{3}$ of its value. The premium paid is \$43.75 and is at a rate of 75 cents per \$100. What is the value of the stock of goods? Ans. \$8,750.

4. A boat load of 8,600 bushels of corn, worth 32 cents per bushel, is insured for $\frac{3}{4}$ of its value at $1\frac{5}{8}$% premium. If the corn is totally destroyed, what will be the owner's loss? Ans. \$721.54.

5. A building worth \$9,600 is insured for $\frac{3}{4}$ of its value in three companies. The first company takes $\frac{1}{3}$ of the risk at $\frac{1}{2}$% premium; the second $\frac{2}{5}$ of it at $\frac{3}{4}$%; and the third the remainder at $\frac{7}{8}$%. What is the total premium? Ans. \$50.40.

6. If it cost \$324 to insure a property for \$21,600, what will it cost at the same rate to insure a similar property for \$35,000, if \$1.50 is charged for the policy in the latter case? Ans. \$526.50.

7. A fire insurance company took a risk of \$42,000 at $\frac{3}{4}$% premium, and reinsured $\frac{1}{3}$ of it in another company at $\frac{1}{2}$%, and $\frac{1}{2}$ of it in a third company at $\frac{5}{8}$%. What did the company gain by reinsuring?

Ans. \$61.25.

TAXES.

74. **Taxes** are sums of money levied on persons, properties, or incomes, for public purposes. Thus, taxes are levied to support the state, county, and city governments; to support schools and charities; and to make improvements, such as paved streets and sewers.

75. A **capitation,** or **poll** tax, is a tax levied on persons; a **property** tax is a tax levied on real estate or personal property; an **income** tax is levied on incomes or salaries. The poll tax is usually a fixed amount for each citizen over 21 years of age. The property tax is estimated at a certain per cent. of the assessed valuation of the property subject to taxation.

76. **Rule.**—*To find a property tax, multiply the assessed value of the property by the rate of taxation.*

EXAMPLE.—What property tax must a person pay who owns real estate assessed at \$34,000 and personal property assessed at \$12,500, the rate of taxation being 8 mills per \$1.00, or rate = .008 ?

SOLUTION.—Total assessed value = \$34,000 + \$12,500 = \$46,500.

Tax = \$46,500 × .008 = \$372. Ans.

77. Rule.—*To find the rate of taxation, subtract from the total tax to be raised the poll tax, if any; divide the remainder by the total assessed valuation of the property to be taxed.*

This rule is derived directly from the rule, Art. **18,** since the tax is the *percentage*, and the assessed valuation is the *base.*

EXAMPLE.—The assessed valuation is \$1,375,000 on real estate and \$575,000 on personal property. The town votes to raise \$12,000 for schools, \$2,100 for streets and highways, \$3,000 for salaries, \$1,500 for support of the poor, and \$650 for contingent expenses. There is a poll tax of \$1.00 each on 3,650 persons. What is the rate of taxation and how much must A pay who has property assessed at \$26,400 and pays for 3 polls ?

SOLUTION.—Total tax = \$12,000 + \$2,100 + \$3,000 + \$1,500 + \$650 = \$19,250. The poll tax is \$3,650; therefore, the property tax is \$19,250 − \$3,650 = \$15,600.

Total assessed value of property = \$1,375,000 + \$575,000 = \$1,950,000.
Rate of taxation = \$15,600 ÷ \$1,950,000 = .008, or 8 mills per \$1.00. Ans.

A's property tax is \$26,400 × .008 = \$211.20; his poll tax is \$3.00.

\$211.20 + \$3.00 = \$214.20, A's total tax. Ans.

EXAMPLES FOR PRACTICE.

78. What is the tax if the assessed value of the property is

(*a*) \$6,300 and the rate of taxation is $1\frac{1}{8}$% ?

(*b*) \$34,300 and the rate of taxation is $6\frac{1}{2}$ mills per \$1.00 ?

(*c*) \$9,430 and the rate of taxation is 85 cents per \$100 ?

Ans. (*a*) $\$70.87\frac{1}{2}$. (*b*) \$222.95. (*c*) $\$80.15\frac{1}{2}$.

What is the rate of taxation (mills per \$1.00) if the assessed valuation is

(*d*) \$625,000 and the gross tax is \$5,230, including 460 polls at \$.50 each ?

(*e*) \$1,364,000 and the gross tax is \$18,130, including 1,440 polls at \$.75 each ?

(*f*) \$34,000 and the gross tax is \$469.20?

Ans. (*d*) 8 mills. (*e*) $12\frac{1}{2}$ mills. (*f*) 13.8 mills.

1. I own real estate worth \$8,500 and personal property worth \$3,750; both are assessed at $\frac{3}{5}$ of their value. The rate of taxation is 1.2%, but I receive a discount of $2\frac{1}{2}$ per cent. of my taxes for prompt payment. How much do I pay the tax receiver? Ans. \$86.

2. A has 5 lots worth \$1,300 each; B has 4 lots worth \$1,000 each; C has 2 lots worth \$1,500 each; D has 7 lots worth \$800 each, and E has 1 lot worth \$1,150. A tax of \$1,417.50 for a street improvement is to be divided among them. What should each pay?

Ans. A, \$455. B, \$280. C, \$210. D, \$392. E, \$80.50.

3. The rate of taxation for a certain state is $3\frac{1}{2}$ mills per \$1.00. How much state tax must be raised by a county whose valuation is fixed by the State Board of Equalization at \$13,876,394? Ans. \$48,567.38.

DUTIES.

79. Duties, or **customs,** are taxes levied by governments on imported goods for the purpose of producing revenue and for the protection of home industries.

80. There are two kinds of duties: *ad valorem* and *specific*. An **ad valorem** duty is estimated at a certain per cent. of the market value of the goods in the country from which they are imported; as, silks 50%, musical instruments 15%, etc. The market value of the goods is the invoice value after deducting discounts and before extra charges, such as commission, freight, boxing, etc., are added.

81. A **specific** duty is a duty levied on imported goods according to the weight, measurement, or number of the articles, without reference to their value; as, wheat 15 cents per bushel, coal 75 cents per ton, etc. Some kinds of merchandise are subject to both ad valorem and specific duties. In computing specific duties, the long ton of 2,240 pounds and the hundredweight of 112 pounds are used.

82. An **invoice** is an itemized statement of the merchandise shipped. It contains the names of purchaser and seller, a description of the quality and quantity of the goods, the price and incidental charges. Invoices are made out in the weights and measures and the currency of the country from which the goods are imported. Thus, the price and cost of goods imported from Germany would be given in *marks;* from France, in *francs;* from England, in £ *s. d.*

83. The following table gives the monetary units of leading foreign nations and their equivalents in United States money. These rates are proclaimed each year by the Secretary of the Treasury, and are used in Custom House computations:

Country.	Monetary Unit.	Value in U. S. Gold.
Canada..........	Dollar = 100 cents....	$1.00
Great Britain.....	Pound = 20 shillings..	$4.86\frac{2}{3}$
France, Belgium, Switzerland.....	Franc = 100 centimes.	.193
Italy.............	Lira = 100 centesimi..	.193
Spain............	Peseta = 100 centimes.	.193
German Empire...	Mark = 100 pfennige..	.238
Denmark, Norway, Sweden.......	Crown = 100 öre......	.268
Japan............	Yen = 100 sen.........	.997

84. Before computing duties the following allowances are made: **Tare,** a deduction for the weight of boxes or crates; **leakage,** an allowance for loss of liquids imported in barrels or casks; and **breakage,** an allowance for loss of liquids imported in bottles. The **net quantity** is what remains after deducting tare, leakage, or breakage.

85. Ad valorem duties are computed by the rules of percentage; the net invoice price is regarded as the *base*, the ad valorem duty as the *percentage*, and the rate of duty as the *rate*.

Duties are not computed on fractions of a dollar; if the

cents are less than 50 they are rejected; if more, they are counted as a dollar.

86. Rule.—*To find the ad valorem duty, reduce the net invoice price to U. S. money, if necessary, deduct allowances, and multiply the remainder (expressed in even dollars) by the rate of ad valorem duty.*

To find the specific duty, multiply the net quantity by the rate of specific duty per unit of quantity.

Example.—What is the duty on an invoice of silks valued at 24,360 francs, the ad valorem rate being 60%?

Solution.—Referring to Art. **83**, 24,360 francs = $24{,}360 \times .193 = \$4{,}701.48$. Duty $= \$4{,}701 \times .60 = \$2{,}820.60$. Ans.

The 48 cents is rejected, being less than 50 cents. (Art. **85**.)

Example.—What is the duty on 820 gallons of brandy at $1.50 per gallon, leakage 3%?

Solution.—
$$\begin{aligned} \text{Leakage} &= 820 \times .03 = 24.6 \text{ gallons.} \\ \text{Net quantity} &= 820 - 24.6 = 795.4 \text{ gallons.} \\ \text{Duty} &= 795.4 \times 1.50 = \$1{,}193.10. \quad \text{Ans.} \end{aligned}$$

EXAMPLES FOR PRACTICE.

87. What is the ad valorem duty on an importation invoiced at

(*a*) £430 12 s. 4 d., allowing 5% breakage, rate of duty 40%?
(*b*) 36,750 lira, allowing 2% for tare, rate of duty 24%?
(*c*) 9,264 marks, rate of duty 85%?
(*d*) 4,700 yen, rate of duty 14%?

Ans. (*a*) $796.40. (*b*) $1,668.24. (*c*) $1,874.25 (*d*) $656.04

What is the specific duty on an importation of

(*e*) 3,200 bu. potatoes at 15 cents per bushel?
(*f*) 60 dozen bottles of wine at $3.00 per dozen, breakage 10%?
(*g*) 125 gross of empty bottles, breakage 6%, duty 10 cents per dozen?
(*h*) 3 T. 6 cwt. of iron castings at $\frac{3}{4}$ cent per pound?

Ans. (*e*) $480. (*f*) $162. (*g*) $141. (*h*) $55.44.

1. What is the duty on 25,670 lb. of pig iron at $5.00 per ton?
Ans. $57.30.

2. What is the duty on 5 blocks of marble each 12 ft. long, 4 ft. wide, and $2\frac{1}{2}$ ft. thick, at 65 cents per cu. ft.? Ans. $390.

3. What is the duty on an importation of 2,650 yards of woolen goods weighing 620 pounds net and valued at 72 cents per yard, the rates of duty being 60 cents per pound and 30% ad valorem? Ans. $944.40.

4. An importation of musical instruments from Germany is valued at 13,670 marks. What is the duty at $17\frac{1}{2}$% ad valorem? Ans. $569.28.

5. An importer buys French silks at $1.80 per yard and pays a duty of 35% ad valorem, and $.60 per yard specific. At what price per yard must the silk be sold to yield a profit of 25%? Ans. $3.79.

6. What is the duty at 65%, upon a consignment of 1,350 dozen kid gloves invoiced at 115 francs per dozen? Ans. $19,475.95.

ARITHMETIC.

INTEREST.

SIMPLE INTEREST.

1. Interest is money paid for the use of money belonging to another.

2. The **principal** is the sum for which interest is paid.

3. The **rate per cent.** is the per cent. of the principal that is paid for its use for a given time, usually a year.

4. The **amount** is the sum of the principal and interest.

5. The **legal rate** is the rate established by law.

6. Usury is a rate that exceeds the legal rate. The penalty for usury is, in some States, the forfeiture of all interest, in others the forfeiture of both principal and interest. In a number of States, no legal notice is taken of usury.

7. In computing interest, a year is usually regarded as consisting of 12 months of 30 days each. Interest so computed is greater than it should be, unless the time is an exact number of years.

8. The elements in interest correspond with those of ordinary percentage as follows:

The *principal* is the *base.*

The *interest* is the *percentage.*

The *product of the rate per year by the time in years* is the *rate.* Thus, if the rate per cent. per year is 4% and the time is 5 years, the rate per cent. is 20%. That is, 20% of the principal

equals the interest. It is, of course, understood that fractions of a year are included in the expression "time in years."

9. From the correspondence between percentage and interest, it is obvious that all methods of computing interest depend upon the principle expressed in the following

Formula: *Interest = principal × rate for 1 year × time in years.*

10. As in percentage, the rate is used in computation instead of rate per cent. When no other rate per cent. is specified, 6% is to be understood.

FIRST METHOD.

Example 1.—Find the interest of \$400 for 2 years 6 months at 5%.

Solution.— 2 years 6 months = $2\frac{1}{2}$ years; $\$400 \times .05 \times 2\frac{1}{2} = \$50$. Ans.

Example 2.—What is the interest of \$878.60 for 3 yr. 5 mo. 15 da. at 6%?

Solution.— $\$878.60 \times .06 \times \frac{83}{24} = \$182.31$. Ans.

Explanation.—The time must first be changed to years. To do this, we begin with the days. 15 da. = $\frac{1}{2}$ mo.; $5\frac{1}{2}$ mo. = $\frac{11}{2}$ mo., or $\frac{11}{2 \times 12}$ yr. = $\frac{11}{24}$ yr.; $3\frac{11}{24}$ yr. = $\frac{83}{24}$ yr.

Example 3.—Find the interest of \$697.23 for 2 yr. 11 mo. 24 da. at $3\frac{1}{2}$%.

Solution.— $\$697.23 \times .035 \times \frac{179}{60} = \$72.80$. Ans.

Explanation.— 24 da. = $\frac{4}{5}$ mo.; $11\frac{4}{5}$ mo. = $\frac{59}{5}$ mo. = $\frac{59}{5 \times 12}$ yr. = $\frac{59}{60}$ yr.; $2\frac{59}{60}$ yr. = $\frac{179}{60}$ yr. The rate per cent. being $3\frac{1}{2}$%, the rate is $.03\frac{1}{2}$, or .035. Performing the multiplication as indicated, and pointing off the result, gives \$72.80.

11. Rule.—*Multiply the principal by the rate, and that product by the time in years. The result will be the interest required.*

SECOND METHOD.

12. A very neat method of computing interest is the following:

Example.—What is the interest at 5% of \$248.80 for 5 yr. 9 mo. 29 da.?

SOLUTION.—

$248.80				
.05				
$12.44000	= Interest for 1 yr.			
62.2000	= Interest for 5 yr.	=	1 yr.	× 5.
6.2200	= Interest for 6 mo.	=	1 yr.	÷ 2.
3.1100	= Interest for 3 mo.	=	6 mo.	÷ 2.
.5200	= Interest for 15 da.	=	3 mo.	÷ 6.
.4150	= Interest for 12 da.	=	6 mo.	÷ 15.
.0690	= Interest for 2 da.	=	12 da.	÷ 6.
$72.5340	= Interest for 5 yr. 9 mo. 29 da. Ans.			

EXPLANATION.—We first find the interest for 1 year, and then for 5 years. Having the interest for 1 year, we take half of that, which is the interest for 6 of the 9 months. Half of the interest for 6 months is the interest for 3 months. The interest for 29 days is found by taking $\frac{1}{6}$ of 3 months' interest, which gives the interest for 15 ($\frac{1}{2}$ mo.) of the 29 days. Since 12 days = $\frac{1}{15}$ of 6 months, we divide the interest for 6 months by 15* and get 12 days' interest. Only 2 of the 29 days remain; but 2 days = $\frac{1}{6}$ of 12 days. Hence, dividing 12 days' interest by 6, we have the interest for 2 days. 5 years + 6 months + 3 months + 15 days + 12 days + 2 days = 5 yr. 9 mo. 29 da., so that the sum of these several interests = the interest required.

It will be noticed that in the foregoing, 30 days are considered equal to 1 month, or 360 days to a year.

EXAMPLE.—What is the interest of $6,400 for 3 yr. 11 mo. 26 da., at $4\frac{1}{2}$%?

SOLUTION.—

$6400				
.045				
$288.00	= Interest for 1 yr.			
864.00	= Interest for 3 yr.	=	1 yr.	× 3.
144.00	= Interest for 6 mo.	=	1 yr.	÷ 2.
72.00	= Interest for 3 mo.	=	6 mo.	÷ 2.
48.00	= Interest for 2 mo.	=	6 mo.	÷ 3.
16.00	= Interest for 20 da.	=	2 mo.	÷ 3.
4.00	= Interest for 5 da.	=	20 da.	÷ 4.
.80	= Interest for 1 da.	=	5 da.	÷ 5.
$1148.80	= Interest for 3 yr. 11 mo. 26 da. Ans.			

*To divide by 15, divide by 5 and then by 3.

13. **Rule.**—*Find the interest at the given rate for one year, and multiply this by the number of years. Find the interest for the given months by taking suitable parts of one year's interest, and for the days, suitable parts of the interest for one or more months. The sum of these partial results will be the total interest for the given time.*

EXAMPLES FOR PRACTICE.

14. Find the interest by the first method, and prove the correctness of your work by the second method:

1. Of \$600 for 1 yr. 4 mo. 15 da. at 6%.
2. Of \$2,400 for 2 yr. 2 mo. 18 da. at 5%.
3. Of \$1,800 for 2 yr. 7 mo. 20 da. at $4\frac{1}{2}$%.
4. Of \$1,250 for 9 mo. 25 da. at $3\frac{1}{2}$%.
5. Of \$87.50 for 5 yr. 10 mo. 27 da. at 8%.
6. Of \$675.60 for 2 yr. 8 mo. 12 da. at $4\frac{1}{4}$%.
7. Of \$1,388.84 for 3 yr. 7 mo. 17 da. at 4%.
8. Of \$725.50 for 7 yr. 5 mo. 23 da. at $7\frac{1}{2}$%.
9. Of \$6,496.40 for 5 yr. 11 mo. 21 da. at 3%.
10. Of \$847.23 for 9 yr. 6 mo. 14 da. at $3\frac{3}{4}$%.

Answers.—(1) \$49.50; (2) \$266; (3) \$213.75; (4) \$35.85+; (5) \$41.36—; (6) \$77.53—; (7) \$201.69; (8) \$407.04—; (9) \$1,164.48—; (10) \$303.06.

SIX-PER-CENT. METHOD.

15. Assuming that a year is composed of 12 months, each consisting of 30 days, it is clear that at 6% the interest of \$1 for 1 year is 6 cents, or \$.06; that for 1 month it is $\frac{1}{12}$ of \$.06, which is equal to 5 mills, or \$.005; and that for $\frac{1}{5}$ of 30 days, or 6 days, it is 1 mill, or \$.001. Or, in tabular form,

Interest of \$1 for 1 year = \$.06.
Interest of \$1 for 1 month = \$.005.
Interest of \$1 for 6 days = \$.001.

Hence, the interest of \$1 for any other time will be \$.06 for each year, \$.005 for each month, and \$.000$\frac{1}{6}$ for each day. The sum of these three results will be the interest of \$1 for the given time.

EXAMPLE.—What is the interest of \$1 for 5 yr. 7 mo. 21 da. at 6% ?

SOLUTION.—

Interest of \$1 for 5 yr.	=	\$.06 × 5	=	\$.3 0
Interest of \$1 for 7 mo.	=	.005 × 7	=	.0 3 5
Interest of \$1 for 21 da.	=	.000$\frac{1}{6}$ × 21	=	.0 0 3 5

Interest of \$1 for 5 yr. 7 mo. 21 da. at 6% = \$.3 3 8 5. Ans.

Now, if we know the interest of \$1, it is simply a matter of multiplication to find the interest of any other number of dollars, or of dollars and cents.

Again, if we know the interest at 6% we may obtain the interest at 1% by dividing the interest at 6% by 6; and having the interest at 1%, we may find it at 4% by multiplying the interest at 1% by 4; similarly, for any other per cent.

EXAMPLE.—What is the interest of \$654 for 3 yr. 9 mo. 28 da. at 5% ?

SOLUTION.—

Interest of \$1 at 6% for 3 yr.	=	.06 × 3	=	\$.1 8
Interest of \$1 at 6% for 9 mo.	=	.005 × 9	=	.0 4 5
Interest of \$1 at 6% for 28 da.	=	.000$\frac{1}{6}$ × 28	=	.0 0 4 $\frac{2}{3}$

Interest of \$1 at 6% for 3 yr. 9 mo. 28 da. = \$.2 2 9 $\frac{2}{3}$

\$.229$\frac{2}{3}$ × 654 = \$150.202 = interest at 6%.

\$150.202 ÷ 6 = \$25.033$\frac{2}{3}$ = interest at 1%.

\$25.033$\frac{2}{3}$ × 5 = \$125.168$\frac{1}{3}$ = interest at 5%. Ans.

16. Rule.—*Find the interest of \$1 at 6% for the given time. To do this, multiply \$.06 by the number of years, \$.005 by the number of months, and \$.000$\frac{1}{6}$ by the number of days. Multiply the sum of these results by the number of dollars in the principal, and the result will be the interest at 6%. For any other rate, divide the interest at 6% by 6, and multiply the quotient by the given rate.*

17. In practice, it is better to find the interest at other rates than 6% by adding to, or subtracting from, the result for 6% suitable parts of itself.

The following partial table will illustrate:

$$\text{Interest at 6\%} + \begin{cases} \frac{1}{6} \text{ of itself} = \text{Interest at 7\%.} \\ \frac{1}{3} \text{ of itself} = \text{Interest at 8\%.} \\ \frac{1}{2} \text{ of itself} = \text{Interest at 9\%.} \\ \frac{1}{4} \text{ of itself} = \text{Interest at } 7\frac{1}{2}\text{\%.} \\ \frac{1}{12} \text{ of itself} = \text{Interest at } 6\frac{1}{2}\text{\%.} \end{cases}$$

$$\text{Interest at } 6\% - \begin{cases} \frac{1}{6} \text{ of itself} = \text{Interest at } 5\%. \\ \frac{1}{4} \text{ of itself} = \text{Interest at } 4\tfrac{1}{2}\%. \\ \frac{1}{3} \text{ of itself} = \text{Interest at } 4\%. \\ \frac{1}{9} \text{ of itself} = \text{Interest at } 5\tfrac{1}{3}\%. \\ \frac{1}{12} \text{ of itself} = \text{Interest at } 5\tfrac{1}{2}\%. \end{cases}$$

In the case of unusual rates per cent., it may be necessary to add or subtract two or more quotients.

Example.—Suppose we have found the interest of a certain principal at 6% to be \$237.68. How shall we find the interest of the same principal at $4\frac{3}{4}\%$?

Solution.—

\$237.68	= Interest at 6%.
59.42	= Interest at $1\frac{1}{2}\% = 6\% \div 4$.
\$178.26	= Interest at $4\frac{1}{2}\% = 6\% - 1\frac{1}{2}\%$.
9.90+	= Interest at $\frac{1}{4}\% = 1\frac{1}{2}\% \div 6$.
\$188.16	= Interest at $4\frac{3}{4}\% = 4\frac{1}{2}\% + \frac{1}{4}\%$. Ans.

18. When the time for which the interest is to be computed is less than a year, it is customary to use the following method for finding the interest at 6%:

Since the interest for 1 day at 6% is $\$.000\frac{1}{6}$ (see Art. **15**), the interest for any number of days may be found by multiplying the principal by the number of days, moving the decimal point *three* places to the left, and dividing the result by 6. It is usually easier, however, to divide the number of days by 6, multiply the quotient by the principal, and move the decimal point three places to the left.

Example.—Find the interest of \$1,215 for 86 days at 6%.

Solution.— $86 \div 6 = 14\frac{1}{3}$; \$1.215 = the principal with the decimal point moved three places to the left; $\$1.215 \times 14\frac{1}{3} = \$17.415$, or \$17.42–. Ans.

When using this method, retain any fraction that may arise from dividing by 6. Thus, for 83 days, multiply by $13\frac{5}{6}$, rather than reduce the fraction to a decimal.

EXAMPLES FOR PRACTICE.

19. By the foregoing method, find the interest

1. Of \$484 for 2 yr. 5 mo. 15 da. at 3%.
2. Of \$768 for 1 yr. 9 mo. 20 da. at 4%.
3. Of \$3,825 for 3 yr. 6 mo. 24 da. at 5%.

4. Of $9,600 for 4 yr. 7 mo. 27 da. at $3\frac{1}{2}$%.
5. Of $168.75 for 2 yr. 11 mo. 23 da. at $4\frac{1}{2}$%.
6. Of $437.50 for 5 yr. 8 mo. 18 da. at $5\frac{1}{2}$%.
7. Of $627.40 for 4 yr. 10 mo. 14 da. at $7\frac{1}{2}$%.
8. Of $969.96 for 3 yr. 9 mo. 22 da. at 7%.
9. Of $1,237.50 for 7 yr. 2 mo. 26 da. at $2\frac{3}{4}$%.
10. Of $1,875.60 for 12 yr. 3 mo. 10 da. at $3\frac{2}{3}$%.
11. Of $784.15 for 57 da.
12. Of $4,225 for 126 da.

Answers.—(1) $35.695; (2) $55.47–; (3) $682.13–; (4) $1,565.20; (5) $22.63+; (6) $137.56–; (7) $229.26+; (8) $258.76+; (9) $246.35–; (10) $844.37–; (11) $7.45–; (12) $88.73–.

SIXTY-DAY METHOD.

20. For 1 year, at 6%, the interest of any principal is .06 of the principal itself, and for 2 months, or 60 days, the interest is .01 of the principal. Hence,

If the decimal point of any sum be moved two places to the left, it will give the interest of that sum for 60 days at 6%.

Thus, the interest of $3,472.75 for 60 days at 6% is $34.73–, and of $692 it is $6.92.

Having the interest for 60 days, it is easy, by operations that will suggest themselves, to find the interest for any other number of days.

Example 1.—Find the interest of $8,368 for 99 days at 6%.

Solution.—

$83.68	= Interest for 60 days.
41.84	= Interest for 30 days = $\frac{1}{2}$ of 60 days.
8.368	= Interest for 6 days = $\frac{1}{10}$ of 60 days.
4.184	= Interest for 3 days = $\frac{1}{2}$ of 6 days.
$138.072	= Interest for 99 days. Ans.

Example 2.—What is the interest at 9% of $1,264.76 for 49 days?

Solution.—

$12.6476	= Interest for 60 days at 6%.
6.3238	= Interest for 30 days = $\frac{1}{2}$ of 60 days.
3.1619	= Interest for 15 days = $\frac{1}{2}$ of 30 days.
.6324	= Interest for 3 days = $\frac{1}{10}$ of 30 days.
.2108	= Interest for 1 day = $\frac{1}{3}$ of 3 days.
$10.3289	= Interest for 49 days, at 6%.
5.1644	= Interest for 49 days, at 3%.
$15.4933	≐ Interest for 49 days, at 9%. Ans.

21. **Rule.**—*Take .01 of the principal for the interest at 6% for 60 days, and then, by the method of aliquot parts, find the interest for the given time at the rate specified.*

EXAMPLES FOR PRACTICE.

22. By the sixty-day method, find the interest

1. Of $8,000 for 87 days at 6%.
2. Of $6,050 for 96 days at 3%.
3. Of $875.28 for 77 days at $3\frac{1}{2}$%.
4. Of $1,468.80 for 123 days at 4%.
5. Of $23,750 for 108 days at $4\frac{1}{2}$%.
6. Of $42,690 for 176 days at $3\frac{2}{3}$%.
7. Of $7,200 for 225 days at 5%.
8. Of $468.24 for 101 days at $5\frac{1}{2}$%.
9. Of $6,880 for 186 days at 7%.
10. Of $7,600 for 143 days at $7\frac{1}{2}$%.

Answers.—(1) $116; (2) $48.40; (3) $6.55+; (4) $20.07+; (5) $320.625; (6) $765.26–; (7) $225; (8) $7.23–; (9) $248.83–; (10) $226.42–.

EXACT INTEREST.

23. When interest is to be computed for one or more entire years at a specified rate per year, the fact that 12 months of 30 days each are usually regarded as a year does not affect the result—it is only when months and days, or days alone, become an element of the given time, that the interest is greater than it should be. The average length of a month in an ordinary year is $30\frac{5}{12}$ days, and in a leap year it is $30\frac{1}{2}$ days. A day is not $\frac{1}{360}$ of a year, but $\frac{1}{365}$ of a common year, and $\frac{1}{366}$ of a leap year. Hence, 360 days $= \frac{360}{365}$, or $\frac{72}{73}$, of a common year, and $\frac{360}{366}$, or $\frac{60}{61}$, of a leap year. By the ordinary method of finding interest, the result is either $\frac{1}{73}$ or $\frac{1}{61}$ greater than it should be.

Thus, the interest of $7,300 for 60 days at 6%, as found by the usual method, is $73. In equity it is $\$7,300 \times .06 \times \frac{60}{365} = \$72$. That is, each $73 interest should be $72.

24. The method practised by the government and by most banks is to compute the interest for the number of

entire years in a period, and then treat the remaining days as so many 365ths of a year.

Example.—Find the exact interest of \$8,000 from Jan. 5, 1873, to July 23, 1880, at 6%.

Solution.—

From Jan. 5, 1873, to Jan. 5, 1880 = 7 yr.

From Jan. 5, 1880, to July 23, 1880 =

Jan. Feb. Mar. Apr. May June July

$26 + 29 + 31 + 30 + 31 + 30 + 23 = 200$ days $= \frac{200}{366} = \frac{100}{183}$ yr.

$\$8,000 \times .06 \times 7\frac{100}{183} = \$3,622.30-$. Ans.

25. The same result is obtained by adding to the interest for 7 years the interest for 200 days less $\frac{1}{61}$ of itself, found by the usual method. Thus,

$\$8,000 \times .06 \times 7 =$		\$3,360.00
$\$8,000 \times .06 \times \frac{200}{360} =$	\$266.67	
Deducting $\frac{1}{61}$ of \$266.67	4.37 =	262.30
Exact interest for 7 yr. 200 da. =		\$3,622.30 Ans.

Example.—What is the exact interest of \$4,800 for 198 days of an ordinary year, at 6% ?

Solution.—

\$ 48.00	= Interest for 60 days.
96.00	= Interest for 120 days.
12.00	= Interest for 15 days.
2.40	= Interest for 3 days.
73) \$ 158.40	= Interest for 198 days, counting 360 days of the year.
2.17	
\$ 156.23	= Exact interest for 198 days. Ans.

Explanation.—The interest is first found by the usual method, and $\frac{1}{73}$ of the result deducted.

26. Rule.—*Find the interest by the ordinary method for the whole number of years included in the period. Count the number of days that remain, and by the same method find the interest for the days. If the days are part of an ordinary year, diminish the interest for the days by $\frac{1}{73}$ of itself; if they are part of a leap year, diminish the interest by $\frac{1}{61}$ of itself. Add the interest for the years to that for the days, and the result will be the exact interest.*

EXAMPLES FOR PRACTICE.

27. Find the exact interest

1. Of \$10,000 for 123 days at 6%.
2. Of \$12,800 for 168 days at 6%.
3. Of \$6,400 for 213 days at 5%.
4. Of \$22,800 for 2 yr. 73 da. at 4%.
5. Of \$960 for 5 yr. 300 da. at $3\frac{1}{2}$%.
6. Of \$484.80 for 6 yr. 202 da. at $5\frac{1}{2}$%.
7. Of \$13,000 from Jan. 17, 1897, to Nov. 29, 1897, at $4\frac{1}{2}$%.
8. Of \$968.40 from Apr. 19, 1865, to July 1, 1896, at 3%.
9. Of \$1,234.60 from Dec. 23, 1888, to Mar. 17, 1890, at 6%.
10. Of \$43,000 from May 29, 1891, to Nov. 3, 1895, at 7%.

Answers.—(1) \$202.19+; (2) \$353.49; (3) \$186.74–; (4) \$2,006.40; (5) \$195.62–; (6) \$174.74; (7) \$506.47–; (8) \$906.41–; (9) \$91.12+; (10) \$13,342.96–.

ANNUAL INTEREST.

28. Unless otherwise specified, interest upon debts is understood to be payable annually. In case it is not so paid, it is permitted in some States to charge interest upon overdue interest. In some other States this practice is illegal. Where it is intended to charge "annual interest," the written obligation should contain the words "interest payable annually."

EXAMPLE.—What is the interest at 6% of \$2,400 for 6 years 6 months, interest payable annually, if no interest is paid until the end of the time?

SOLUTION.—

$$\begin{aligned} \text{Interest for 1 year} &= \$2{,}400 \times .06 = \$144.00 \\ \text{Interest for 6 yr. 6 mo.} &= \$144 \times 6\tfrac{1}{2} = \$936.00 \\ \text{Interest of \$144 for } 5\tfrac{1}{2} + 4\tfrac{1}{2} + 3\tfrac{1}{2} + 2\tfrac{1}{2} + 1\tfrac{1}{2} + \tfrac{1}{2} & \\ = \text{Interest for 18 yr.} &= \$144 \times .06 \times 18 = 155.52 \\ & \$1{,}091.52 \quad \text{Ans.} \end{aligned}$$

EXPLANATION.—The first year's interest remains unpaid for $5\frac{1}{2}$ yr., the second for $4\frac{1}{2}$ yr., etc. The sum of these periods is 18 yr. One year's interest of the principal is \$144, and the interest of this for 18 yr. is \$155.52. The sum of \$936 and \$155.52 is the entire interest due.

29. **Rule.**—*Find the interest of the principal for one year, and for the entire time the debt runs. Find the sum of the several periods that the annual interest remains unpaid, and for this time find the interest of one year's interest of the debt. The sum of the interest of the main debt and that of the unpaid annual interest will be the result required.*

EXAMPLES FOR PRACTICE.

30. In the following examples, assume that the annual interest is payable but unpaid, and find the entire interest due at the end of the given time:

1. Of \$240 for 2 yr. 6 mo. at 6%.
2. Of \$380 for 3 yr. 8 mo. at 5%.
3. Of \$1,000 for 4 yr. 9 mo. at 7%.
4. Of \$1,200 for 3 yr. 3 mo. 15 da. at 6%.
5. Of \$387.50 for 5 yr. 6 mo. 20 da. at 4%.
6. Of \$7,625 for 8 yr. 7 mo. 18 da. at 5%.

Answers.—(1) \$37.73– ; (2) \$74.42– ; (3) \$376.60; (4) \$253.74; (5) \$94.03+ ; (6) \$3,921.79+.

PROBLEMS IN INTEREST.

31. Given the interest, rate, and time, to find the principal.

We know that I = Prin. × rate × time (in years); or, more briefly,

$$\frac{P\,r\,t}{100} = I,$$

when r is the rate per cent. Hence, multiplying each side by 100, we have $P\,r\,t = 100\,I$. Now, if we divide each side by $r\,t$, we have $\frac{P\,r\,t}{r\,t} = \frac{100\,I}{r\,t}$. Canceling $r\,t$ of the left member of this equation, we have

$$P = \frac{100\,I}{r\,t}.$$

That is, *the principal is equal to 100 times the interest divided by the product of the rate per cent. and the time.*

EXAMPLE.—What principal in 4 yr. 9 mo. at 5% will give $152 interest ?

SOLUTION.—Applying the formula and noticing that 9 mo. = $\frac{3}{4}$ yr.,

$$P = \frac{152 \times 100}{5 \times 4\frac{3}{4}} = \frac{\overset{8}{\cancel{152}} \times \overset{20}{\cancel{100}} \times 4}{\cancel{5} \times \cancel{19}} = \$640. \quad \text{Ans.}$$

EXPLANATION.— $4\frac{3}{4}$ being in the divisor, the fraction must be reduced to the form of an improper fraction and inverted. The rate also appears in the inverted form of $\frac{100}{5}$, since .05 cannot be inverted.

EXAMPLE.—What principal at $3\frac{2}{5}$% will, in 2 yr. 3 mo. 15 da., give $374 interest ?

SOLUTION.—Changing the time to $\frac{55}{24}$ years, writing $3\frac{2}{5}$% as $\frac{17}{5}$, and applying the formula, we have

$$P = \frac{374 \times 100}{\frac{17}{5} \times \frac{55}{24}} = \frac{\overset{\overset{2}{\cancel{22}}}{\cancel{374}} \times 100 \times \cancel{5} \times 24}{\cancel{17} \times \underset{\cancel{5}}{\cancel{55}}} = \$4,800. \quad \text{Ans.}$$

32. Rule.—*To find the principal when the interest, rate, and time are given, divide 100 times the interest by the product of the time in years and the rate per cent.*

EXAMPLES FOR PRACTICE.

33. Find the principal, when

1. Interest = $96, time = 2 years, rate = 4%.
2. Interest = $131.25, time = 2 yr. 6 mo., rate = 6%.
3. Interest = $62.25, time = 2 yr. 3 mo. 20 da., rate = $4\frac{1}{2}$%.
4. Interest = $60, time = 180 days, rate = 5%.
5. Interest = $546, time = 3 yr. 5 mo. 18 da., rate = $3\frac{1}{2}$%.
6. Interest = $23.75, time = 1 yr. 8 mo. 24 da., rate = $5\frac{1}{2}$%.
7. Interest = $43.60, time = 2 yr. 11 mo. 12 da., rate = 7%.
8. Interest = $124.30, time = 3 mo. 20 da., rate = $3\frac{2}{3}$%.

Answers.—(1) $1,200; (2) $875; (3) $600; (4) $2,400; (5) $4,500; (6) $249.13– ; (7) $211.14– ; (8) $11,094.55–.

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34. Given the principal, the interest, and the time, to find the rate per cent.

Resuming the equation,

$$P r t = 100\ I;$$

dividing each side by $P t$, we have

$$\frac{P r t}{P t} = \frac{100\ I}{P t}, \text{ or } r = \frac{100\ I}{P t}.$$

That is, rate per cent. = (interest $\times$ 100) $\div$ (principal $\times$ time).

Example.—At what rate per cent. will \$480 in 3 years 10 months give \$92 interest ?

Solution.—Applying the formula,

$$r = \frac{100 \times 92}{480 \times 3\frac{5}{6}} = \frac{\overset{5}{\cancel{100}} \times \overset{4}{\cancel{92}} \times \cancel{6}}{\underset{\underset{\cancel{20}}{\cancel{120}}}{\cancel{480}} \times \cancel{23}} = 5\%. \quad \text{Ans.}$$

35. Rule.—*Change the time to years, and divide 100 times the interest by the product of the principal and the time.*

EXAMPLES FOR PRACTICE.

36. Find the rate per cent. when

1. Principal = \$2,875, time = 4 yr. 7 mo. 6 da., interest = \$529.
2. Principal = \$760, time = 3 yr. 9 mo. 18 da., interest = \$144.40.
3. Principal = \$1,260, time = 2 yr. 1 mo. 10 da., interest = \$119.70.
4. Principal = \$2,340, time = 2 yr. 6 mo. 20 da., interest = \$328.90.
5. Principal = \$4,870, time = 3 yr. 5 mo. 24 da., interest = \$1,017.83.
6. Principal = \$7,200, time = 123 days, interest = \$114.80.
7. Principal = \$1,500, time = 1 yr. 9 mo. 18 da., interest = \$99.
8. Principal = \$1,600, time = 5 yr. 7 mo. 6 da., interest = \$380.80.

Answers.—(1) 4%; (2) 5%; (3) $4\frac{1}{2}$%; (4) $5\frac{1}{2}$%; (5) 6%; (6) $4\frac{2}{3}$%; (7) $3\frac{2}{3}$%; (8) $4\frac{1}{4}$%.

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37. Given the principal, interest, and rate per cent., to find the time.

If we divide the equation $P r t = 100\ I$ by $P r$, we shall have

$$\frac{P r t}{P r} = \frac{100\ I}{P r}, \text{ or } t = \frac{100\ I}{P r}.$$

Otherwise expressed, this means that

time (in years) = (100 × interest) ÷ (principal × rate per cent.)

EXAMPLE.—In what time will \$4,480 at 6% give \$871.36 interest?

SOLUTION.—Applying the formula,

$$\text{time} = \frac{100 \times 871.36}{4{,}480 \times 6} = \frac{\overset{389}{\cancel{87136}}}{\underset{20}{\cancel{4480}} \times 6} = \frac{389}{120}\text{ yr.}$$

$$\tfrac{389}{120}\text{ yr.} = 3\tfrac{29}{120}\text{ yr.};\ \frac{29}{120} \times 12 = \frac{29}{10}\text{ mo.} = 2\tfrac{9}{10}\text{ mo.};\ \frac{9}{10} \times 30 = 27\text{ da.}$$

EXPLANATION.—The result is obtained in years, and must be reduced to years, months, and days. $\frac{389}{120}$ years $= 3\frac{29}{120}$ years. Since there are 12 months in one year, $\frac{29}{120}$ of a year $= \frac{29 \times 12}{120}$ $= \frac{29}{10}$ months, or $2\frac{9}{10}$ months. Since there are 30 days in one month, $\frac{9}{10}$ of a month $= \frac{9 \times 30}{10} = 27$ days. Hence, the required time is 3 yr. 2 mo. 27 da. Ans.

38. Rule.—*To find the time, when the principal, interest, and rate are given, divide 100 times the interest by the product of the principal and the rate per cent., and reduce the result, which is years, to years, months, and days.*

EXAMPLES FOR PRACTICE.

39. Find the time, when

1. Principal = \$4,800, interest = \$652, rate = 6%.
2. Principal = \$680, interest = \$163.20, rate = 5%.
3. Principal = \$360, interest = \$26.325, rate = $4\frac{1}{2}$%.
4. Principal = \$338.75, interest = \$35.23, rate = 4%.
5. Principal = \$1,080, interest = \$112.50, rate = 3%.
6. Principal = \$1,800, interest = \$63, rate = $3\frac{1}{2}$%.
7. Principal = \$1,050, interest = \$45.50, rate = 5%.
8. Principal = \$1,000, interest = \$143, rate = $6\frac{1}{2}$%.
9. Principal = \$2,400, interest = \$74.80, rate = $5\frac{1}{2}$%.

Answers.—(1) 2 yr. 3 mo. 5 da.; (2) 4 yr. 9 mo. 18 da.; (3) 1 yr. 7 mo. 15 da.; (4) 2 yr. 7 mo. 6 da.; (5) 3 yr. 5 mo. 20 da.; (6) 1 yr.; (7) 10 mo. 12 da.; (8) 2 yr. 2 mo. 12 da.; (9) 6 mo. 24 da.

40. Given the amount, time, and rate, to find the principal.

Since at 6% the interest of \$1 for, say, 2 years, is 12 cents, the amount is \$1 + 12 cents = \$1.12. Each \$1 of the principal will, in like manner, amount to \$1.12. Hence, if we divide the amount of any principal in a given time at a specified rate by the amount of \$1 for that time and rate, it will give the number of times \$1 is contained in the principal.

Or using the rate per cent. instead of the rate, we may, by means of the following formula, express the process of finding the principal:

$$P = \frac{100\,A}{100 + rt}.$$

That is, (100 × amount) ÷ (100 + rate per cent. × time) = principal.

Example.—What principal will, in 5 yr. 6 mo., at 4%, amount to \$591.70?

Solution.—Applying the formula,

$$P = \frac{100 \times 591.70}{100 + 4 \times 5\frac{1}{2}} = \frac{59{,}170}{122} = \$485. \quad \text{Ans.}$$

41. Rule.—*To find the principal, when the amount, rate, and time are given, divide 100 times the amount by 100 increased by the product of the rate per cent. and the time.*

Note.—In using this rule, and the formula given in Art. **40**, care must be taken that the time be expressed in years and fractions of a year.

EXAMPLES FOR PRACTICE.

42. Find the principal that will amount

1. To \$1,005 in 5 yr. 8 mo. at 6%.
2. To \$3,459 in 3 yr. 4 mo. 24 da. at $4\frac{1}{2}$%.
3. To \$2,985 in 2 yr. 1 mo. 10 da. at 5%.
4. To \$1,443.60 in 2 yr. 10 mo. 24 da. at 7%.
5. To \$2,353.50 in 1 yr. 6 mo. 12 da. at 3%.

Answers.—(1) \$750; (2) \$3,000; (3) \$2,700; (4) \$1,200; (5) \$2,250.

TRUE DISCOUNT.

43. The student has learned that any deduction made from a debt or other obligation is a *discount.* In making such deductions, the element of *time* may or may not be considered. When time is considered, we have one of the applications of interest. **True discount** is discount when time is considered and *no interest is allowed on the discount.* True discount corresponds exactly to the problems of Interest given in Arts. **40–42**—the case in which the amount, rate, and time are given, to find the principal. The terms employed, however, are different.

The *principal* is called the **present worth.**

The *rate* is called the **rate of discount.**

The *interest* is called the **true discount.**

The *amount* is called the **debt,** or **obligation.**

True discount is so called to distinguish it from *bank discount*, which will be treated later.

44. The *present worth* of an obligation is a sum such that, if it be put at interest at a specified rate for a given time, it will amount to the obligation.

Thus, if the specified rate is 5%, a debt of $105 due in one year is worth $100 *now*, since $100 placed at interest at 5% will in one year amount to $105.

45. *True discount* is the difference between a debt due at a future time and its present worth.

Thus, $5 in the illustration given above is the true discount of $105 due in one year, when the rate of discount is 5%.

46. Given the debt, rate of discount, and time, to find the present worth and the discount.

The present worth may be found by means of the formula of Art. **40,** or by the rule of Art. **41.** Thus,

$$P = \frac{100\,A}{100 + r\,t}, \text{ or present worth } = \frac{100 \times \text{debt}}{100 + r\,t}.$$

EXAMPLE.—A debt of \$773, which has 1 yr. 5 mo. 20 da. yet to run, is discounted at 5%. What is (*a*) the present worth, and (*b*) the discount?

SOLUTION.—1 yr. 5 mo. 20 da. $= 1\frac{17}{36}$ years.

$$(a)\quad \text{Present worth} = \frac{100 \times 773}{100 + 1\frac{17}{36} \times 5}$$

$$= \frac{77{,}300 \times 36}{3{,}865} = \$720. \quad \text{Ans.}$$

(*b*) Discount $= \$773 - \$720 = \$53$. Ans.

47. If the debt bears interest, its amount for the time it has to run must first be found, and that sum discounted at the specified rate.

EXAMPLE.—A debt of \$1,200 having 1 yr. 6 mo. to run bears interest at 6%. What is its present worth discounted at 5%?

SOLUTION.—Amount of \$1,200 in 1 yr. 6 mo. = \$1,308.

Applying the formula,

$$\text{Present worth} = \frac{100 \times 1{,}308}{100 + 1\frac{1}{2} \times 5} = \frac{130{,}800}{107.5} = \$1{,}216.74+. \quad \text{Ans.}$$

48. Rule.—I. *Divide 100 times the amount of the debt when it is due, by 100 increased by the product of the time and the rate of discount. The result will be the present worth.*

II. *Subtract the present worth from the debt, and the remainder will be the discount.*

EXAMPLES FOR PRACTICE.

49. What is the present worth and the discount

1. Of \$900 due in 9 months, discounted at 4%?
2. Of \$1,000 due in 1 yr. 6 mo., discounted at 6%?
3. Of \$800 due in 5 mo., discounted at 5%?
4. Of \$2,800 due in 1 yr. 3 mo. 12 da., discounted at 8%?
5. Of \$625 due in 2 yr. 5 mo. 15 da., discounted at 7%?
6. Of a note for \$600 bearing interest at 5%, having 2 yr. 10 mo. to run, and discounted at 6%?

Answers.—(1) \$873.79–, \$26.21+; (2) \$917.43+, \$82.57–; (3) \$783.67+, \$16.33–; (4) \$2,539.30–, \$260.70+; (5) \$533.24–, \$91.76+; (6) \$585.47, \$99.53.

COMPOUND INTEREST.

50. If the interest of a principal is added to the principal at regular intervals to form by each addition a new principal for the next interval, the resulting interest is called **compound interest.**

Thus, if $100 be placed at compound interest at 6%, with the understanding that the interest is to be compounded annually, the principal will be $100 for the first year, $106 for the second year, $112.36 for the third year, etc.

51. Most savings banks allow compound interest, although in most States its payment cannot be legally enforced, even though it be specified in a contract.

Unless otherwise stated, interest is understood to be compounded annually. If it be compounded semiannually, one-half the annual rate is taken as the rate; if quarterly, one-fourth the annual rate is taken; etc.

52. When the time is given in years, months, and days, the interest is compounded for the greatest number of entire periods included in the time, and the simple interest of the last principal is found for the remaining time.

EXAMPLE.—Find the compound interest of $800 for 1 yr. 9 mo. 20 da. at 6%, interest compounded semiannually.

SOLUTION.—

$800	= prin. 1st 6 mo.
24	= int. 1st 6 mo. = $800 × .03.
$824	= prin. 2d 6 mo.
24.72	= int. 2d 6 mo. = $824 × .03.
$848.72	= prin. 3d 6 mo.
25.46	= int. 3d 6 mo. = $848.72 × .03.
$874.18	= prin. for 3 mo. 20 da.
16.03	= int. for 3 mo. 20 da. = $874.18 × .06 × $\frac{11}{36}$.
$890.21	= amt. for 1 yr. 9 mo. 20 da.
800	= original prin.
$90.21	= comp. int. for 1 yr. 9 mo. 20 da. Ans.

EXPLANATION.—In 1 yr. 9 mo. 20 da. there are three complete periods of 6 mo. each, and 3 mo. 20 da. besides. Since the

annual rate is 6%, for 6 mo. the rate per cent. is 3%. Finding the interest at 3%, and adding the principal for these three periods, gives \$874.18. The amount of this sum for the remaining 3 mo. 20 da. is \$890.21, from which we subtract the original principal. The remainder, \$90.21, is the required compound interest.

53. Compound interest is calculated in actual business by means of a table. The table shows the amount of \$1 at all the different rates, and for all the different times that are likely to occur. Having the amount of \$1 at any given rate and for any number of periods, we multiply it by the number of dollars in any given principal. The result will be the amount of that sum for the given time. If the original principal be subtracted from this amount, the remainder is the compound interest required.

The formula for compound interest is, therefore,

$$\text{Comp. Int.} = P(1+r)^n - P.$$

In this formula, r is the rate, and n the number of periods —it is an exponent expressing the power to which $1+r$ must be raised to give the amount of \$1 for that number of periods.

EXAMPLE.—What is the compound interest of \$600 for 4 years at 6%, interest compounded annually?

SOLUTION.—Applying the formula,

$$\text{Comp. int.} = \$600\,(1+.06)^4 - \$600.$$
$$1.06^4 = 1.262477;\ \$600 \times 1.262477 = \$757.4862;$$
$$\$757.4862 - \$600 = \$157.49. \quad \text{Ans.}$$

54. In the foregoing formula, r is the rate for one of the equal intervals of time—it is not necessarily the rate per year. For instance, if the interest is compounded quarterly, r is one-fourth of the rate per year; if compounded semiannually, r is one-half the rate per year. In the following table, the numbers in the columns headed "$2\frac{1}{2}$ per cent.," "3 per cent.," etc., are the values obtained by raising $1+r$ to the power whose exponent is the number of complete periods during which the interest is compounded. Thus, in the column headed "4 per cent.,"

COMPOUND INTEREST TABLE.

Showing the amount of $1, at various rates, compound interest, from 1 to 20 years, interest compounded annually.

Yr.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.
1	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000
2	1.050625	1.060900	1.071225	1.081600	1.102500	1.123600
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519
7	1.188686	1.229874	1.272279	1.315932	1.407100	1.503630
8	1.218403	1.266770	1.316809	1.368569	1.477455	1.593848
9	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479
10	1.280085	1.343916	1.410599	1.480244	1.628895	1.790848
11	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299
12	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928
14	1.412974	1.512590	1.618695	1.731676	1.979932	2.260904
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339
19	1.598650	1.753506	1.922501	2.106849	2.526950	3.025600
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136

Yr.	7 per cent.	8 per cent.	9 per cent.	10 per cent.	11 per cent.	12 per cent.
1	1.070000	1.080000	1.090000	1.100000	1.110000	1.120000
2	1.144900	1.166400	1.188100	1.210000	1.232100	1.254400
3	1.225043	1.259712	1.295029	1.331000	1.367631	1.404908
4	1.310796	1.360489	1.411582	1.464100	1.518070	1.573519
5	1.402552	1.469328	1.538624	1.610510	1.685058	1.762342
6	1.500730	1.586874	1.677100	1.771561	1.870414	1.973822
7	1.605781	1.713824	1.828039	1.948717	2.076160	2.210681
8	1.718186	1.850930	1.992563	2.143589	2.304537	2.475963
9	1.838459	1.999005	2.171893	2.357948	2.558036	2.773078
10	1.967151	2.158925	2.367364	2.593742	2.839420	3.105848
11	2.104852	2.331639	2.580426	2.853117	3.151757	3.478549
12	2.252192	2.518170	2.812665	3.138428	3.498450	3.895975
13	2.409845	2.719624	3.065805	3.452271	3.883279	4.363492
14	2.578534	2.937194	3.341727	3.797498	4.310440	4.887111
15	2.759031	3.172169	3.642482	4.177248	4.784588	5.473565
16	2.952164	3.425943	3.970306	4.594973	5.310893	6.130392
17	3.158815	3.700018	4.327633	5.054470	5.895091	6.866040
18	3.379932	3.996019	4.717120	5.559917	6.543551	7.689964
19	3.616527	4.315701	5.141661	6.115909	7.263342	8.612760
20	3.869684	4.660957	5.604411	6.727500	8.062309	9.646291

1.872981 is the value of 1.04^{16} to 6 places of decimals; it is also the amount at compound interest of $1 for 16 years at 4%,

compounded annually, of \$1 for 8 years at 8%, compounded semiannually, and of \$1 for 4 years at 16%, compounded quarterly, since in each case the number of periods is 16 and the rate is .04. Hence, if it was required to find the amount at compound interest of \$1 compounded annually, it may be obtained for any rate per cent. from 1 to 20 years from the table; if compounded semiannually, find the number of periods, divide the annual rate per cent. of interest by 2, and treat these results as "years" and "rate per cent.," respectively, when using the table; if compounded quarterly, divide the annual rate per cent. of interest by 4.

EXAMPLE.—Find the amount of \$1 at compound interest for 3 yr. 6 mo. at 7% when compounded (*a*) annually; (*b*) semiannually; (*c*) quarterly.

SOLUTION.—(*a*) Referring to the table, the amount for 3 yr. at 7% is \$1.225043. The simple interest for 6 mo. on \$1.225043 at 7% is \$1.225043 $\times$ .035 = \$.042877–. The total amount is \$1.225043 + \$.042877– = \$1.267920–. Ans.

(*b*) When the interest is compounded semiannually, there are $3\frac{1}{2} \times 2 = 7$ periods in 3 yr. 6 mo.; $7\% \div 2 = 3\frac{1}{2}\%$. Referring to the table, we find in the column headed "$3\frac{1}{2}$ per cent.," opposite 7, in column of years, 1.272279; hence, when the interest is compounded semiannually, the amount of \$1 for 3 yr. 6 mo. at 7% is \$1.272279. Ans.

(*c*) When the interest is compounded quarterly, there are $3\frac{1}{2} \times 4 = 14$ periods; $7\% \div 4 = 1\frac{3}{4}\%$. Since there is no column in the table headed "$1\frac{3}{4}$ per cent.," the amount must be calculated directly. The amount of \$1 is, then, $\$1\,(1 + .0175)^{14} = \$1 \times 1.0175^{14} = \$1.274913$. Ans.

55. Rule.—*Multiply the amount of \$1 for the given time and rate by the principal, and from the product subtract the principal. The remainder is the compound interest.*

If there is a partial period, find the amount of the last principal for this partial period, and then subtract the original principal.

EXAMPLES FOR PRACTICE.

56. Find the compound interest of the following amounts for the times mentioned, and prove the correctness of your result by using the table. Unless otherwise stated the interest is to be compounded annually.

1. Of $1,600 for 2 yr. at 8%.

2. Of $960 for 3 yr. 8 mo. at 6%.

3. Of $1,280 for 5 yr. 6 mo. at 4%.

4. Of $2,400 for 3 yr. 10 mo. 20 da. at 5%.

5. Of $360 for 4 yr. 4 mo. 12 da. at 6%.

6. Of $480 for 3 yr. 9 mo. 24 da. at 8%, interest compounded semi-annually.

7. Of $237.50 for 1 yr. 11 mo. 15 da. at 12%, interest compounded quarterly.

8. Of $3,875 for 2 yr. 11 mo. 12 da. at 6%, interest compounded semi-annually.

Answers.—(1) $266.24; (2) $229.11; (3) $308.46; (4) $501.78; (5) $104.49; (6) $167.65–; (7) $61.90–; (8) $738.48–.

PARTIAL PAYMENTS.

57. A debt or obligation may be discharged at one payment; or, from time to time, payments *in part* may be made, and finally at a time of *settlement* the remainder of the debt may be paid. Now, it is obvious that interest should be allowed upon such payments as are made, since interest is charged upon the obligation itself. But, if a payment should be less than the interest upon the debt since a previous payment had been made, to subtract such payment from the debt with accrued interest would result in increasing the principal. This would be a species of compound interest which, in many States, is illegal.

58. When a partial payment of a note is made, the date of payment and its amount are written upon the back of the note, and this record of it is called an **indorsement.**

The following rule for partial payments has been formulated by the Supreme Court of the United States, and has been adopted by most of the States:

59. United States Rule.—I. *Find the amount of the principal to the time when the payment, or the sum of the*

payments, is greater than the interest then due. From the amount subtract the payment or the sum of the payments, and treat the remainder as a new principal.

II. *Proceed in this manner to the date of settlement, and the last amount will be the sum still due.*

60. To compute partial payments by the United States rule.

EXAMPLE.—

$1,200. *New York, Sept. 16, 1895.*

On demand I promise to pay John Crawford, or order, Twelve Hundred Dollars, with interest at 6%, value received.

Edward G. Carson.

Indorsements: Jan. 1, 1896, $120; May 7, 1896, $300; Dec. 22, 1896, $16; Sept. 19, 1897, $400. What was due Jan. 1, 1898?

SOLUTION.—

Principal	$1200
Interest from Sept. 16, '95, to Jan. 1, '96 (3 mo. 15 da.)	21
Amount	1221
First payment	120
New principal	1101
Interest from Jan. 1, '96, to May 7, '96 (4 mo. 6 da.)	23.12
Amount	1124.12
Second payment	300
New principal	824.12
Interest from May 7, '96, to Sept. 19, '97 (1 yr. 4 mo. 12 da.)	67.58
Amount	891.70
Sum of third and fourth payments	416.00
New principal	475.70
Interest from Sept. 19, '97, to Jan. 1, '98 (3 mo. 12 da.)	8.09
Amount due at time of settlement	$ 483.79 Ans.

In this example, 360 days are considered as 1 year. The third payment of $16 is less than the interest due at the time it was made; hence, according to the rule, it is added to the next payment of $400 and the interest is computed to the time of the fourth payment.

EXAMPLES FOR PRACTICE.

61. Find the amount due at the time of settlement on each of the following:

1. *$2,000.* *Philadelphia, July 1, 1896.*

One year after date, for value received, I promise to pay Wm. Gray, or order, Two Thousand Dollars, with interest at 6%.

Henry G. Brown.

Indorsements: Dec. 16, 1896, $350; Mar. 1, 1897, $25; Oct. 25, 1897, $400; June 14, 1898, $275. Time of settlement, July 1, 1899.

2. A note for $3,000 at 6% dated Mar. 5, 1892, bore the following indorsements: Dec. 20, 1892, $400; Mar. 14, 1893, $60; Nov. 30, 1893, $360; July 15, 1894, $600. Time of settlement, Jan. 1, 1895.

3. A note for $4,000 at 6% dated Sept. 1, 1886, is indorsed as follows: Jan. 1, 1887, $500; July 1, 1887, $450; Jan. 1, 1888, $90; Sept. 1, 1888, $800. Time of settlement, Jan. 1, 1889.

4. Face of note, $3,600, rate 8%, dated May 1, 1890. Indorsements: Dec. 1, 1890, $600; Mar. 1, 1891, $100; Dec. 1, 1891, $800; July 1, 1892, $1,000. Time of settlement, Dec. 1, 1892.

Answers.—(1) $1,216.80; (2) $2,024.38; (3) $2,625.50; (4) $1,691.36.

When the time from the date of a note or other obligation is less than a year, settlement is usually made by a method called the *merchants' rule*.

62. The Merchants' Rule.—I. *By the method of exact interest, find the amount of each of the several payments from the time each is made to the date of settlement.*

II. *Subtract the sum of these amounts from the amount of the obligation from its date to the time of settlement. The remainder will be the amount still due.*

EXAMPLE.—Face of note, $2,000; rate, 6%; date of note, Dec. 31, 1888; time of settlement, Nov. 15, 1889. Indorsements: Mar. 10, 1889, $200; June 1, 1889, $300; Aug. 20, 1889, $400; Oct. 1, 1889, $500. What was due at time of settlement?

SOLUTION.—

Principal..	$2000
Interest of $2,000 for *319 da............................	104.88
Amount...	$2104.88

* Table XX, § 4, will be found very useful for finding the number of days between two dates.

Amount brought forward		$ 2 1 0 4.8 8
Amount of $200 for 250 da	$ 2 0 8.2 2	
Amount of $300 for 167 da	$ 3 0 8.2 4	
Amount of $400 for 87 da	$ 4 0 5.7 2	
Amount of $500 for 45 da	$ 5 0 3.7 0	
Sum of payments, with interest		$ 1 4 2 5.8 8
Amount due at time of settlement		$ 6 7 9.0 0 Ans.

EXAMPLES FOR PRACTICE.

63. Solve the following examples by the merchants' rule:

1. Debt, $5,000; rate, 5%; date of note, July 1, 1894; date of settlement, June 15, 1895. Indorsements: Sept. 10, 1894, $1,000; Dec. 12, 1894, $800; Mar. 12, 1895, $1,200; May 1, 1895, $1,200. What is due at the time of settlement?

2. Face of note, $8,400; rate, 4%; date of note, Jan. 1, 1897; time of settlement, Oct. 20, 1897. Indorsements: Mar. 30, 1897, $1,600; June 1, 1897, $800; Aug. 10, 1897, $2,000; Sept. 15, 1897, $3,000. What is due on the note at time of settlement?

3. Debt, $5,000; rate, 6%; date of note, Nov. 1, 1897; time of settlement, May 20, 1898. Indorsements: Dec. 20, 1897, $900; Jan. 1, 1898, $1,200; Mar. 24, 1898, $1,200; Apr. 20, 1898, $400. What is due at time of settlement?

Answers.—(1) $957.67; (2) $1,193.60; (3) $1,401.41.

PROMISSORY NOTES.

64. A **promissory note** is a written promise to pay a certain sum at a certain time.

65. The **maker** or **drawer** of a note is the person that promises to pay; the **payee** is the person to whom the note is payable; and the **holder** is the person that owns it.

66. The **face** of a note is the sum promised to be paid. This sum should be written both in figures and in words.

67. Notes are of two kinds—notes *bearing interest*, and notes *not bearing interest*. When no rate of interest is specified, the legal rate in the state or county where the note is made is to be understood. If a note not bearing

interest is not paid when due, it bears interest at the legal rate after that time until paid.

68. The following table is given for reference:

INTEREST LAWS.

Laws of each state and territory regarding rates of interest and penalties for usury, with the law or custom as to days of grace on notes and drafts. Compiled from the latest state and territorial statutes.

States and Territories.	Legal Rate of Interest. Per Cent.	Rate Allowed by Contract. Per Cent.	Penalties for Usury.	Grace or No Grace.
Alabama	8	8	Forfeiture of entire interest.	Grace.
Arizona	10	Any rate.	None.	Grace.
Arkansas	6	10	Forfeiture of principal and int.	Nostatute.
California	7	Any rate.	None.	No grace.
Colorado	10	Any rate.	None.	Grace.
Connecticut	6	6	None.	Grace.
Dakota	7	12	Forfeiture of excess.	Grace.
Delaware	6	6	Forfeiture of principal.	Grace.
Dist. of Columbia	6	10	Forfeiture of entire interest.	Grace.
Florida	8	Any rate.	None.	Nostatute.
Georgia	7	8	Forfeiture of excess.	Grace.
Idaho	10	18	Forf. of 3 times excess of int.	No grace.
Illinois	6	8	Forfeiture of entire interest.	Grace.
Indiana	6	8	Forfeiture of excess of interest.	Grace.
Iowa	6	10	Forf. of 10% per year on amt.	Grace.
Kansas	7	12	Forfeiture of excess of interest.	Grace.
Kentucky	6	10	Forfeiture of excess over 10%.	Grace.
Louisiana	5	8	Forfeiture of entire interest.	Grace.
Maine	6	Any rate.	None.	Grace.
Maryland	6	6	Forfeiture of excess of interest.	Grace.
Massachusetts	6	Any rate.	None.	Grace.
Michigan	7	10	Forfeiture of excess of interest.	Grace.
Minnesota	7	10	Forfeiture of excess over 10%.	Grace.
Mississippi	6	10	Forfeiture of excess of interest.	Grace.
Missouri	6	10	Forfeiture of entire interest.	Grace.
Montana	10	Any rate.	None.	No grace.
Nebraska	7	10	Forfeiture of interest and cost.	Grace.
Nevada	10	Any rate.	None.	Grace.
New Hampshire	6	6	Forfeiture of thrice the excess.	Grace.
New Jersey	6	6	Forfeiture of entire interest.	No grace.
New Mexico	6	12	None.	No grace.
New York*	6	6	Forfeiture of principal and int.	No grace.
North Carolina	6	8	Forfeiture of entire interest.	Grace.
Ohio	6	8	Forfeiture of excess above 6%.	Grace.
Oregon	8	10	Forfeiture of principal and int.	No grace.
Pennsylvania	6	6	Forfeiture of excess of interest.	No grace.
Rhode Island	6	Any rate.	None.	Grace.
South Carolina	7	Any rate.	None.	Grace.
Tennessee	6	10	Forf. of exc. int. and $100 fine.	Grace.
Texas	8	12	Forfeiture of entire interest.	Grace.
Utah	10	Any rate.	None.	No grace.
Vermont	6	6	Forfeiture of excess of interest.	No grace.
Virginia	6	8	Forfeiture of excess over 6%.	Grace.
Washington	10	Any rate.	None.	......
West Virginia	6	6	Forfeiture of excess of interest.	Grace.
Wisconsin	7	10	Forfeiture of entire interest.	No grace.
Wyoming	12	Any rate.	None.	Grace.

* Upon call loans of $5,000 or upwards, on collateral security, any rate of interest is legal.

69. A note should be so written as to show where it was made and when, the sum promised to be paid, whether it does or does not bear interest, and the words "value received." The law assumes that no one is to be compelled to pay unless he has received what he deems an equivalent. If "value received" is omitted, the holder may have to prove that the maker of the note did actually receive value for the money promised in it.

70. A note usually specifies where it is to be paid—usually at a bank. If no place is designated, the holder's place of business is understood.

71. If a note contains the words "or order," it is a **negotiable note,** and may pass like a bank note from one person to another. If the holder of a negotiable note wishes to dispose of it, he is generally required to guarantee its payment by *indorsing* it—that is, by writing his name across the back of the note. There are several kinds of indorsements. Thus, if the holder is John Smith, he may, on the back of the note, write *John Smith.* This is an indorsement *in blank*, and makes John Smith responsible for the payment of the note.

He may write, *Pay to William Jones.* The note is then payable only to William Jones.

If he indorses it, *Pay to William Jones, or order*, it is payable to William Jones, or to any one to whom William Jones may order it to be paid.

He may indorse it, *Pay to William Jones, or bearer*, and it is payable to any person that presents it.

If it be indorsed, *Pay to bearer*, it is payable to the person that presents it for payment.

72. A **joint and several** note is a note signed by two or more persons, who become collectively and individually responsible for its payment.

A note is **protested** when the holder of it notifies the indorsers in legal form and within the time prescribed by law that the note is unpaid. Unless such protest is legally

made, the indorsers are not responsible for its payment. This protest must reach the indorser not later than the day when the note is payable.

73. Some forms of notes used in actual business are given below.

$250. *New York, Sept. 17, 1896.*

On demand I promise to pay George Camp, or order, Two Hundred and Fifty Dollars, value received.

Howard Gray.

$1,000. *Scranton, July 5, 1898.*

Three months after date, for value received, I promise to pay Stephen Girard, or order, One Thousand Dollars, with interest at 5%. *Charles Goldwin.*

$3,000. *Philadelphia, July 5, 1898.*

Six months after date, we, or either of us, will pay to George Owen, Three Thousand Dollars, value received, with interest at 6%.

George Kirwin.
Henry Potter.
Erastus Kirby.

Payable at the First National Bank.

74. In most states, *three days of grace* are allowed before a note must be paid. If a bank discounts a note, interest is charged for days of grace in States where days of grace are legal.

75. If a note falls due on a Sunday, or on a legal holiday, it is usually payable during banking hours on the business day preceding its maturity. Interest, however, is charged for three days of grace in such cases. In some States, a note falling due on a Sunday or a legal holiday is payable on the first business day thereafter.

EXAMPLES FOR PRACTICE.

76. Solve the following examples:

1. Write a negotiable demand note for $600, with interest at 6%, and make Brown, Jones & Co. the payee.

2. Write a non-negotiable note for $4,000, with interest at 5%, payable in 30 days, yourself being the maker and Howard Crosby the payee.

3. How much will pay the following note when due without grace?

575\frac{50}{100}$. *New York, Sept. 19, 1898.*

Sixty days after date, for value received, I promise to pay Ralph Newton, or order, Five Hundred Seventy-Five and $\frac{50}{100}$ Dollars, with interest at 4½%. *Henry Miles.*

Ans. $579.82.

BANK DISCOUNT.

77. **Bank discount** is the charge made by a bank for paying a note or other obligation before it is due. This charge is the interest on the amount of the obligation from the time it is discounted until its maturity. This interest is subtracted from the face of the obligation, and its holder receives for it the remainder, which is called the **proceeds.** Hence, bank discount is inequitable, since interest is charged, not only upon the sum actually paid for the obligation, but also upon the discount.

78. In states where days of grace are allowed, bank discount is calculated for 3 days more than the time specified in the note.

Thus, if a 60-day note for $1,000 is discounted at a bank, the interest of $1,000 is found for 63 days, and is subtracted from $1,000. If the rate of discount is 6%, the holder will receive as proceeds $1,000 – $10.50 = $989.50.

79. It is evident that the owner of the note should receive for it the *true present worth* of $1,000 payable in 63 days, or $989.61. The bank gives him only $989.50, or 11 cents less than he should get. When it is considered that the sums annually discounted by most banks mount up into millions, it will be seen how much of their gain is unearned.

80. The **maturity** of a note is on the last day of grace. The time of maturity is generally indorsed on the note, thus, Mar. 7/10, which means that it matures nominally on Mar. 7, and legally on Mar. 10.

The **term of discount** is the time from the discounting of the note to its maturity.

81. In the case of an interest-bearing note, the sum discounted is the amount of the note at maturity.

82. Banks usually require that a discounted note shall be payable at the bank that discounted it, and they rarely discount notes having more than 90 days to run.

83. To find the time when a note matures, the term of discount, the discount, and the proceeds.

EXAMPLE.—Find (*a*) the discount, and (*b*) the proceeds of the following note:

484\frac{60}{100}$. *Newark, N. J., Oct. 4, 1897.*

Sixty days after date, for value received, I promise to pay William Hall, or order, Four Hundred Eighty-Four and $\frac{60}{100}$ *Dollars, at the Ninth National Bank.* *Henry Parshall.*

Discounted, Oct. 20, 1897, at 6%.

SOLUTION.—

(*a*) Maturity, Dec. 3/6, 1897.
Term of discount, 47 days.
Discount, \$3.80. Ans.

(*b*) Proceeds, \$484.60 – \$3.80 = \$480.80. Ans.

EXPLANATION.—Sixty days after Oct. 4 is Dec. 3, and three days of grace make the date of legal maturity Dec. 6. From the time of discount, Oct. 20, to Dec. 6 is 47 days, for which the interest at 6% is \$3.80. Subtracting the discount, \$3.80, from the face of the note, \$484.60, gives \$480.80, the proceeds.

EXAMPLE.—Find (*a*) the discount and (*b*) the proceeds of the following note:

\$1,060. *Chicago, Ill., August 6, 1898.*

For value received, I promise to pay, three months after date, to Ernest Hazard, or order, One Thousand Sixty Dollars, with interest at 5%. *Emil Reeves.*

Discounted, Sept. 1, 1898, at 6%.

SOLUTION.—

(*a*)	Maturity, Nov. 6/9, 1898.		
	Amount of note at maturity.................	\$1073.69	
	Term of discount, 69 days..................		
	Discount....................................	12.35	Ans.
(*b*)	Proceeds....................................	\$1061.34	Ans.

84. Rule.—I. *If the note bears interest, find its amount at the time of maturity.*

II. *Find the interest on the face of the note, or, if it is an interest-bearing note, on the amount of the note at maturity at the given rate of discount for 3 days more than the time it has to run until its nominal maturity, and the result will be the bank discount.*

III. *Subtract the bank discount from the face of the note, or from its amount at maturity, and the remainder will be the proceeds.*

EXAMPLES FOR PRACTICE.

85. Solve the following examples:

1. Find (*a*) the bank discount and (*b*) the proceeds of a note for $5,000, due in 60 days, discounted at 6%.

Ans. { (*a*) $52.50. (*b*) $4,947.50.

2. Find (*a*) the bank discount and (*b*) the proceeds of a note for $4,000, due in 90 days, discounted at 5%.

Ans. { (*a*) $51.67. (*b*) $3,948.33.

3. Required, (*a*) the bank discount and (*b*) the proceeds of a note for $7,600, due in 30 days discounted at 8%.

Ans. { (*a*) $55.73. (*b*) $7,544.27.

4. A note for $8,000 dated July 5, 1898, is discounted at 5% on Sept. 7, 1898. If it is a 90-day note, what are the proceeds?

Ans. $7,967.78.

5. A note for $2,800 bearing interest at 6%, and due in 60 days, is discounted at 5%. What are the proceeds? Ans. $2,804.64.

6. $8,476$\frac{00}{100}$. *St. Louis, Mo., Jan. 8, 1898.*

Six months after date I promise to pay to Charles Brown, or order, Eight Thousand Four Hundred Seventy-Six Dollars, value received, with interest at 6%. *Howard Bristow.*

Find the proceeds of the foregoing note if discounted April 25, 1898, at 5%. Ans. $8,641.11.

7. A note for $2,800, due in 3 months, is dated June 5, 1899, and bears interest at 7%. It is discounted July 20, at 6%. Find the proceeds. Ans. $2,826.87.

8. A note for $96,000, with interest at 8%, is dated Nov. 16, 1897, and is due in 90 days. It is discounted Dec. 20, at 6%. Find how much the proceeds differ from the true present worth. Ans. $9.38.

86. To find the face of a note when the proceeds, time, and rate are given.

EXAMPLE.—The proceeds of a note discounted at a bank for 45 days at 6% were $1,488. What was the face of the note?

SOLUTION.—

Proceeds of $1 for 45 + 3 days = $.992.
Face of the note = $1,488 ÷ $.992 = $1,500. Ans.

87. Rule.—*Divide the proceeds by the proceeds of $1 for 3 days more than the given time.*

EXAMPLES FOR PRACTICE.

88. Solve the following examples:

1. A 60-day note discounted at a bank at 6% yields $1,246.77. What is its face? Ans. $1,260.

2. The proceeds of a note for 2 mo. 12 da., discounted at a bank at 7%, are $4,079.625. Find the face of the note. Ans. $4,140.

3. Given, rate of discount, 8%; time, 90 days; proceeds, $3,613.74. Find the face of the note. Ans. $3,690.

4. Given, rate of discount, 7%; time, 30 days; proceeds, $86,382.135. Find the face of the note. Ans. $86,940.

5. Given, time, 51 days; rate, 4%; proceeds, $484.575. What is the face of the note? Ans. $487.50.

6. A note for 60 days discounted at $4\frac{1}{2}$% yields $81,815.335. What is its face? Ans. $82,464.75.

7. Write in proper form a 60-day note payable at the Chemical Bank of New York, which, when discounted when the note is made, will yield at 5%, $7,850 proceeds. Ans. Face, $7,915.97.

ARITHMETIC.

STOCKS AND BONDS.

1. If any work involving a large expenditure of money is to be undertaken, it is usual to organize a company, and procure a charter under the laws of some state. The chartered company then issues shares, which are sold to any persons having money to invest, and willing to incur the chances of loss. The advantage of having a charter for the company is that the shareholders, in case the business is unprofitable, are liable for the debts of the company only to the amount of their shares. Otherwise, any member of the company can be compelled to pay all of its debts. Moreover, a chartered company can do business just as an individual—that is, it may sue or be sued for debts, enter into contracts, etc.

2. The **par value** of shares is the amount—usually \$25, \$50, or \$100—specified in the certificates issued to its subscribers.

When the business of a company is profitable, its shares sell *above par*, or for more than their face value; if it is unprofitable, the shares sell at a *discount*, or *below par*.

3. When a company gains in its business, it pays its shareholders, from time to time, part of its profits, called **dividends.**

4. Dividends are *declared* quarterly, semiannually, or annually, and are usually paid at the general office of the company.

5. If the business is at a loss, the shareholders may be required to make good the loss. Such payments are called **assessments.**

6. If a man buys stocks of a company and makes partial payments on them, such payments are called **instalments.**

7. A **bond** is a written obligation under seal to pay a certain sum at a specified time.

8. Bonds to furnish money for the national or any state government, or for a city, county, town, village, or for an incorporated company, are prepared and sold in the open market. The money accruing from such sales may be used for current expenses, or for such improvements as may be desired. The bonds are secured by the property of those who issue them, and bear interest payable quarterly, semi-annually, or annually.

9. Registered bonds are numbered, and the names of their purchasers are recorded. To sell registered bonds, the transfer must be recorded on the books of the company that issued them. Sometimes bonds have **coupons** attached, stating the amount of interest due at certain times. These coupons may be detached, and exchanged for money at the general office of the company, or at a bank acting for the company.

10. Government or State bonds are usually designated by the interest they bear, or by the time when they are payable. Thus, "U. S. $3\frac{1}{2}$'s, 1907" are bonds of the United States Government bearing interest at $3\frac{1}{2}\%$, and payable in 1907.

11. A **stock broker** is a person whose business consists in buying and selling bonds or stocks for others. His compensation is a certain per cent. of the *par value* of the stocks bought or sold. The compensation of a broker is called **brokerage.**

12. The *par value* of stocks is to be understood as 100, unless some other value is given. Whatever may be the market price of stocks and bonds, brokerage is calculated on their par value.

EXAMPLE 1.—Find the cost of 480 shares of Canadian Pacific stock bought at $123\frac{1}{2}$, if the brokerage is $\frac{1}{8}$%.

SOLUTION.— $(123\frac{1}{2}+\frac{1}{8})\times 480 = \$59{,}340$. Ans.

EXAMPLE 2.—How many shares of bank stock selling at 112 can be bought for \$89,700, if the brokerage is $\frac{1}{8}$%? The par value of the shares is \$50.00.

SOLUTION.—The cost of 1 share at the market price is $1.12\times\$50 = \$56$; the brokerage per share is $.00\frac{1}{8}\times\$50 = \$0.06\frac{1}{4}$. Therefore, total cost of 1 share $= \$56+\$0.06\frac{1}{4} = \$56.06\frac{1}{4}$, and the number of shares bought $= \$89{,}700\div\$56.06\frac{1}{4} = 1{,}600$ shares. Ans.

13. NOTE.—Unless otherwise stated, the par value of each share will always be assumed to be \$100.00. Then, in example 1 above, the cost of 1 share is \$123.50. But the broker gets $\frac{1}{8}$%, or $\frac{1}{8}$ of \$1.00 per share for selling them. Therefore, the total cost to the purchaser of 480 shares is $(123\frac{1}{2}+\frac{1}{8})\times 480 = \$59{,}340$.

14. Rule.—I. *To find the cost of any number of shares of stock, multiply the sum of the market price per share and the brokerage by the number of shares, and the product will be the cost.*

II. *To find the number of shares that can be bought for a given sum, divide the given sum by the cost of one share, including the brokerage, and the quotient will be the number of shares.*

15. EXAMPLE.—How much must be invested in railroad stock that pays a quarterly dividend of $2\frac{1}{2}$%, in order to have an income of \$4,000, if they are bought at $104\frac{1}{2}$, brokerage being $\frac{1}{8}$%?

SOLUTION.—The expression, "a dividend of $2\frac{1}{2}$%," means $2\frac{1}{2}$ per cent. on the *par* value of the stock. Hence, since the *quarterly* dividend is $2\frac{1}{2}$%, the *annual* income per share of \$100 will be \$10. Consequently, to obtain an annual income of \$4,000, there must be bought $4{,}000\div 10 = 400$ shares; then each share will cost $\$104\frac{1}{2}+\$\frac{1}{8}$, and their total cost will be 400 times as much, or $\$104\frac{5}{8}\times 400 = \$41{,}850$. Ans.

EXAMPLE.—What per cent. is realized by buying 4% bonds at $89\frac{7}{8}$, brokerage being $\frac{1}{8}$%?

SOLUTION.—Since each share costs $\$89\frac{7}{8}+\$\frac{1}{8} = \$90$, and each share yields \$4 annual income, the per cent. realized will be found by dividing the gain, \$4, by the entire cost of one share. Hence,

$$\$4\div(\$89\tfrac{7}{8}+\$\tfrac{1}{8}) = .04\tfrac{4}{9} = 4\tfrac{4}{9}\%. \quad \text{Ans.}$$

16. Rule.—I. *To find the investment that will yield a given income, divide the income by the gain from one share,*

and the quotient will be the number of shares that must be bought; then multiply the cost of one share by the number of shares, and the product will be the investment.

II. *To find the rate per cent. of income from money invested in stocks or bonds, divide the gain yielded by one share by the cost of a share, and multiply the quotient by 100.*

EXAMPLES FOR PRACTICE.

17. 1. What must be paid for 128 shares of Standard Oil stock at $128\frac{1}{4}$, brokerage $\frac{1}{8}$%? Ans. $16,432.

2. How many shares of Union Gas Co. stock at $98\frac{3}{4}$ can be bought for $39,550, the brokerage being $\frac{1}{8}$%? Ans. 400 shares.

3. How much will 68 U. S. 4% bonds of 1907 cost at $116\frac{1}{2}$, brokerage being $\frac{1}{8}$%? Ans. $7,930.50.

4. The cost of some railroad stock was $18,150, for which a man paid $\$137\frac{3}{8}$ per share, and $\frac{1}{8}$% brokerage. How many shares did he buy? Ans. 132 shares.

5. Find the cost of 240 shares of mining stock at $98\frac{3}{8}$, brokerage being $\frac{1}{4}$%. Ans. $23,670.

6. How much must be paid for 5% city bonds to yield an annual income of $1,250, if they cost $104\frac{7}{8}$, brokerage $\frac{1}{8}$%? Ans. $26,250.

7. What is the rate per cent. received on the foregoing investment? Ans. $4\frac{16}{21}$%.

8. Bank stock that pays an annual dividend of 10% is bought for $109\frac{7}{8}$, brokerage $\frac{1}{8}$%. What per cent. is realized by investing in it? Ans. $9\frac{1}{11}$%.

9. How much must be paid for the bank stock mentioned above, to endow a college professorship with an annual income of $5,000? Ans. $55,000.

10. How much more is the rate per cent. of income on 8% stock bought at $119\frac{7}{8}$ than on 6% stock bought at $107\frac{7}{8}$, brokerage in each case being $\frac{1}{8}$%? Ans. $1\frac{1}{9}$%.

11. How much per cent. better is an investment in 8% securities bought at 80 than 7% stock at 84, not regarding brokerage? Ans. $1\frac{2}{3}$%.

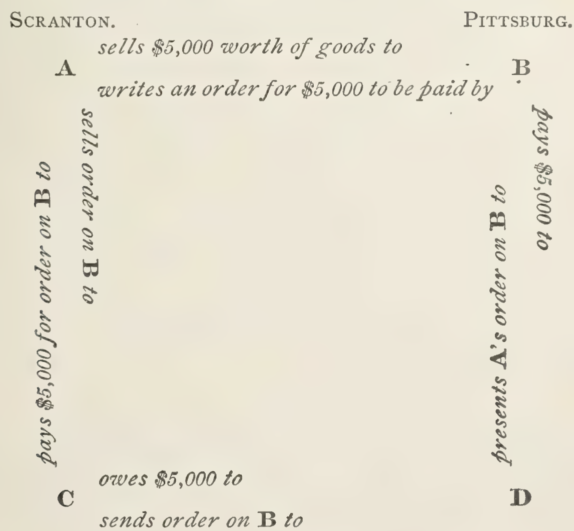
12. How much better is a gain of 20% on an investment at 80 than a gain of 18% on an investment at 90? Ans. 5%.

13. A man left his wife an annual income of $8,000 by an investment in 5% bonds bought at $111\frac{1}{4}$, brokerage being $\frac{1}{8}$%. What did they cost? Ans. $178,200.

EXCHANGE.

18. **Exchange** is a method of paying debts in a distant place without transmitting money. It is done by means of *drafts*, or *bills of exchange.*

19. Thus, suppose that A, who does business in Scranton, *sells* $5,000 worth of goods to B, who does business in Pittsburg; also, that C in Scranton *owes* $5,000 to D in Pittsburg. Then, B in Pittsburg owes A in Scranton, and C in Scranton owes D in Pittsburg. A writes an order directing B to pay to C $5,000, and sells this order to C, who pays A $5,000 for it. C endorses the order, and makes it payable to D; he sends the order to D, who presents it to B, and B pays D $5,000. A and D thus have the money that was due them, B and C have the goods, and no money had to be sent outside of Scranton or outside of Pittsburg. The diagram below will serve to make the operation clearer. The order directing payment to be made is called a **draft,** or a **bill of exchange.**



The foregoing explanation explains the purpose and convenience of a draft. Usually, however, the transaction is

conducted through a bank or broker. Thus, suppose that B in Pittsburg owes A in Scranton $5,000, and that B wishes to pay A without incurring the danger and risk of actually sending the money. He goes to a bank in Pittsburg and buys a draft, say, on New York, paying $5,000 and as much more as the bank charges him for its trouble. The Pittsburg bank then makes out a draft directing a New York bank to pay to A's order $5,000. B sends draft to A, who endorses it and presents it either at the New York bank, or, more conveniently, at a Scranton bank, where it is cashed. The Scranton bank sends the draft to some other bank (preferably a New York bank) with which the Scranton bank has dealings, and this bank in turn presents it at the bank drawn on, which credits the bank presenting the draft, and debits the Pittsburg bank. At stated intervals the various banks balance their accounts, at which time the debtor banks forward to the creditor banks the amounts due them.

Or, again, suppose that B in Pittsburg owes A in Scranton $5,000, and that A desires to collect this amount from B. A draws* a draft, or order, on B, directing him to pay the cashier of A's bank the $5,000 due A. A then leaves this draft at his bank, and the bank sends it to some bank in Pittsburg, which bank presents the draft to B, who pays that bank the money. This bank then notifies the Scranton bank that it has collected the draft and has placed the money to the Scranton bank's credit. The Scranton bank then notifies A that the draft has been collected and that the money has been placed to A's credit.

20. Exchange between different parts of the *same* country is **domestic exchange,** and between *different* countries is **foreign exchange.**

21. A **sight draft** is payable when presented for payment; a **time draft** is payable after a specified time. Time drafts are usually made to run 30, 60, or 90 days, and in some places days of grace are allowed on them.

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* To **draw** a draft is to request the payment of a certain sum, at a certain time, to a certain person, his order, or to bearer.

When a time draft is presented to the drawee, if he desires to pay it, he writes across its face the word "Accepted," the date of acceptance, and his name. The draft then becomes due, the number of days stated in the draft, from the date of acceptance, instead of from the date of the draft. A time draft, thus accepted, in reality becomes a promissory note. Sight drafts do not need to be accepted, as they are usually paid when presented.

22. The **drawer** of a draft is the person that requests payment to be made; the **drawee** is the person requested to make such payment; and the **payee** is the person to whom the money is to be paid.

23. A draft is **accepted** by the drawee if he writes on its face the word "Accepted" and the date of acceptance, together with his name. He thus becomes liable for its payment.

24. **Discount** is allowed on time drafts, and is computed on the amount or on the face of the draft.

FORM OF A DRAFT.

$465\frac{25}{100}$. *Scranton, Pa., Jan. 3, 1898.*
At sight, pay to the order of Henry Hudson, Four Hundred Sixty-five $\frac{25}{100}$ Dollars, value received, and charge to the account of *The Colliery Engineer Company.*
To Brown & Bird,
Chicago, Ill.

Explanation.—The Colliery Engineer Co. corresponds to A in Art. **19,** Brown & Bird to B, and Henry Hudson to C. Henry Hudson endorses the draft, "Pay to the order of J. R. Robinson," and signs his name; he forwards the draft to J. R. Robinson, of Chicago (who corresponds to D), and he presents it to Brown & Bird for acceptance.

DOMESTIC EXCHANGE.

25. Exchange may be at *par*, *above par*, or *below par*. For example, consider two cities, as New York and New Orleans. If New Orleans owes more money in New York than New York owes in New Orleans, there will be more persons in New Orleans who wish to buy drafts on New York than there are sellers; hence, a buyer in New Orleans will be willing to pay a seller more than the face value for a draft on New York. Therefore, in New Orleans, exchange on New York will be above par, or at a *premium*. In New York, however, there will be more sellers than buyers, and the seller will be willing to take less than the face of a draft on New Orleans. Therefore, in New York, exchange on New Orleans will be below par, or at a *discount*. If there is an equality of debts between New York and New Orleans, exchange will be at par. The premium or discount is only so much as will suffice to cover the cost of safely transferring the money from the debtor city to the creditor city.

26. To find the cost of a sight draft.

EXAMPLE 1.—Find the cost of a sight draft on Baltimore for \$2,800, exchange being at $\frac{1}{2}$% premium.

SOLUTION.— \$2,800 × .005 = \$14; \$2,800 + \$14 = \$2,814. Ans.
Or, \$2,800 × 1.005 = \$2,814. Ans.

EXAMPLE 2.—What must be paid for a sight draft on New York for \$3,675, at $\frac{3}{4}$% discount?

SOLUTION.— \$3,675 × .0075 = \$27.5625;
\$3,675 − \$27.56 = \$3,647.44. Ans.
Or, \$3,675 × (1 − .0075) = \$3,647.44. Ans.

27. Rule.—*Find the premium or the discount. The sum of the face of the draft and the premium, or the difference between the face of the draft and the discount, will be the cost of the draft.*

EXAMPLES FOR PRACTICE.

28. Solve the following examples:

1. Find the cost of a sight draft for \$1,876 at (*a*) $1\frac{1}{4}$% premium; (*b*) $\frac{1}{2}$% discount.

Ans. { (*a*) \$1,899.45.
{ (*b*) \$1,866.62.

2. The face of a sight draft is \$7,875.56, and the premium is $\frac{4}{5}\%$. Find its cost. Ans. \$7,938.56.

3. How much will it cost to pay, by a sight draft on San Francisco, a bill of \$7,528, when exchange is at $1\frac{1}{2}\%$ discount? Ans. \$7,415.08.

29. To find the cost of a time draft.

Example.—Find the cost of the following draft at $\frac{3}{4}\%$ premium, when money is worth 5% interest.

\$4,800$\frac{00}{100}$. *New York, July 1, 1898.*
Ninety days after sight, pay to William Wood, or order, Four Thousand Eight Hundred Dollars, value received, and charge to my account.
John Steinway.
To Henry Brothers,   *New Orleans, La.*

Solution.—In states where days of grace are allowed, 3 days must be added to the number of days specified in the draft. Hence, since the draft is *payable* in Louisiana, a state where days of grace are allowed (Art. **68**, § 9), 90 days + 3 days of grace = 93 days.

$\$4,800 \times .0129\frac{1}{6} = \$62 =$ int. of \$4,800 at 5% for 93 da.
$\$4,800 - \$62 = \$4,738 =$ proceeds of \$4,800.
$\$4,800 \times .0075 = \$36$; $\$4,738 + \$36 = \$4,774$. Ans.

Explanation.—The bank discount at 5% for 93 days on \$4,800 is \$62; hence, the present worth, or proceeds, of \$4,800, payable in 93 days, is \$4,738. The premium on the face of a draft is \$36. This, with the proceeds, is the cost of the draft.

30. Rule.—*Find the proceeds of the face of the draft for three days more than the time the draft has to run. Find, also, the premium or the discount, on the face of the draft. The sum of the proceeds and the premium, or the difference between the proceeds and the discount, will be the cost of the draft.*

If days of grace are not allowed in the state where the draft is payable, find the proceeds for the time the draft has to run, and then proceed as before.

EXAMPLES FOR PRACTICE.

31. Solve the following examples:

1. The face of a draft is $5,000; the discount is ½%; the time to elapse before it is payable is 60 days. Find the cost of the draft when money is worth 6%. Ans. $4,922.50.

2. How much will it cost to pay 30 days after sight a bill of $3,250 in Charleston, S. C., at a premium of 1½%, money being worth 6%? Ans. $3,280.87.

3. When exchange is at a discount of ¾%, and money is worth 4%, what must be paid for a 60-day time draft for $6,000? Ans. $5,913.

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32. To find the face of a sight or a time draft.

Example.—What is the face of a sight draft bought for $3,000, exchange being ½% discount?

Solution.— $1.00 − $.005 = $.995.
$3,000 ÷ .995 = $3,015.08. Ans.

Explanation.—One dollar of the face will cost $0.995; hence, there must be as many times one dollar of the face as .995 is contained times in the entire cost, or 3,015.08.

Example.—Find the face of a 60-day draft costing $2,343.51 when money is worth 7% and exchange is at 1⅜% premium.

Solution.—When the place where the draft is to be paid is not stated, days of grace should be allowed. Hence, the time is 60 + 3 = 63 days.

Interest of $1 for 63 days at 7% = $0.01225.
Proceeds of $1 = $1 − $0.01225 = $0.98775.
Premium on $1 = $0.01375.
Proceeds + premium = $0.98775 + $0.01375 = $1.0015.
Face of draft = $2,343.51 ÷ 1.0015 = $2,340. Ans.

33. Rule.—*Divide the amount paid for the draft by the amount that will pay for $1 of its face.*

EXAMPLES FOR PRACTICE.

34. Solve the following examples:

1. Find the face of a 90-day draft costing $6,000, when discount is 1¼% and money is worth 5%. Ans $6,156.48

2. A man paid $484.72 for a 60-day draft, premium being 1%, and money worth 6% interest. What was the face of the draft?

Ans. $484.96.

3. If a draft that is payable 30 days after sight costs $2,800 when discount is $\frac{3}{4}$% and money worth 6%, what is its face? Ans. $2,836.88.

4. Find the face of a sight draft costing $1,200, when exchange is at 1$\frac{3}{8}$% discount. Ans. $1,216.73

FOREIGN EXCHANGE.

35. Foreign bills of exchange are drawn in sets of two, called a **set of exchange.** These are numbered *1* and *2*, and are sent by different mails; only the first presented for payment has any value. Formerly, foreign bills of exchange were drawn in sets of three, and are frequently so drawn now.

36. Exchange is at a premium or at a discount according to the balance of trade, or to the time that must elapse before payment is to be made.

Thus, let A and B denote two countries engaged with each other in commerce. Suppose that the balance of trade is in B's favor. By this is meant that A owes B more than B owes A. Now, it is clear that A must send money at some risk and expense to B to equalize matters. If one were to get in A a draft payable in B, it would put A more deeply in debt to B. This fact would put a premium upon the draft, and a discount upon a draft drawn in B upon A.

Again, a sight draft upon either country would cost more than a time draft.

37. The Secretary of the Treasury of the United States issues, on the first of January of each year, a statement showing, in terms of its own gold monetary unit, the value of the monetary unit of each other country, that is, in dollars and cents.

38. The daily papers of our commercial cities give quotations showing the rates of exchange from day to day. One of these follows:

Sterling exchange* was again weak and lower. Continental exchange was also lower. Rates are: Long bills, $\$4.82\frac{1}{2}$ @ $\$4.82\frac{3}{4}$; sight drafts, $\$4.84\frac{3}{4}$ @ $\$4.85$, and cable transfers, $\$4.85\frac{1}{4}$ @ $\$4.85\frac{1}{2}$. Francs are quoted at $5.21\frac{7}{8}$ for long and 5.20 for short; reichsmarks, $94\frac{1}{2}$ @ $94\frac{9}{16}$ for long and $95\frac{1}{8}$ @ $95\frac{3}{16}$ for short; guilders, $39\frac{7}{8}$ @ $39\frac{15}{16}$ for long and 40 @ $40\frac{1}{16}$ for short.

NOTE.—A reichsmark (mark of the empire) is the same as a mark, about $23\frac{3}{4}$ cents. The exchange value of 4 marks, or reichsmarks, is given in commercial quotations in the daily newspapers.

39. A person going from New York to England carries with him, instead of money, a draft like the following:

New York, Oct. 1, 1897.

Exchange for £820-12-6 sterling.

At sight pay this First of Exchange, second of same tenor and date unpaid, to Edward Howe, or order, the sum of Eight Hundred Twenty Pounds £820-12-6 sterling.

Value received, and charge to the account of

Smith, Jones & Co.

To Baring Bros. & Co.,
London, England.

EXAMPLE 1.—Find the cost of the foregoing draft in New York when exchange on London is $4.84\frac{3}{4}$.

SOLUTION.— £820-12-6 = £820 12s. 6d. = £820.625.
$\$4.8475 \times 820.625 = \$3,977.98$. Ans.

EXAMPLE 2.—What must be paid for a draft on Paris of 8,000 francs, when \$1 is quoted at $5.21\frac{1}{2}$?.

SOLUTION.— 1 franc = $\$1 \div 5.215$;
$(\$1 \div 5.215) \times 8,000 = \$1,534.04$. Ans.

40. Rule.—*To find the cost of a draft upon a foreign country, multiply the quoted value of a foreign monetary unit by the given number of such units.*

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*Sterling exchange is Bills of Exchange payable in English money called Pounds Sterling. Long bills are those payable 30, 60, 90 or more days after being received. Short bills are those payable from sight to 30 days after being received. Sight drafts are payable at sight, that is, as soon as received.

EXAMPLES FOR PRACTICE.

41. Solve the following examples:

1. Find the cost of a draft on London, at 60 days' sight, for £987 16s., exchange being $\$4.82\frac{3}{4}$. Ans. $4,768.60.

2. When exchange on Paris is quoted at 5.23, what must be paid for a sight draft for 2,800 francs? Ans. $535.37.

3. I bought a long draft on Berlin for 8,425 reichsmarks when exchange was quoted at $94\frac{3}{4}$ per 4 reichsmarks. What did it cost? Ans. $1,995.67.

4. What must be paid for a draft on Amsterdam for 8,000 guilders, exchange being $40\frac{1}{16}$? Ans. $3,205.

ARBITRATION OF EXCHANGE.

42. Arbitration of exchange is the process of finding the cost of a draft on one place through one or more intermediate places.

Thus, the quoted rates between New York and Vienna may be high, while those between New York and London, and between London and Vienna may be low. It may be cheaper for a man in New York to purchase a draft on Vienna through London than to purchase directly on Vienna.

In this roundabout or circuitous method, the intermediate brokers charge *brokerage* for their services, usually $\frac{1}{8}\%$.

43. Example 1.—When exchange between New York and Paris is 5.22, between New York and London is $4.83, and between London and Paris 24.84 francs to the pound, which is cheaper—direct or circuitous exchange from New York upon Paris for 10,000 francs, London brokerage being $\frac{1}{8}\%$?

Solution.—By direct exchange, the cost of a draft for 10,000 francs is

$$10{,}000 \div 5.22 = \$1{,}915.71.$$

By circuitous exchange, the cost of the draft in pounds is

$$10{,}000 \div 24.84 = £402.5765.$$

$$\text{Cost of draft} + \text{brokerage} = £402.5765 \times 1.00\tfrac{1}{8} = £403.0797.$$

$$\text{Cost of draft in dollars} = \$4.83 \times 403.0797 = \$1{,}946.87.$$

Therefore, direct exchange is cheaper by

$$\$1{,}946.87 - \$1{,}915.71 = \$31.16. \quad \text{Ans.}$$

EXAMPLE 2.—A merchant sends 12,000 reichsmarks from New York to Berlin through London and Amsterdam. Exchange on London is \$4.85, between London and Amsterdam 11.86 guilders to the pound, and between Amsterdam and Berlin 1.72 reichsmarks (.43 of 4 reichsmarks) to the guilder. What is the cost of the draft in dollars, if brokerage at each place is $\frac{1}{8}\%$?

SOLUTION 1.—First change the reichsmarks to guilders, then the guilders to pounds, and finally the pounds to dollars, adding each time the commission.

$12{,}000 \times 1.00\frac{1}{8} = 12{,}015$, the number of reichsmarks required to pay the debt and the broker's commission in Berlin.

$(12{,}015 \div 1.72) \times 1.00\frac{1}{8} = 6{,}994.197$, the number of guilders required in Amsterdam.

$(6{,}994.197 \div 11.86) \times 1.00\frac{1}{8} = 590.467+$, the number of pounds required in London.

$590.467 \times 4.85 \times 1.00\frac{1}{8} = 2{,}867.34$, the number of dollars required to purchase the bill of exchange in New York. \$2,867.34. Ans.

SOLUTION 2.—Another solution is the following, in which the vertical line is one of the signs of division, and indicates that the product of the numbers on the left of the line is to be divided by the product of the numbers on the right.

$$\begin{array}{r|l} \$4.85 \times 1.00\frac{1}{8} & £1. \\ £1 \times 1.00\frac{1}{8} & 11.86 \text{ guilders.} \\ 1 \text{ guilder} \times 1.00\frac{1}{8} & 1.72 \text{ marks.} \\ 12{,}000 \text{ marks} \times 1.00\frac{1}{8} & \$\,? \end{array}$$

$$\frac{\$4.85 \times 1.00125^4 \times 12{,}000}{11.86 \times 1.72} = \$2{,}867.34. \quad \text{Ans.}$$

To avoid the use of decimals having such an inconveniently large number of figures, it is better to use common fractions. Thus, $1.00\frac{1}{8} = \frac{8.01}{8} = \frac{801}{800}$; hence, the last expression above becomes

$$\frac{\$4.85 \times (\frac{801}{800})^4 \times 12{,}000}{11.86 \times 1.72} = \frac{\$4.85 \times 801^4 \times 12{,}000}{11.86 \times 800^4 \times 1.72} = \$2{,}867.34.$$

By using common fractions instead of the decimals, the results will be more accurate and the principle of cancelation can be more readily employed.

44. It will be noticed that in the above arrangement the various monetary units appear once on each side of the vertical line of division. The same method may be employed with other than monetary units.

EXAMPLE.—If 4 apples are worth 3 peaches, and 9 peaches are worth 5 oranges, how many apples must be given for 60 oranges?

SOLUTION.—

$$\begin{array}{l|l} \overset{12}{\cancel{60}} \text{ oranges} & x \text{ apples.} \\ \overset{3}{\cancel{9}} \text{ peaches} & \cancel{5} \text{ oranges.} \\ 4 \text{ apples} & \cancel{3} \text{ peaches.} \end{array}$$

$$x = 12 \times 3 \times 4 = 144 \text{ apples. Ans.}$$

EXPLANATION.—The unknown quantity x should always be placed on the right-hand side of the vertical line of division, and the given quantity to which x is equivalent on the left-hand side. Then arrange the other quantities so that each quantity of the same kind appears on each side of the line. In the present example, 60 oranges are equivalent to a certain number of apples; hence, we place 60 oranges on the left and x on the right. Now, since 5 oranges are equivalent to 9 peaches, and we already have oranges on the left-hand side, we place the 5 oranges on the right-hand side and the 9 peaches on the left-hand side. For the same reason, we place 4 apples on the left-hand side and 3 peaches on the right-hand side. Canceling, we find that $x = 144$ apples.

EXAMPLES FOR PRACTICE.

45. Solve the following examples:

1. I sent from New York to Christiania, Norway, 4,740 crowns through London and Amsterdam. If 1 guilder = .66$\frac{7}{8}$ crowns, 11.85 guilders = £1, and \$4.85 = £1, what does the draft cost, brokerage at London on Amsterdam, and at Amsterdam on Christiania, being $\frac{1}{8}$%?

Ans. \$2,908.19+.

2. If 5 cords of oak wood are worth 3 cords of hickory, and 4 cords of hickory are worth 10 cords of pine, what should be paid for oak when pine is \$1.25 per cord? Ans. \$1.87$\frac{1}{2}$.

3. If 9 bushels of wheat are worth 14 bushels of rye, 12 bushels of rye are worth 17 bushels of corn, and 3 bushels of corn are worth 5 bushels of oats, how many bushels of oats should be given for 324 bushels of wheat? Ans. 1,190 bu.

AVERAGE OR EQUATION OF PAYMENTS.

46. Suppose that A owes B any sum, say \$100, due in 10 days, a second \$100, due in 20 days, and a third \$100, due in 30 days, there is evidently a time when he may pay B the entire \$300 without loss of interest to either party. Clearly, this time is 20 days after A incurs the debts. The conditions are that A may retain and use the first \$100 for 10 days, the second \$100 for 20 days, and the third \$100 for 30 days. But, so far as the interest is concerned, the use of \$100 for 10 days is equivalent to the use of \$100 × 10, or \$1,000, for 1 day; \$100 for 20 days equals \$2,000 for 1 day; \$100 for 30 days equals \$3,000 for 1 day. A's privilege, therefore, equals \$1,000 + \$2,000 + \$3,000, or \$6,000 for 1 day. But \$6,000 for 1 day is the same as \$300 for 20 days. Expressing this argument more briefly, we have

$$\begin{array}{lcrl}
\$100 \text{ for } 10 \text{ days} & = & \$1{,}000 & \text{for 1 day.} \\
\$100 \text{ for } 20 \text{ days} & = & \$2{,}000 & \text{for 1 day.} \\
\underline{\$100} \text{ for } 30 \text{ days} & = & \underline{\$3{,}000} & \text{for 1 day.} \\
\$300 & &)\underline{\$6{,}000} & \\
 & & 20 & \text{days.}
\end{array}$$

47. Average, or equation, of payments is the process of finding the equitable time when payment of several sums, due at different times, may be made in one payment.

48. The equated, or average, time of payment is the time when several debts with different *terms of credit* may be equitably made in one payment.

49. Example 1.—At a certain time A agrees to pay \$1,000 as follows: \$300 in 30 days, \$200 in 60 days, and \$500 in 90 days. Find the equated time of payment; that is, the time at which the entire debt may be paid without interest and still be fair to both parties.

Solution.—

$$\begin{array}{rcr}
\$300 \times 30 & = & \$9000 \\
\$200 \times 60 & = & \$12000 \\
\underline{\$500} \times 90 & = & \underline{\$45000} \\
\$1000 & &)\underline{\$66000} \\
 & & 66 \text{ days.}
\end{array}$$

The whole sum may equitably be paid 66 days after the obligation is incurred. Ans.

EXAMPLE 2.—A man owes \$250 due Mar. 1, \$300 due Apr. 20, \$450 due May 5, and \$500 due June 25. When is the equated time of payment?

SOLUTION.—

$$\begin{array}{rr}
\$250 \times 0 = & 0 \\
\$300 \times 50 = & \$15000 \\
\$450 \times 65 = & \$29250 \\
\$500 \times 116 = & \$58000 \\
\hline
\$1500 \quad &)\$102250 \\
 & 68+
\end{array}$$

68 days after Mar. 1, or May 8. Ans.

EXPLANATION.—Taking Mar. 1, the time when the first debt is due, as the *date of reference*, or the time from which to determine the terms of credit, the term of credit for \$250 is 0 days. The term of credit for \$300 is from Mar. 1 to Apr. 20, or 50 days; for \$450 the term of credit is from Mar. 1 to May 5, or 65 days; and for \$500 the term of credit is from Mar. 1 to June 25, or 116 days. We multiply each debt by its term of credit, and divide the sum of the products by the sum of the debts. The quotient, 68 days, is the number of days after Mar. 1 when one payment of the whole indebtedness may equitably be made, or May 8.

50. In the example just given, the length of time between the date when the equated time was computed and Mar. 1 was not stated. This does not matter, since the only effect that would be produced by introducing the number of days between this date and Mar. 1 would be to increase the numbers on the right of the signs of equality, and, since the total debt remains the same, the number of days obtained for the equated time will be increased by an amount just equal to the number of days between this date and Mar. 1, which, of course, does not change the date of settlement. For example, suppose that the equated time was computed 30 days preceding Mar. 1, that is on Jan. 29. Then, the first debt falls due in 30 days; the second, in $30 + 50 = 80$ days; the third, in $30 + 65 = 95$ days; and the fourth, in $30 + 116 = 146$ days.

In order, therefore, to find the equated time, we proceed as follows:

$$\begin{array}{rcr}
\$250 \times 30 = & & \$7500 \\
300 \times 80 = & & 24000 \\
450 \times 95 = & & 42750 \\
500 \times 146 = & & 73000 \\
\hline
\$1500 &) & \$147250 \\
\hline
 & & 98.1+
\end{array}$$

or 98 days after Jan. 29.

But 98 days after Jan. 29 is the same as 68 days after Mar. 1.

The number of days as computed above is always taken to the nearest integer; that is, if the fractional part is .5 or greater, 1 day is added. For instance, had the above result been 98.6, the number of days would have been taken as 99.

51. Rule.—*Taking as the date of reference the date when the first debt is due, find the term of credit for each debt.*

Multiply each debt by its term of credit, and divide the sum of the products by the sum of the debts. The quotient to the nearest integer will be the number of days from the date of reference to the equated time.

EXAMPLES FOR PRACTICE.

52. Solve the following examples:

1. A owes B $500 due in 8 months, and $900 due in 4 months. When may he equitably pay B both debts in one payment?
Ans. 5 mo. 13 da.

2. Find the equated time for paying $400 due May 10, $500 due June 20, $900 due July 30, and $1,000 due Aug. 15. Ans. July 17.

3. Tefft, Weller & Co. sold to E. King & Co. goods as follows: on June 15, $2,500 on 30 days' credit, and, on June 30, $3,600 on 20 days' credit. Find the equated date of payment. Ans. July 18.

4. What is the equated time for the payment of three notes: one for $600, dated Aug. 9, 1897, for 3 months; the second for $800, dated Oct. 1, 1897, for 2 months; the third for $1,200, dated Dec. 21, 1897, for 6 months? Ans. Feb. 27, 1898.

5. On Jan. 1, 1898, a merchant sold a bill of goods amounting to $3,600, payable as follows: $\frac{1}{2}$ in 30 days, $\frac{1}{3}$ in 60 days, and the remainder in 90 days. Find the equated time of payment. Ans. Feb. 20, 1898.

AVERAGE OR EQUATION OF ACCOUNTS.

53. Goods are usually sold on credit, the term of credit being commonly 30 days, 60 days, or 90 days. The prices of the goods are fixed for the time on which they are sold, interest being charged if payment is not made at the end of the time specified, and a discount being given if the debt is paid before the end of the term of credit. Now, if one or more payments are made on the bill before it is due, it is evident that a rebate ought to be given the purchaser, or else his term of credit on the remainder of the bill ought to be extended. For example, suppose that Wm. Marshall buys a bill of goods amounting to \$1,525.86 on Jan. 5, on 90 days' credit. The price was so fixed that the seller would not lose anything by selling the goods on 90 days' credit. Suppose that, on Jan. 27, Mr. Marshall pays \$425.40 on account, and on Feb. 24, \$506.62. There still remains unpaid $\$1,525.86 - (\$425.40 + \$506.62) = \$593.84$. Mr. Marshall ought either to receive a rebate, or his term of credit on the \$593.84 still unpaid ought to be extended, in order to compensate him for having paid a part of the bill before it was due. To determine how long the term of credit should be extended, we reason as follows: The first payment was made 22 days, and the second payment 50 days, after the goods were bought. Hence, Mr. Marshall lost the use of \$425.40 for $90 - 22 = 68$ days, and of \$506.62 for $90 - 50 = 40$ days; or of $\$425.40 \times 68 = \$28,927.20$ for 1 day, and of $\$506.62 \times 40 = \$20,264.80$ for 1 day. Consequently, altogether, he lost the use of $\$28,927.20 + \$20,264.80 = \$49,192$ for 1 day. Therefore, to make things equal all around, the term of credit on the \$593.84 still unpaid should be extended $49,192 \div 593.84 = 83$ days, and the date of settlement should be $90 + 83 = 173$ days after Jan. 5, or June 27.

In equation of payments, only one side of the account is considered, the items being either all debits or all credits; but, when both sides of the account are considered, the process of finding the equated time is called **equation of accounts.** In averaging accounts, the method of finding

the equated time is nearly the same as in equation of payments; the method is shown in the following examples:

54. EXAMPLE.—Find the equated time for the settlement of the following account:

HENRY WARDELL.

1889.				1889.			
Jan.	20	Mdse., 30 days,	800	Mar.	1	Cash,	400
Feb.	18	" 90 "	600		20	"	600
Mar.	14	" 60 "	1,000	Apr.	1	"	1,000
Apr.	10	" 30 "	1,200		20	"	500

SOLUTION.—The date of reference is Feb. 19, since 30 days after Jan. 20 is Feb. 19.

Feb. 19,	$800 \times 0 =$	0	Mar. 1,	$400 \times 10 =$	4000
May 19,	$600 \times 89 =$	53400	Mar. 20,	$600 \times 29 =$	17400
May 13,	$1000 \times 83 =$	83000	Apr. 1,	$1000 \times 41 =$	41000
May 10,	$1200 \times 80 =$	96000	Apr. 20,	$500 \times 60 =$	30000
	3600	232400		2500	92400
	2500	92400			
	1100	)140000			
		127			

Average term of credit, 127 days.
Equated time, 127 days after Feb. 19, or June 26. Ans.

EXPLANATION.—As in equation of payments, the debts are multiplied by the number of days from the date of reference to the dates when they respectively become due. It is most convenient to take as the date of reference (usually called the **focal date**) the date when the first debt becomes due; or, if a payment is made before the first debt becomes due, take the date of the first payment as the focal date. The same operation is performed on the payments, *using the same focal date.* The sum of the credits is then subtracted from the sum of the debits, the sum of the products on the credit side from the sum of the products on the debit side, and the second remainder is divided by the first remainder.

EXAMPLE.—In New York, what will be the cash balance of the following ledger account Jan. 10, 1898, interest at 6%?

WM. BONNER.

1897.					1897.			
Sept.	1	Mdse., 90 days,	800		Sept.	12	Cash,	600
	20	" 60 "	900		Oct.	20	Draft, 30 days,	500
Oct.	25	" 60 "	1,000		Nov.	15	Cash,	800
Nov.	1	" 30 "	2,000		Dec.	12	"	1,000

SOLUTION.—In this case it will be more convenient to take as the focal date Sept. 1. It will be noticed that one of the payments, that of \$500 on Oct. 20, is a 30-day draft. Since this draft can be cashed for its face value only after 30 days, i. e., on Nov. 19, this payment must be considered as having been made on Nov. 19; and, as days of grace are not allowed in New York, they are not added to the time the draft has to run. Proceeding as follows to find the equated time of settlement,

$$\begin{array}{llr|llr}
800 \times & 90 = & 72000 & 600 \times & 11 = & 6600 \\
900 \times & 79 = & 71100 & 500 \times & 79 = & 39500 \\
1000 \times & 114 = & 114000 & 800 \times & 75 = & 60000 \\
\underline{2000} \times & 91 = & \underline{182000} & \underline{1000} \times & 102 = & \underline{102000} \\
4700 & & 439100 & 2900 & & 208100 \\
\underline{2900} & & \underline{208100} & & & \\
1800 & &)\underline{231000} & & & \\
 & & 128\tfrac{1}{3} \text{ days.} & & &
\end{array}$$

we find it to be 128 days after Sept. 1, or Jan. 7, 1898. If the bill is not settled until Jan. 10, it is clear that in equity 3 days' interest should be charged on the \$1,800 yet unpaid. The interest of \$1,800 for 3 days at 6% is \$.90; hence, the total amount that should be paid is \$1,800.90. Ans.

EXAMPLE.—When should interest begin on the balance of the following account?

HENRY WELLINGTON.

1902.					1902.			
June	16	Mdse.,	1,200		July	21	Cash,	800
July	21	"	1,000		Aug.	11	"	1,200
Aug.	13	"	2,000			30	Draft, 30 days,	1,600
Sept.	15	"	3,600		Sept.	20	" 10 "	2,400

SOLUTION.—We take, as the focal date, June 1st. The reason for taking the first day of the month as the focal date is that it is easier to reckon the number of days between the first day of the month and some later date than it is to reckon the number of days between some other

day of the month and a later date. Days of grace are allowed on the two time drafts, because no state is mentioned in the example.

1200 ×	15 =	18000	800 ×	50 =	40000
1000 ×	50 =	50000	1200 ×	71 =	85200
2000 ×	73 =	146000	1600 ×	123 =	196800
3600 ×	106 =	381600	2400 ×	124 =	297600
7800		595600	6000		619600
6000					595600
1800				1800)	24000
					13 + days.

We find the products of the items and number of days as in the two previous examples. It will be noticed that the sum of the products on the credit side is greater than the sum of the products on the debit side, while the sum of the payments is less than the sum of the debts. We divide the difference between the sums of the products by the difference between the sum of the debts and the sum of the payments, obtaining for our result 13 days. Now, instead of *adding* this 13 days to the focal date, we subtract. The reason for subtracting will be evident when we consider that the last three payments were made a comparatively long time after the merchandise was bought. In other words, the merchant lost the use of the money due him in payment of the goods for a time equivalent to the difference between 13 days preceding June 1st (or May 19th) and the date of settlement, and he should receive interest on $1,800 for this time. If, however, the sum of the products on the credit side had exceeded the sum of the products on the debit side, and the sum of the payments had exceeded the sum of the debts, the number of days obtained would have been *added* to the focal date, as in the two preceding examples. Hence, the interest on the balance of the above account should begin on May 19th. Ans.

55. Rule.—*Find the date on which each item of the account matures, and take the first day of the month in which the earliest of these dates occurs, as the focal date. Find the number of days between the focal date and the date of maturity of the different items, and multiply each item by the number of days so found. Divide the difference between the sums of the products by the difference between the sum of the debts and the sum of the payments, and the quotient will be the equated time. If the greater sum of the items and the greater sum of the products are both on the same side of the account,* **add** *the equated time to the focal date. But, if the greatest sum of the items and the greatest sum of the products*

are on opposite sides of the account, **subtract** *the equated time from the focal date. The result obtained by adding or subtracting the equated time from the focal date will be the date when the balance of the account is equitably due.*

EXAMPLES FOR PRACTICE.

56. Solve the following examples, allowing 3 days of grace on time drafts:

1. Find the cash balance of the following account, July 1, 1903, interest at 6%; also, the equated time of settlement.

GEORGE GRIFFIN.

1903.					1903.				
Jan.	10	Mdse.,	2,500		Feb.	20	Cash,	4,000	
Feb.	10	"	4,800		April	24	Mdse.,	1,800	
Mar.	10	"	2,000		May	7	Draft, 10 days,	2,800	
May	20	"	6,800		June	12	Cash,	5,000	

2. Find the equated time of settlement of the following account:

WALTER ROBERTS.

1901.					1901.				
Jan.	1	Mdse., 90 days,	360		Feb.	4	Cash,	300	
Mar.	4	" 30 "	480		April	6	Draft, 30 days,	900	
May	13	" 60 "	640		May	7	" 60 "	600	
June	12	" 30 "	960						

3. Find the equated time of settlement of the following account:

WILLARD SMITH.

1896.					1896.				
Jan.	21	Mdse., 60 days,	4,000		Jan.	31	Cash,	2,500	
Feb.	3	" 60 "	5,000		Feb.	14	Real Estate,	3,500	
May	13	" 90 "	8,000		April	15	Draft, 60 days,	6,000	
June	15	" 30 "	7,800		May	1	Cash,	7,500	
					July	1	"	2,000	

Answers.—(1) $2,626.67; Aug. 31, 1902; (2) Aug. 16; (3) Apr. 27, 1897.

57. NOTE.—The student will find the table given in Art. **68**, § 4, of great assistance to him in working examples in equation of accounts.

PARTNERSHIP.

58. **Partnership** is an association of two or more persons for the transaction of business with joint capital.

59. The **capital** is the money or other property invested. The persons associated are **partners,** and these may be *active partners* or *silent partners.* An **active partner** is one who has a voice in the management of the enterprise. A **silent partner** is one having money invested in the business, but having no voice in its management.

60. The **assets** or **resources** of a firm are the property it owns and the debts owing to it.

The **liabilities** of a firm are the debts it owes.

WHEN THE PARTNERS INVEST FOR THE SAME PERIOD OF TIME.

61. The gains and the losses of a firm are apportioned among its members in proportion to each partner's investment, and the time during which it is invested. The operation is best performed by the method known as **distributive proportion.** When the shares of the several partners are invested for equal times, the apportionment of gains or losses is exactly similar to the division of a number into given proportional parts.

62. Example 1.—Divide the number 324 into three parts that are to one another as 5, 6, and 7.

1st Solution.—The sum of the proportional parts is 18. It is required, therefore, to find 5, 6, and 7 of the 18 equal parts of 324.

$$\left.\begin{array}{l} 5+6+7 : 5 = 324 : x;\ x = 90. \\ 5+6+7 : 6 = 324 : x;\ x = 108. \\ 5+6+7 : 7 = 324 : x;\ x = \underline{126.} \\ \hphantom{5+6+7 : 7 = 324 : x;\ x = }\ 324. \end{array}\right\}\text{Ans.}$$

2d Solution.—It is evident that the required numbers must be multiples of 5, 6, and 7; hence, the sum of these multiples (which is 324

according to the conditions of the example) must also be a multiple of the sum of the given numbers 5, 6, and 7, i. e., a multiple of 18. Dividing 324 by 18, the quotient is 18, and 18 multiplied in succession by 5, 6, and 7 gives 90, 108, and 126, the required numbers. Ans.

EXAMPLE 2.—A, B, and C invest $500, $600, and $700, respectively, in an enterprise by which they gain $1,620. Apportion the gain.

1ST SOLUTION.—

$$\$500 + \$600 + \$700 = \$1,800, \text{ the total amount invested.}$$

$$\left.\begin{array}{l} \$1,800 : \$500 = \$1,620 : \text{A's share; A's share} = \$450. \\ \$1,800 : \$600 = \$1,620 : \text{B's share; B's share} = \$540. \\ \$1,800 : \$700 = \$1,620 : \text{C's share; C's share} = \$630. \end{array}\right\} \text{Ans.}$$

2D SOLUTION.—Total amount invested is $1,800. Since each partner shares the gain in proportion to the amount he invests, it is clear that A's share must be $\frac{500}{1,800}$ of $1,620, or $450; B's share $\frac{600}{1,800}$ of $1,620, or $540; and C's share $\frac{700}{1,800}$ of $1,620, or $630. Ans. This method is exactly the same as in the 2d solution of the last example; for, dividing 1,620 by 1,800, the result is .9, and .9 multiplied successively by $500, $600, and $700, gives $450, $540, and $630.

63. Rule.—*Divide the amount each partner invests by the total amount invested, and multiply the quotient by the gain or loss. The result will be each partner's gain or loss.*

EXAMPLES FOR PRACTICE.

64. Solve the following examples:

1. Divide $1,440 into 3 parts that shall be to one another as 3, 5, and 8. Ans. $270; $450; $720.

2. Divide the number 1,058 into 3 parts that shall be to one another as $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$. Ans. 276; 368; 414.

SUGGESTION.—Change the fractions to a common denominator, and divide the number into parts that are proportional to the numerators.

3. A and B engage in trade, and gain $2,340. A's investment is to B's as 5 is to 8. Apportion the gain.

$$\text{Ans.} \begin{cases} \text{A's gain, \$900.} \\ \text{B's gain, \$1,440.} \end{cases}$$

4. A puts $8,000 into an enterprise, B $1,100, and C $1,300. They lose by it $3,520. What does each lose?

Ans. A loses $2,707.69. B loses $372.31. C loses $440.00.

5. A man fails in business with assets amounting to $7,080. He owes A $7,500, B $9,500, and C $12,500. How much of the assets should each receive?

Ans. A, $1,800. B, $2,280. C, $3,000.

6. A, B, and C rent a pasture for $63. A puts in 9 horses, B 12 horses, and C 15 horses. How much of the rent should each pay?

Ans. A, $15.75. B, $21.00. C, $26.25.

7. A's debts are to B's as $\frac{2}{3}$ to $\frac{3}{4}$, and together they owe $4,003.50. Find the debt of each.

Ans. A, $1,884.00. B, $2,119.50.

WHEN PARTNERS INVEST FOR DIFFERENT PERIODS OF TIME.

65. Example.—Three men engage in business. A puts in $8,000 for 8 months, B $10,000 for 12 months, and C $9,000 for 10 months. They lose $10,960. What does each man lose?

1st Solution.— $\$8,000 \times 8 + \$10,000 \times 12 + \$9,000 \times 10 = \$274,000.$

$$\left.\begin{array}{l} \$274,000 : \$64,000 = \$10,960 : \text{loss of A, or } \$\ 2\,5\,6\,0. \\ \$274,000 : \$120,000 = \$10,960 : \text{loss of B, or } \$\ 4\,8\,0\,0. \\ \$274,000 : \$90,000 = \$10,960 : \text{loss of C, or } \$\ 3\,6\,0\,0. \end{array}\right\} \text{Ans.}$$

$$\$\,1\,0\,9\,6\,0.$$

Explanation.— $8,000 for 8 months equals $64,000 for 1 month; $10,000 for 12 months equals $120,000 for 1 month; $9,000 for 10 months equals $90,000 for 1 month. The remainder of the operation is as before explained.

2d Solution.—

$$\begin{array}{rcr} \$\ 8,000 \times 8 &=& \$\ \ 6\,4\,0\,0\,0 \\ \$10,000 \times 12 &=& \$\ 1\,2\,0\,0\,0\,0 \\ \$\ 9,000 \times 10 &=& \$\ \ 9\,0\,0\,0\,0 \\ \hline & & \$\ 2\,7\,4\,0\,0\,0 \text{ for 1 month.} \end{array}$$

$$\left.\begin{array}{l} \text{A's loss} = \dfrac{64,000}{274,000} \times \$10,960 = \$2,560. \\[2ex] \text{B's loss} = \dfrac{120,000}{274,000} \times \$10,960 = \$4,800. \\[2ex] \text{C's loss} = \dfrac{90,000}{274,000} \times \$10,960 = \$3,600. \end{array}\right\} \text{Ans.}$$

EXAMPLE.—A and B engage in business for 1 year. During the first 5 months A's capital was $8,000, at the end of which time he increased it by $1,000. B's capital the first 8 months was $10,000; he then drew out $5,000 of it. At the end of the year they had gained $8,120. Find each man's share.

1ST SOLUTION.—Reducing the amounts invested to an equal interval of time, as 1 month, A had $8,000 invested for 5 months, and $\$8,000 + \$1,000 = \$9,000$ for 7 months; B had $10,000 invested for 8 months, and $\$10,000 - \$5,000 = \$5,000$ for 4 months.

$$\$8,000 \times 5 + \$9,000 \times 7 = \$103,000.$$
$$\$10,000 \times 8 + \$5,000 \times 4 = \$100,000.$$

$$\left.\begin{array}{l} 203,000 : 103,000 = \$8,120 : \text{A's share, or } \$\,4\,1\,2\,0 \\ 203,000 : 100,000 = \$8,120 : \text{B's share, or } \underline{\$\,4\,0\,0\,0} \end{array}\right\}\text{Ans.}$$
$$\$\,8\,1\,2\,0$$

2D SOLUTION.—Proceeding as before, the total amount invested for 1 month was $203,000; hence,

$$\left.\begin{array}{l} \text{A's gain was } \dfrac{103,000}{203,000} \times \$8,120 = \$4,120. \\ \\ \text{B's gain was } \dfrac{100,000}{203,000} \times \$8,120 = \$4,000. \end{array}\right\}\text{Ans.}$$

66. Rule.—*Reduce the amounts invested to equivalent amounts for an equal interval of time, as 1 year, 1 month, or 1 day; then proceed as in Art.* **63.**

EXAMPLES FOR PRACTICE.

67. Solve the following examples:

1. A and B entered into partnership. A put in $6,000 for 5 months, and B put in $5,000 for 8 months. They gained $3,500. What was each man's share of the gain? Ans. $\begin{cases} \text{A, \$1,500.} \\ \text{B, \$2,000.} \end{cases}$

2. A began business with a capital of $12,000. At the end of 4 months he took in B with $20,000, and 2 months later they took in C with $30,000. At the end of the year, their gain was $18,150. How much of the gain should each receive? Ans. $\begin{cases} \text{A, \$5,400.} \\ \text{B, \$6,000.} \\ \text{C, \$6,750.} \end{cases}$

3. A, B, and C with a combined capital of $73,000 gained $10,000. A's capital was in the business 4 months, B's 5 months, and C's 10 months. If A's capital was $18,500, B's $23,800, and C's $30,700, how much did each gain? Ans. $\begin{cases} \text{A, \$1,480.} \\ \text{B, \$2,380.} \\ \text{C, \$6,140.} \end{cases}$

4. A contractor had 12 men on a piece of work for 10 days, when they went on a strike. He then employed 15 new men at the same daily wages, and they finished the work in 12 days. If he paid in wages for the entire work $555, how much did each set of men receive, and how much did each man get per day?

Ans. { 1st set, $222.
2d set, $333.
$1.85 a day.

5. Two men, A and B, hire a pasture for $56. A puts in 12 horses for 15 days, and B puts in 24 oxen for 18 days. What should each pay, assuming that 4 oxen eat as much as 5 horses?

Ans. { A, $14.
B, $42.

6. A and B are in partnership for 1 year. A begins the year with $2,400; at the end of 3 months he increases his capital by $1,000; and 4 months later he draws out $600. B begins 3 months after A with $3,600, and 5 months later he adds $1,200 to his capital. At the end of the year they have gained $6,120. How much of the gain should each receive?

Ans. { A, $2,958.
B, $3,162.

ALLIGATION.

68. **Alligation** is a process of computation in which the *proportion* of the ingredients in a mixture, their respective *prices*, and the *price of a unit* of the mixture are the elements considered.

69. Alligation is of two kinds—*alligation medial* and *alligation alternate.*

ALLIGATION MEDIAL.

70. **Alligation medial** is the process of finding the value of a unit of a combination made up of several ingredients when the number of units and the price or quality per unit of each ingredient are given.

71. Example.—A farmer sold 30 bushels of wheat at 75 cents a bushel, 72 bushels of rye at 45 cents a bushel, and 60 bushels of corn at 30 cents a bushel. Find the average price per bushel.

Solution.—

$$\begin{array}{rcr}
\$.75 \times 30 & = & \$22.50 \\
\$.45 \times 72 & = & \$32.40 \\
\$.30 \times 60 & = & \$18.00 \\
\hline
162 &) & \$72.90 \\
\hline
 & & \$.45 \text{ Ans.}
\end{array}$$

EXPLANATION.—The operation is performed by the method of products, exactly as in equation of payments. The entire cost of the three kinds of grain is first found, and this sum is divided by the number of bushels sold.

72. Rule.—*Multiply the price or quality of a unit of each element of the combination by the number of such units, and divide the sum of the products by the entire number of units in the combination. The quotient will be the average cost or quality of a unit.*

EXAMPLES FOR PRACTICE.

73. Solve the following examples:

1. A grocer made a mixture of 15 lb. of 17-cent coffee, 15 lb. of 20-cent coffee, and 39 lb. of 30-cent coffee. For how much should it be sold per pound? Ans. $.25 per lb.

2. A goldsmith melted together gold as follows: 48 oz. of 15 carats, 24 oz. of 17 carats, 12 oz. of 20 carats, and 36 oz. of 22 carats. How many carats fine was the mixture? Ans. 18 carats.

3. Teas worth $.40, $.50, $.60, and $.75 per pound are made into a mixture containing 15 lb. of the first kind, 60 lb. of the second, 45 lb. of the third, and 15 lb. of the fourth. Find the value of a pound of the mixture. Ans. $.55 per lb.

4. A merchant sold silk in quantity and price as follows: 72 yd. at $1.50 per yard, 24 yd. at $1.87½, 12 yd. at $2.25, and 48 yd. at $2.75. What was the average price per yard? Ans. $2 per yd.

5. A mixture of candy consists of 20 lb. worth 18 cents a pound, 40 lb. worth 25 cents a pound, 48 lb. worth 35 cents a pound, and 20 lb. worth 40 cents a pound. What is a pound of the mixture worth? Ans. $.30 per lb.

ALLIGATION ALTERNATE.

74. Alligation alternate is the process of determining the proportional parts of the several ingredients in a combination or mixture having a given average unit value.

Thus, it may be required to find in what proportional parts we must mix coffee, tea, candies, etc. at different prices, so that the combination may be sold without loss at a given price per unit.

75. Example.—Teas worth 38, 45, 52, and 60 cents per pound are mixed so as to be sold at 50 cents per pound. Find the proportional weights of the several kinds.

Solution.—

I.

50	38	$\frac{1}{12}$		5	
	45		$\frac{1}{5}$	2	Ans.
	52		$\frac{1}{2}$	5	
	60	$\frac{1}{10}$		6	

$$\text{Proof.}\begin{cases} \$.38 \times 5 = \$\,1.90 \\ \$.45 \times 2 = \$\ \ .90 \\ \$.52 \times 5 = \$\,2.60 \\ \$.60 \times 6 = \$\,3.60 \end{cases}$$

$$18\ )\,\$9.00$$
$$.50$$

II.

50	38	$\frac{1}{12}$		1	
	45		$\frac{1}{5}$	2	Ans.
	52	$\frac{1}{2}$		6	
	60		$\frac{1}{10}$	1	

$$\text{Proof.}\begin{cases} \$.38 \times 1 = \$\ \ .38 \\ \$.45 \times 2 = \$\ \ .90 \\ \$.52 \times 6 = \$\,3.12 \\ \$.60 \times 1 = \$\ \ .60 \end{cases}$$

$$10\ )\,\$5.00$$
$$\$.50$$

Explanation.—The average price and the ingredient prices are written as shown. The ingredient prices are then linked in pairs, so that one of each pair may be greater, and one less, than the average price. In Solution I, 38 is linked with 60, and 45 with 52. Each different linking gives a different result. Considering Solution I, a pound worth \$0.38 sold at \$0.50 gains \$0.12, and to gain \$0.01 there must be sold $\frac{1}{12}$ of a pound. A pound worth \$0.60 sold at \$0.50 loses \$0.10, and to lose \$0.01 there must be sold $\frac{1}{10}$ of a pound. A gain of 1 and a loss of 1 is, on the whole, neither gain nor loss. It is clear that the sale of 60 times $\frac{1}{12}$ lb. and 60 times $\frac{1}{10}$ lb. will produce neither gain nor loss—that is, a sale of 5 and 6.

Applying the same reasoning to the other pair, we find that the sale of $\frac{1}{5}$ lb. and $\frac{1}{2}$ lb., or of 10 times these, will, in like manner, result neither in gain nor in loss. Hence, if 5, 2, 5, and 6 lb. of teas worth 38, 45, 52, and 60 cents per pound, respectively, be mixed, the mixture will be worth 50 cents per pound.

Any multiple of 5, 2, 5, and 6 will have the same average value per pound.

Solution II should be evident after considering the explanation of Solution I.

76. The fractions may be avoided by writing the difference between the average price and each ingredient price opposite the other ingredient price with which it is linked. This will be shown in the following example:

EXAMPLE.—In what proportional parts must candies worth 25, 30, 36, and 50 cents a pound occur in a mixture worth 40 cents a pound?

SOLUTION.—

40	25	10			10	Ans.
	30		10		10	
	36			10	10	
	50	15	10	4	29	

PROOF.

$$\begin{aligned} \$.25 \times 10 &= \$\ \ 2.50 \\ \$.30 \times 10 &= \$\ \ 3.00 \\ \$.36 \times 10 &= \$\ \ 3.60 \\ \$.50 \times \underline{29} &= \underline{\$14.50} \\ 59\ )\, &\underline{\$23.60} \\ &\ \ \$.40 \end{aligned}$$

EXPLANATION.—Here, then, are three kinds of candy worth less than the average, 40 cents, and one kind worth more. Hence, we link 25 with 50, 30 with 50, and 36 with 50, as shown. We draw a vertical line and subtract 25 from 40, obtaining 15, which we write opposite 50; subtracting 40 from 50 we get 10, which we write opposite 25. Drawing another vertical line, we subtract 30 from 40, getting 10, which we write opposite 50; subtracting 40 from 50, we get 10, which we write opposite 30. Drawing another vertical line, we subtract 36 from 40, getting 4, which we write opposite 50; subtracting 40 from 50, we get 10, which we write opposite 36. Now, adding horizontally the numbers included between the vertical lines, the required proportion is 10 lb. of 25-cent, 10 lb. of 30-cent, 10 lb. of 36-cent, and 29 lb. of 50-cent candy.

77. The same method may be applied to examples in Art. **75**, thus:

50	38	10		10	Ans.
	45		2	2	
	52		5	5	
	60	12		12	

78. There are usually many ways of linking the quantities in alligation alternate, but the answers will all be true, provided the two following conditions have been observed:

1. *That each pair must consist of one term below the average and one above.*

2. *That every quantity except that denoting the average must be included in the linking.*

79. When, however, only one quantity is greater or less than the average, it must be linked with each other quantity. In this case, there is only one method of linking, and only one set of numbers with its multiples will then meet the conditions of the example.

When there are two elements above the average and two below, there are several ways in which they may be linked. As the number of elements increases, the ways in which they may be linked become very numerous.

It is obvious that the conditions of an example will be met not only by any multiple of the several answers, but also by the sum of any two or more answers, provided that their elements are taken in the same order.

EXAMPLES FOR PRACTICE.

80. Solve the following examples:

1. A miller mixes wheat worth \$.80, \$.95, \$1.02, and \$1.08 per bushel so as to sell the mixture at \$1.00 per bushel. Find the proportional number of bushels of each kind in the mixture.

Ans. $\begin{cases} 8, 2, 5, 20. \\ 2, 2, 5, 5. \\ \text{etc.} \end{cases}$

2. A butcher bought lambs at \$4 each, sheep at \$7, hogs at \$9, and calves at \$12. In what proportion were they, if he paid for them an average of \$$8\frac{1}{2}$ each?

Ans. $\begin{cases} 7, 1, 3, 9. \\ \text{etc.} \end{cases}$

3. A merchant bought different kinds of cloth at \$1.50, \$1.65, \$1.85, and \$2.00 per yard. If the average price was \$1.75 per yard, in what proportion were the different kinds?

Ans. $\begin{cases} 2, 5, 5, 2. \\ \text{etc.} \end{cases}$

4. A mixture of 14, 19, 20, and 21 carat gold has an average fineness of 18 carats. Of what proportional parts is the mixture?

Ans. 3, 2, 2, 2.

5. The sum of certain multiples of 19, 23, 29, and 31 is exactly divisible by 24. What is the least possible sum of these numbers?

Ans. 240.

SUGGESTION.—First find the multipliers; then multiply the four given numbers by these multipliers, and add the products.

6. A pile of wood worth \$504 contains hickory worth \$8 a cord, oak worth \$5, maple worth \$4, and pine worth \$2. How many cords are there of each kind, if the average price per cord is $\$4\frac{1}{2}$?

Ans. $\begin{cases} 40, 8, 8, 56. \\ \text{etc.} \end{cases}$

SPELLING.

RULES FOR SPELLING.

1. The plural of nouns is generally formed by adding *s* to the singular; as, field, fields; friend, friends; temple, temples.

2. Nouns ending in *s*, *sh*, *ch* (soft), *x*, or *z*, add *es* for the plural; as, class, classes; brush, brushes; watch, watches; tax, taxes; buzz, buzzes.

3. Nouns ending in *y*, with a vowel* before the *y*, add *s* to form the plural; as, monkey, monkeys; day, days; valley, valleys.

4. Nouns ending in *y*, with a consonant before the *y*, change *y* to *i* and add *es* to form the plural; as, story, stories; army, armies; colony, colonies.

5. Nouns ending in *f* and *fe* generally change *f* or *fe* into *ves* to form the plural; as, calf, calves; knife, knives; thief, thieves.

FAMILIAR WORDS.

1.

clothes	cloak	clean	a′pron	words
shoe	your	slate	col′lar	spell
stock′ing	sharp	jack′et	rib′bon	les′son
coat	write	mit′ten	here	pen′cil

2.

peel	stem	mouth	tree	teeth
core	fore′head	chin	pulp	check
seeds	eye	flow′er	rind	ear
juice	nose	fruit	tongue	hair

* The *vowels* are *a*, *e*, *i*, *o*, *u*, and sometimes *w* and *y*. The other letters of the alphabet are called *consonants*.

3.

leaves	board	masts	e nough′	skip′ping
aunt	green	fence	through	eve′ning
their	mean	howls	dread′ful	cheer′ful
claws	learn	ap′ples	ven′ture	sum′mer

4.—THE CLOCK.

face	key	case	fig′ures
hour	glass	i′ron	min′ute
hands	brass	di′al	ham′mer
wheel	weights	wood	pen′du lum

5.—RELATIVES.

pa′pa	son	aunt	wife
fa′ther	sis′ter	niece	kin′dred
moth′er	broth′er	un′cle	hus′band
par′ents	chil′dren	cous′in	grand′fa ther
mam′ma	daugh′ter	neph′ew	grand′daugh ter

6.

earn	wrong	guide	weak	were
blue	knee	cough	col′or	worse
lamb	piece	debt	read′y	man′y
bus′y	ov′en	lose	broad	strong

7.

lunch	break′fast	but′ter	poul′try	fruit
bread	beef	oat′meal	sau′sage	buns
din′ner	sug′ar	pork	ven′i son	pud′ding
sup′per	ba′con	game	pies	pre serves′

8.

sand	drank	ham′mock	tramp	wag′on
hash	sad′dle	hand′some	stamp	scant′y
back	glad′ly	gas	scratch	ram′ble
latch	stag′ger	crab	rat′tle	prac′tice
catch	nar′row	plank	trav′el	pal′ace

9.—THE HOUSE.

floor	nurs'er y	chim'ney	man'tel	en'try
porch	clos'et	cup'board	fur'nace	pan'try
par'lor	gar'ret	cel'lar	door'step	li'bra ry
kitch'en	fire'place	hearth	at'tic	stair'case

10.—IN THE SCHOOLROOM.

ru'ler	wall	satch'el	pens
cray'on	class	tran'som	globe
pic'ture	shelf	reg'is ter	pa'per
teach'er	seats	pro'gram	shut'ters
schol'ars	chart	pen'-wi per	neat'ness
black'board	chairs	waste'-bas ket	at ten'tion

11.

cit'y	switch	kit'ten	bring	click
quit	skip	glimpse	width	wink
script	milk	still	fringe	cling
kissed	whist	miss	stiff	print
bridge	which	prim	knit	a bout'

12.

edge	wedge	rem'e dy	dense	when
fence	smell	wheth'er	meant	necks
hedge	shel'ter	cent	health	sweat
tempt	red'dish	deaf	next	depth

13.

cof'fin	con'cert	dodge	trot'ted	con vey'
for'est	shop	knock	hol'low	of fense'
bot'tle	knob	mock	pock'et	be yond'
gloss'y	notch	rock'et	fond	con verse'
bon'net	lodge	pop'py	bod'y	coun'tries

14.—BIRDS.

gull	grouse	swal'low	vul'ture	crane
swan	ra'ven	par'tridge	bob'o link	ea'gle
wren	par'rot	lark	night'in gale	cuck'oo

quail	os′trich	her′on	owl	spar′row
stork	pig′eon	o′ri ole	crow	con′dor
hawk	pea′cock	blue′jay	dove	lin′net

15.—WHAT BIRDS DO AND HAVE.

coo	car′ol	whis′tle	poise	tal′on
caw	hov′er	mi′grate	perch	wings
chirp	war′ble	soar	pin′ion	feath′ers
cheep	twit′ter	whir	beak	plu′mage

16.

crust	bunch	crutch	buzz	luck
ug′ly	jump	rub′ber	snug	mush
drum	brush	just	crumb	shrub
dumb	struck	dust	hut	pump

17.

live	space	moves	called	sur′face
ball	earth	through	breast	beau′ti ful
light	might	great	curled	won′der ful
gives	world	round	dressed	beau′ti ful ly

18.

po′ny	mus′tard	char′coal	tor′ment	hat′ter
med′al	par′ing	floun′der	scrib′ble	stick′y
cor′ner	mor′tar	hoe′ing	pres′ence	crip′ple
med′dle	pea′nuts	peel′ing	din′gy	grate′ful

19.—GEOGRAPHY.

sea	o′cean	sound	rap′ids	ca nal′
bay	pond	chan′nel	riv′u let	glac′i er
gulf	riv′er	lake	cat′a ract	cas cade′
strait	brook	creek	pool	foun′tain

20.—LAND.

cape	isth′mus	prai′rie	plain
is′land	head′land	pla teau′	o′a sis
con′ti nent	moun′tain	vol ca′no	val′ley
pen in′su la	prom′on to ry	ta′ble-land	des′ert

21.

lean	mid′dle	mis take′	rum′ple	south
lame	on′ly	naugh′ty	quar′rel	sor′ry
lamp	noise	ripe	pres′ent	sev′en
met′al	of′ten	or′der	soul	shad′ow
mer′ry	my self′	re fuse′	rust	shov′els

22.

taste	spo′ken	tum′ble	whale	worth
teach	thief	thought	up set′	wreck
stood	thirst	wake	up′per	whose
sprang	un less′	un til′	year	young
sto′ries	try′ing	waste	wool	bathe

23.

sure	talk′a tive	groan	shiv′er
quite	re la′tion	kite	whisk′ers
reach	mil′i ta ry	strength	gal′lon
speech	in ter rupt′	though	sal′low

KITCHEN.—24.—DINING ROOM.

tongs	sieve	urn	fork
ba′sin	stove	chi′na	spoon
pok′er	broom	plates	ta′ble
shov′el	ket′tle	sau′cer	cru′et
dip′per	scut′tle	gob′let	cast′ers
buck′et	dust′pan	tea′cup	nap′kin
coal′-hod	skim′mer	tu reen′	tum′bler
grid′i ron	sauce′pan	plat′ter	side′board

BEDROOM.—25.—PARLOR.

soap	sheet	vase	so′fa
bowl	lounge	car′pet	stool
tow′el	pil′low	mir′ror	scarf
bol′ster	bu′reau	cur′tain	mu′sic
pitch′er	blank′et	has′sock	pi an′o
scis′sors	mat′tress	book′case	por′trait
nee′dles	bed′stead	arm′chair	cab′i net
thim′ble	cov′er let	ot′to man	cush′ions

26.

balk	al′mond	tomb	dumb′ly
calk	salm′on	jamb	plumb′er
balm	be calm′	plumb	numb′ness
alms	em balm′	doubt	climb′er
salve	balm′y	debt′or	comb′ing
halves	psalm	crumbed	doubt′ful

27.

friend	like	sha′dy	sev′er al
glad	brook	cro quet′	ber′ries
birds	woods	ten′nis	ev′er y
fish	re ceive′	gained	moun′tains

28.

seal	er′mine	floe	moss′es
whale	rein′deer	Lapp	li′chens
sa′ble	po′lar bear	au ro′ra	red′snow
wal′rus	ei′der duck	snow hut	ice′bergs

29.—TREES.

pine	ash	al′der	birch
larch	elm	ma′ple	lo′cust
hol′ly	oak	eb′o ny	lin′den
ce′dar	beech	wil′low	wal′nut
spruce	ol′ive	hick′o ry	rose′wood
cy′press	pop′lar	chest′nut	pal met′to

30.—TOOLS.

file	gouge	lev′er	chis′el
adz	square	au′ger	lev′el
rake	lathe	gim′let	trow′el
spade	wrench	mal′let	hatch′et
plane	shears	pin′cers	cork′screw

31.

juice	loaf	liq′uid	sev′er al
cane	re fine′	hogs′head	plan ta′tion
mill	joints	mo las′ses	blos′som
scum	leaves	boiled	stom′ach

32.

know	va′por	er′rand	ques′tion
bruise	bal loon′	nei′ther	scare′crow
sneeze	no′tice	thir′teen	of′fi cer
rinse	e lev′en	be cause′	an oth′er
can′dy	e nough′	fair′y	nec′es sa ry

33.

On your way home you may see—

men	roads	gates	wom′en
stores	streets	parks	av′e nues
hous′es	gar′dens	hors′es	lamp′-posts
church′es	chil′dren	sta′bles	car′riag es

34.—CLOTHING.

hose	mack′in tosh	veil	muff
gloves	gai′ters	scarf	boots
cra vat′	col′lar	hood	tip′pet
skirt	trou′sers	ruf′fle	par′a sol
shawl	hand′ker chief	neck′tie	o′ver alls

35.—WORDS OF SIMILAR MEANING.

arch	curve	hank	skein
bluff	cliff	neat	ti′dy
bench	set tee′	quaint	strange
blithe	joy′ous	queer	droll
fleet	nim′ble	twain	coup′le
gruff	glum	tenth	tithe
gang	crew	van	front
giv′er	do′nor	wont	hab′it
pris′on	dun′geon	peo′ple	per′sons
late	tar′dy	soul	spir′it
snug	co′zy	rob	plun′der
rash	ha′sty	rich	fer′tile
grim	sur′ly	e rect′	up′right
hurt	in′jure	loose	un bound′
sole	sin′gle	com′fort	con sole′

36.—REVIEW.

beech	squash	sen′tence	pal′ace
sieve	fields	car′riage	lil′y
those	knife	pic′ture	dai′sy
wren	through	ea′gle	bu′reau
strait	tongs	chest′nut	tu reen′
pear	peach	on′ion	Eu′rope
sleeve	pen′cil	ba′sin	po′ny
fruit	tongue	re fuse′	os′trich
birch	Wednes′day	mi′grate	mo′ment

37.—REVIEW.

rob′in	let′tuce	meant	scis′sors
pig′eon	rai′sin	wrote	pump′kin
wal′nut	cur′rant	please	thim′ble
pop′lar	for′ward	ev′er y	pitch′er
for′est	ber′ries	qui′et	ba na′na
gob′let	rhu′barb	ache	a′pri cot
mat′tress	spin′ach	Christ′mas	beau′ti ful
cru′et	Eng′land	pil′low	Feb′ru a ry
min′ute	chis′el	mead′ow	po ta′toes

38.—IN THE COUNTRY.

farm′house	ditch	sheep	swing
vines	knoll	lambs	in′sects
gar′den	ar′bor	cat′tle	or′chard
flow′ers	hay-loft	fields	ber′ries
barn	mead′ow	wheat	but′ter flies

39.—VEGETABLES.

kale	tur′nip	car′rot	cu′cum ber
beet	rhu′barb	let′tuce	as par′a gus
bean	pars′nip	pump′kin	cau′li flow er
cress	cab′bage	spin′ach	egg′plant
gar′lic	mush′room	to ma′to	rad′ish

40.

glass	smooth	brit′tle	trans par′ent
win′dow	sash	panes	weights

hemmed	gath′ered	stitched	seam′stress
sleeves	a′pron	emp′ty	rode
learned	eas′i ly	rap′id ly	per′fect ly

41.

palms	ca ca′o	li′on	el′e phant
cof′fee	cam′phor	jag u ar′	ser′pent
cot′ton	in′di go	gi raffe′	go ril′la
sa′go	In′di a-rub′ber	cam′el	croc′o dile
spi′ces	sug′ar-cane	mon′key	leop′ard

42.

We should be—

good	cor′dial	help′ful	civ′il
frank	sin cere′	thought′ful	o blig′ing
prompt	lov′ing	ge′ni al	gen′er ous
hon′est	truth′ful	stu′di ous	o be′di ent
no′ble	care′ful	pa′tient	tem′per ate
po′lite	hope′ful	court′e ous	in dus′tri ous

43.

We should not be—

mean	stin′gy	rude	im po lite′
curt	cru′el	sur′ly	dis hon′est
proud	self′ish	care′less	cow′ard ly
la′zy	un kind′	haugh′ty	quar′rel some
sulk′y	fret′ful	de ceit′ful	dis hon′or a ble
sau′cy	sul′len	tat′tling	dis o be′di ent

44.—ABBREVIATIONS.

Jan′u a ry	Jan.	Ju ly′	July
Feb′ru a ry	Feb.	Au′gust	Aug.
March	Mar.	Sep tem′ber	Sept.
A′pril	Apr.	Oc to′ber	Oct.
May	May	No vem′ber	Nov.
June	June	De cem′ber	Dec.

WINDS.—45.—BOATS.

gale	cy′clone	barge	ca noe′
gust	si moom′	yawl	cut′ter
breeze	ty phoon′	sloop	schoon′er
squall	tor na′do	yacht	ves′sel
zeph′yr	whirl′wind	do′ry	frig′ate
tem′pest	hur′ri cane	gon′do la	steam′er

46.

goat	ot′ter	oats	flax
deer	pan′ther	corn	hemp
wolf	squir′rel	maize	to bac′co
moose	buf′fa lo	bar′ley	tim′o thy
rab′bit	an′te lope	ce′re al	mul′ber ry

47.

stealth	vague	whiff	ooze	wrought
trough	voice	trait	cruise	squeeze
thrift	vogue	grate	browse	grudge
twilled	vault	fierce	shrimp	sword

48.—FRUITS.

fig	peach	cher′ry	cran′ber ry
prune	grape	cit′ron	blue′ber ry
date	cur′rant	dam′son	straw′ber ry
plum	lem′on	pine′ap ple	rasp′ber ry
quince	mel′on	a′pri cot	goose′ber ry
pear	rai′sin	ba na′na	huck′le ber ry

49.—FLOWERS.

tu′lips	lark′spurs	pan′sies	sun′flow ers
as′ters	lil′ies	sweet peas	hol′ly hocks
li′lacs	dai′sies	hy′a cinths	vi′o lets
clo′vers	mar′i golds	dan′de li ons	ge ra′ni ums

50.

fell	cause	stum′bled	troub′le
lack	killed	horse′man	neg′li gence

steed	caught	in′ju ry	un fast′ened
foe	slight	loos′ened	at ten′tion

51.

co here′	e′qual	need′y	wear′y
se crete′	se vere′	speed′y	drear′y
im pede′	de′cent	feed′ing	trea′son
con vene′	fe′male	heed′ful	cheap′ly
ex treme′	pre′cept	free′dom	year′ling
su preme′	de scribe′	cheer′ful	mean′ing

52.

toil	gov′ern	pawn	scrub	trick
moat	heat	pledge	scour	trench
dirt	jest	quoth	di rect′	yawn
frown	joke	ruse	sell	wan′der
filth	stray	scheme	seize	be fore′
gape	pile	la′bor	scowl	col′umn

GEOGRAPHY.—53.—ARITHMETIC.

source	po′lar	add	mul′ti pli er
ax′is	cli′mate	plus	prod′uct
or′bit	sav′age	sum	di vide′
cir′cle	e qua′tor	sub tract′	div′i dend
mo′tion	ho ri′zon	min′u end	di vi′sor
dai′ly	ze′nith	sub′tra hend	quo′tient
year′ly	par′al lel	re main′der	proof
trop′ic	hem′i sphere	mul′ti ply	a rith′me tic
pole	ge og′ra phy	mul ti pli cand′	prob′lem

54.—USED IN COOKING.

rice	so′da	all′spice	va nil′la
sage	gin′ger	gel′a tine	vin′e gar
mace	pars′ley	choc′o late	hom′i ny
yeast	pep′per	sal e ra′tus	tap i o′ca
cloves	nut′meg	buck′wheat	cin′na mon

55.

Write the plurals of the words in this exercise:

skein	loaf	con′cert	man′sion
cit′y	dai′sy	sheaf	buf′fa lo
o′cean	tur′key	of′fi cer	jour′ney
wretch	suit′or	sur′face	os′trich
box	scratch	sand′wich	at tor′ney

56.

se rene′	rus′tic	waste′ful	si′lent
com′ic	tac′it	fear′ful	gloom′y
pu′trid	tur′bid	ru′ral	mud′dy
ea′ger	stur′dy	plac′id	flor′id
rud′dy	som′ber	ar′dent	rot′ten
lav′ish	tim′id	hard′y	mirth′ful

57.

gift	geese	gib′bous	gey′ser	guard
girt	ga′ble	gew′gaw	gal′lant	gid′dy
gild	gir′dle	tar′get	gaunt′let	gig′gle
gear	giz′zard	gal′lop	guest	guin′ea
gimp	gher′kin	gar′gle	gauze	gor′geous

58.—OPPOSITES.

right	wrong	dry	moist
find	lose	near	dis′tant
strong	weak	sweet	sour
gay	sad	quick′ly	slow′ly
rare	com′mon	a like′	un like′
ac cept′	de cline′	a′ged	youth′ful
in′door	out′door	a part′	to geth′er
cease	con tin′ue	prop′er	im prop′er
o bey′	dis o bey′	some′where	no′where

59.—FOOD.

toast	gru′el	broth	dump′ling
tarts	crack′er	sir′loin	sand′wich
veal	cook′y	om′e let	dough′nut

bis′cuit	waf′fle	muf′fin	sauer′kraut
sal′ad	hon′ey	por′ridge	cus′tard
mut′ton	catch′up	chow′der	suc′co tash

60.

ag′ile	rig′id	im′age	fidg′et
viv′id	ac′tive	jus′tice	res′pite
doc′ile	fes′tive	den′tist	cor′nice
hos′tile	rep′tile	ser′vice	des′tine
mis′sile	crev′ice	prom′ise	doc′trine

61.

cowl	crowd	rouse	de vour′
prowl	bow′er	shout	foun′dry
clown	dow′er	crouch	scoun′drel
crown	cow′ard	mound	com′pound
drown	drow′sy	flounce	pro nounce′

62.

cleft	flare	flaw	liege	rouge
pyre	dealt	goad	lapse	rogue
brief	freak	farce	weird	shone
copse	fraud	guile	grease	prism
corpse	fledge	gloat	mold	pierce
clench	chasm	gorge	mourn	plague
cleanse	dredge	gourd	league	scourge

63.—OCCUPATIONS.

tai′lor	flo′rist	farm′er	mil′li ner
ba′ker	join′er	weav′er	min′is ter
doc′tor	gro′cer	build′er	gar′den er
law′yer	bank′er	butch′er	car′pen ter
cob′bler	mer′chant	drug′gist	black′smith

64.—OUTDOOR SPORTS.

fish′ing	ri′ding	dri′ving	ska′ting
sail′ing	ten′nis	play′ing	mar′bles

row′ing	leap′ing	bowl′ing	coast′ing
boat′ing	cro quet′	jump′ing	base′ball
ba′thing	nut′ting	swing′ing	sleigh′ing

65.

so′lar	nec′tar	dro′ver	dif′fer	lar′der
lu′nar	lat′ter	can′ker	hin′der	blis′ter
tar′tar	an′ger	la′ter	gan′der	blub′ber
stel′lar	bet′ter	cof′fer	fil′ter	mem′ber

66.

loi′ter	coil	an noy′	in′voice
toi′let	broil	de coy′	re joice′
poi′son	spoil	boy′ish	pur loin′
coin′age	hoist	em ploy′	oint′ment
ap point′	choice	boy′cott	em broid′er

67.

cen′ser	cir′cus	ac′id	ceil′ing
cen′sus	ci′pher	cir′cuit	cym′bal
cen′tral	cin′der	cyl′in der	cen′tu ry
ce ment′	cer′tain	cel′e brate	ce les′tial
cen′taur	cen′sure	cem′e ter y	cen ten′ni al

68.—HOMONYMS.

1 { cent, a coin; scent, an odor; sent, did send

2 { plum, a fruit; plumb, perpendicular

3 { stare, to look earnestly; stair, a step

4 { fore, in front; four, a number

5 { to, as in "Give it to me"; too, as in "too cold"; two, a number

6 { threw, did throw; through, as in "through the air"

7 { fir, a tree; fur, fine, soft hair

8 { earn, to get or merit by labor; urn, a vase

Write the following sentences, putting the right word in the right place. Underline the words inserted.

It is not what we (8), but what we save that makes us rich. —*Prov.* A bird in the hand is worth (5) in the bush.—*Prov.*

We (1) the silver (8) (5) them. I remember, the (7) trees, dark and high.—*Hood.* A (1) is one-hundredth part of a dollar. Prunes are dried (2)s. A dog has (4) feet, but the (4) feet are the two front feet. And all the world would (3). —*Cowper.* The (1) of the roses will hang round it still.—*Moore.* Who (6) the stone (6) the window? Russian sable is a costly (7). (5) many cooks spoil the broth. Masons test a wall with a (2)-line. White marble (3)s lead to the Capitol.

69.

nev′er	neth′er	fel′on	men′tal
tem′per	skep′tic	zeal′ot	plen′ty
per′ish	cher′ub	stead′y	pet′rel
cher′ish	her′ald	peas′ant	jeal′ous
wed′lock	blem′ish	threat′en	weath′er
ped′dler	thread′bare	pleas′ure	meas′ure

70.

com′pass	cac′tus	cha′os	chord
cul′prit	com′rade	chrome	chyle
cur′ry	cab′in	chro′mo	chyme
cur′few	ca′ble	chron′ic	chol′er a
com′et	com pute′	cho′ral	chron′i cle
com plete′	cu′pola	chem′ist	char′ac ter
col′umn	com plex′ion	Chris′tian	cha me′le on

71.—ARITHMETIC.

rate	cu′bic	in′te ger	a′mount
terms	fac′tor	dec′i mal	prin′ci pal
prime	frac′tion	mul′ti ple	di vis′i ble
dig′it	al′i quot	in′ter est	in sur′ance
ze′ro	dis′count	com pos′ite	bro′ker age
a′cre	ex am′ple	nu′mer ator	per cent′age
naught	hun′dredth	de nom′i na tor	av oir du pois′

72.—ANIMAL SOUNDS.

purr	yelp	bleat	snort	squeak
hum	howl	cluck	cack′le	roar

low	quack	neigh	whin′ny	scream
grunt	growl	croak	bel′low	buzz
squeal	mew	gob′ble	chir′rup	screech

73.—INSECTS.

bee	gnat	ant	drag′on-fly
wasp	moth	wee′vil	bum′ble bee
flea	roach	mos qui′to	but′ter fly
lo′cust	bee′tle	glow′worm	ka′ty did
hor′net	crick′et	silk′worm	grass′hop per

74.

a′li as	fa′cial	ef face′	fa′tal
a′gen cy	pa′tron	va′cant	ha′zy
ma′ni ac	sta′tion	en gage′	ba′bel
brace′let	an′cient	be came′	az′ure
fa′vor ite	pa rade′	pro fane′	ha′tred
va′por ize	dra′per y	dis place	man′ger

75.

gym′nast	gib′lets	hom′age	gen′u ine
gyp′sy	gen′ius	gib′bet	mag′is trate
en′gine	gen teel′	gen′e sis	gym nas′tics
mar′gin	herb′age	gest′ure	gym na′si um

76.—REVIEW.

ache	bruise	bu′reau	could
ac cept′	Arc′tic	care′ful	dai′sy
a fraid′	bis′cuit	car′riage	cur′tain
a gain′	au′tumn	cel′er y	col′ored
al′mond	ba na′na	chim′ney	con′ti nent
an′i mal	break′fast	Christ′mas	cran′ber ry

77.

dif′fer ent	friend	health′y	juice
dough′nut	gi raffe′	help′ful	knife
ear′nest	gru′el	hom′i ny	knuck′le

ei′ther	e qua′tor	ho ri′zon	laugh
el′e phant	er′rand	hy′a cinth	learn
e nough′	Feb′ru a ry	in′ter est ing	isth′mus

78.—REVIEW.

maize	mo las′ses	o′a sis	rai′sin
li′chen	mos qui′to	o blige′	pic′ture
liq′uid	moun′tain	om′e let	pitch′er
mat′tress	nei′ther	os′trich	pleas′ant
mi′grate	neph′ew	oys′ter	prai′rie
min′u end	niece	par′al lel	pump′kin

79.

rhu′barb	skein	tongue	zone
rough	sleigh	tor′rid	whose
salm′on	sev′er al	un til′	would
schol′ar	skel′e ton	tem′per ate	wrong
scis′sors	spin′ach	thous′and	ze′nith
sen′tence	squir′rel	vol ca′no	writ′ten

80.—SYNONYMS.

ef′fort	en deav′or	lack′ing	de fi′cient
re past′	col la′tion	out′ward	ex ter′nal
in close′	en vel′op	down′cast	de ject′ed
de cide′	de ter′mine	a tone′	ex′pi ate
re vere′	ven′er ate	con fuse′	be wil′der
spring′y	e las′tic	de ride′	rid′i cule

81.

fool	priest	dra′ma	han′dle	lad′der
lock	del′ta	fel′low	hel′met	lob′ster
feast	an′gel	mask	freck′le	mar′ket
hook	se′cret	fos′sil	liq′uor	ma′tron

82.

rul′er	an′cient	tow′er ing	for′tress es
sto′ry	ver′dant	ex ten′sive	at tract′ive

roams	fruit′ful	sep′ul cher	ap pa ri′tion
stream	slum′bers	be stow′ing	pic tur esque′
stat′ed	en tombed′	re main′der	ben e dic′tion

83.—MASCULINE AND FEMININE.

he′ro	her′o ine	beau	belle
host	host′ess	wiz′ard	witch
act′or	ac′tress	sir	mad′am
god	god′dess	bach′e lor	maid, spin′ster
heir	heir′ess	wid′ow er	wid′ow
jan′i tor	jan′i tress	man serv′ant	maid serv′ant
proph′et	proph′et ess	land′lord	land′la dy

84.—HOMONYMS.

1 { break, to part by force; brake, for stopping wheels; a fern

2 { week, seven days; weak, feeble

3 { waist, part of the body; waste, a desert; to squander

4 { piece, a part; a composition; peace, quiet

5 { hail, frozen rain; to salute; hale, healthy

6 { wait, to stay; weight, heaviness

7 { heel, part of the foot; heal, cure

8 { peal, a loud sound; peel, to strip off the skin

In the following sentences put the right word in the right place. Underline the words inserted.

Achilles was slain by being wounded in the (7). A (4) of banana (8) should not be thrown on the pavement. The (1)ing waves dashed high.—*Hemans.* (5), holy light.—*Milton.* The engine whistled "Down (1)s." What is your (6)? I lay me down in (4) to sleep.—*Willard.* Physician, (7) thyself.—*Bible.* If you are (5), you cannot be (2). The deep thunder, (8) on (8), afar.—*Byron.* There was a belt about her (3). Sunday is the first day of the (2). Learn to labor and to (6).—*Longfellow.* (3) not, want not.—*Prov.*

85.

jui′cy	tip′sy	po′sy	ma′zy
spi′cy	fuss′y	pal′sy	diz′zy

fan′cy	moss′y	pro′sy	cra′zy
flee′cy	mass′y	flim′sy	breez′y
mer′cy	drop′sy	glass′y	driz′zly

86.—SYNONYMS.

a base′	de grade′	ad′age	max′im
ab hor′	de test′	ac cost′	sa lute′
a bide′	so′journ	a dieu′	good-by′
ac quit′	ab solve′	a dorn′	dec′o rate
ac cede′	com ply′	ad vice′	coun′sel
a ban′don	for sake′	ac quaint′	in form′

87.—OPPOSITES.

ab′sent	pres′ent	guilt′y	in′no cent
be stow′	re ceive′	stub′born	yield′ing
de stroy′	con struct′	in te′ri or	ex te′ri or
free	cap′tive	fail′ure	suc cess′
dis perse′	as sem′ble	de crease′	aug ment′
e merge′	im merge′	em′i grate	im′mi grate

88.

sa li′va	i′ci cle	con fide′	pi′rate
en vi′ron	pli′a ble	com bine′	bri′ny
en ti′tle	di′a ry	re cite′	mi′nus
vi′o late	li′a ble	sur mise′	li′bel
ri′val ry	si′phon	re quire′	fi′nal
pi′e ty	bi′ped	com prise′	vi′per

89.

braid	ex am′ine	coax	ra′zor
beard	ex cur′sion	skull	ven′ture
bloom	de light′ful	hoarse	whith′er
bleach	in struc′tion	a piece′	smoth′er

90.—MINING.

zinc	mi′ca	gran′ite	lode
lead	sul′phur	gyp′sum	shaft

quartz	car′bon	plat′i num	min′er
cop′per	salt pe′ter	min′er al	tun′nel
ni′ter	mer′cu ry	me tal′lic	der′rick

91.—FARMING.

plow	har′row	ster′ile	fer′tile
reap′er	thresh′er	clay′ey	bog′gy
mow′er	fal′low	bar′ren	ar′a ble
scythe	swamp′y	loam′y	al lu′vi al
sick′le	fruit′ful	marsh′y	gua′no

92.

mu′ti ny	flu′id	dis pute′	se clude′
du′ti ful	mu′sic	pro cure′	de lude′
pu′ri fy	glu′ten	in sure′	as sure′
cru′el ty	tu′mor	di lute′	a buse′
mu′tu al	fu′ture	en dure′	a muse′
lu′di crous	cu′ri ous	pre sume′	al lure′

93.

doub′le	jew′el er	su pe′ri or	sleet
har′ness	tomor′row	va ca′tion	tacks
mat′ting	which ev′er	twi′light	sphere
val′u a ble	scen′er y	moun′tains	ce′dar

94.

blouse	cis′tern	knuck′l e	steppes
breathe	clean′ly	col′an der	tru′ly
el lipse′	dah′lia	fa mil′iar	vil′lain
de spair′	cru sade′	des′o late	cut′ler y
um′pire	cou′pon	de pos′it	hap′pened
jun′ior	com′merce	tran′som	oc curred′
a′corn	cour′te sy	ven′ti late	prec′i pice

95.

daunt	twelfth	come′ly	bel′lows
brooch	a dult′	com mand′	as sume′

nymph	de sist′	a′re a	cask′et
crea′ture	ad vance′	dis arm′	cit′i zen
con sume′	di gest′	cha grin′	duc′tile

96.—HOMONYMS.

1 air, what we breathe; e'er, ever; ere, before; heir, one who inherits

2 sail, of a ship; sale, a selling

3 ho′ly, sacred; whol′ly, completely

4 plain, level ground; clear; plane, flat surface; tree; tool

5 rain, water from clouds; reign, rule; rein, for a horse

6 coarse, rough; course, way

7 col′lar, band for the neck; chol′er, anger

8 dy′ing, ceasing to live; dye′ing, coloring

Write the following sentences, putting the right word in the right place:

Westward the (6) of empire takes its way.—*Berkeley.* O, there is sweetness in the morning (1).—*Byron.* (3) angels guard thy bed.—*Watts.* How gladly would we buy time were it for (2). How beautiful is the (5) after the dust and heat.—*Longfellow.* The top of my desk is a (4) surface. What! drunk with (7).—*Shakespeare.* The (7) was made of cloth. We thought her (8) when she slept. The Prince of Wales is (1) to the English throne. Russia is almost (3) a vast (4). Write home (1) the ship (2)s. The (5) guides the horse. The cochineal insects furnish a red color for (8). Shakespeare lived during the (5) of Queen Elizabeth.

97.—CLOTH.

jean	sat′in	si le′sia	cal′i co
baize	vel′vet	me ri′no	al pac′a
serge	flan′nel	cam′bric	dam′ask
plush	mus′lin	bro cade′	cor′du roy
lin′en	mo′hair	chev′i ot	cas′si mere
tweed	de laine′	ging′ham	vel vet een′
chintz	broad′cloth	cash′mere	seer′suck er

98.

monk	piv′ot	pub′lic	tas′sel
mur′mur	part′ner	rum′mage	trel′lis
mys′ter y	phys′ic	skir′mish	tus′sle
par′a ble	pi′ous	sol′emn	weap′on
par′cel	pi′ra cy	spe′cie	wel′fare
mes′sage	pit′i ful	stam′mer	syr′inge

99.

tu′bu lar	bear′er	vic′tor	ju′ror
tab′u lar	lodg′er	val′or	fla′vor
pop′u lar	cor′o ner	tu′tor	ru′mor
cir′cu lar	mourn′er	tre′mor	or′a tor
cal′en dar	strag′gler	trai′tor	stu′por
sec′u lar	vend′er	tor′por	splen′dor
mus′cu lar	in tru′der	suit′or	sur vey′or

100.—ON THE WRITING DESK.

ream	ru′ler	let′ter	pa′per-weight
quill	tab′let	e ra′ser	port fo′li o
quire	blot′ter	ink′stand	mu′ci lage
stamps	wa′fer	fools′cap	en′vel ope
pen′knife	cal′en dar	dic′tion a ry	seal′ing-wax

101.—FISH.

eel	trout	dol′phin	her′ring
cod	pike	sar′dine	mack′er el
carp	shark	had′dock	pick′er el
perch	shad	suck′er	stur′geon
bass	smelt	min′now	hal′i but

102.

un just′	un just′ly	re′al	re′al ly
lan′guid	lan′guid ly	meek	meek′ly
se cure′	se cure′ly	court	court′ly
se′ri ous	se′ri ous ly	an′nu al	an′nu al ly
spite′ful	spite′ful ly	in tent′	in tent′ly
un u′su al	un u′su al ly	for′mer	for′mer ly
dread′ful	dread′ful ly	fre′quent	fre′quent ly

103.

grief	griev′ous	pore	po′rous
joy	joy′ous	ri′ot	ri′ot ous
vice	vi′cious	glo′ry	glo′ri ous
stud′y	stu′di ous	dan′ger	dan′ger ous
la′bor	la bo′ri ous	mur′der	mur′der ous
in′dus try	in dus′tri ous	ma la′ri a	ma la′ri ous

104.—HOMONYMS.

1 { pair, two; pare, to cut off; pear, a fruit

2 { their, belonging to them; there, in that place

3 { cap′i tal, chief town; stock; cap′i tol, building

4 { hear, to listen; here, in this place

5 { meat, animal food; meet, proper; to come together; mete, to measure

6 { as cent′, a rising; as sent′, agreement

7 { pane, a plate of glass; pain, an ache

8 { rye, a grain; wry, crooked

Write the following sentences, putting the right word in the right place:

(2) is nothing new under the sun.—*Bible.* (8) grows in cold countries. When shall we three (5) again?—*Shakespeare.* A (7) of glass. (4) rests his head upon the lap of earth.—*Gray.* Can you (1) a (1) with a (1) of scissors? The (3) is a white marble building. The (6) of Mont Blanc is full of danger. Be silent that you may (4).—*Shakespeare.* Sweet is pleasure after (7).—*Dryden.* Birds in (2) little nests agree.—*Watts.* Washington is the (3) of the United States. Is not the life more than (5)?—*Bible.* He gave his (6) to the proposal. There is a bird called the (8) neck. With what measure ye (5), it shall be measured to you again.—*Bible.*

105.—KNOWN BY

Seeing.	*Smelling.*	*Tasting.*
squal′id	ran′cid	lus′cious
un couth′	fra′grant	pun′gent

col′ored	per′fumed	bit′ter
o paque′	o′dor ous	in sip′id
ra′di ant	ar o mat′ic	de li′cious

Touching. *Hearing.*

warm	smooth	clear	ech′o ing
rough	tep′id	loud	muf′fled
sleek	un e′ven	in dis tinct′	noi′sy

106.

height	count′er	la′zi ly	ach′ing
re sult′	awk′ward	hes′ i tate	sur prise′
con fess′	bus′i ly	spe′cial	quan′ti ty
wrig′gle	re mark′a ble	mus′cles	syl′la ble
fir′kin	gen′tle man	vil′lage	scold′ed
anx i′e ty	u′su al ly	pro tect′	gos′sa mer

107.—REVIEW.

e ra′ser	dic′tion a ry	scythe	com′mon
bar′y tone	mack′er el	feign	tac′it
or′ches tra	jew′el ry	zeph′yr	gyp′sy
cam paign′	veg′e ta ble	liq′uor	ges′ture
glac′i er	cas′si mere	giz′zard	salm′on
moun′tain	sar sa pa ril′la	gey′ser	wool′en

108.

al pac′a	par′lia ment	thief	dai′sy
sal′a ry	cyl′in der	toi′let	doc′ile
boy′cott	cem′e ter y	au′tumn	rep′tile
ox′y gen	am′e thyst	ac′id	im′age
em′er y	sep′a rate	pum′ice	cor′nice
tur quoise′	hun′dredth	ber′yl	ci′pher

109.—REVIEW.

skep′tic	sol′emn	weird	can′cel
squal′id	ver′ti cal	myrrh	con cede′
squir′rel	de li′cious	which	pleas′ure

schol′ar	syl′la ble	straight	ped′dler
myr′i ad	ker′o sene	weap′on	vil′lage
wiz′ard	med′i cine	syr′inge	griz′zly

110.

i′ci cle	ver mil′ion	a dieu′	tus′sle
rum′mage	Ni ag′a ra	sep′a rate	cer′tain
lunch′eon	nec′es sa ry	twelfth	cha grin′
sur′name	pen i ten′tia ry	be lieve′	cot′tage
car′a mel	re ceived′	the′a ter	sup pose′
span′iel	ex hi bi′tion	res er voir′	de vel′op

111.—HISTORY.

char′ter	squaw	scalped	wil′der ness
spear	pap poose′	ship′wreck	suf′fer ing
wig′wam	war′rior	ex plore′	set′tle ment
wam′pum	cal′u met	col′o ny	trea′ty
moc′ca sin	dis cov′er y	per′ma nent	per′se cute
tom′a hawk	pi o neers′	nav′i ga tor	mas′sa cre

112.—HOMONYMS.

1 { ewe, a female sheep; you, person addressed; yew, an evergreen tree

2 { cru′el, unkind; crew′el, soft yarn

3 { choir, a band of singers; quire, 24 sheets of paper

4 { bough, a branch; bow, to bend; front of ship

5 { knead, to work dough; need, want

6 { stake, a post; a wager; steak, a slice of meat

7 { gait, manner of walking; gate, a kind of door

8 { main, chief; mane, hair on horse's neck; Maine, one of the U. S.

Write the following sentences, putting the right word in the right place:

I had most (5) of blessing.—*Shakespeare.* May I join the (3) invisible ?—*George Eliot.* (1)s and bleating lambs.—*Milton.* Added woes may (4) me to the ground.—*Pope.* (2)s are used in embroidery. (1) must (5) the dough to make good bread. The man who walked through the (7) had a peculiar (7). Have a care for the (8) chance.—*Butler.* Twenty (3)s make a

ream. He bought a slice of sirloin (6). Me. is the abbreviation of (8). The Mistletoe (4) is a poem. Like a dewdrop from the lion's (8). Joan of Arc was burned at the (6). The wood of the (1) was used for making bows.

113.

cu'ti cle	lyr'ic al	cler'ic al	pay'a ble
mir'a cle	spher'ic al	crit'ic al	bla'ma ble
ob'sta cle	sur'gi cal	cyn'ic al	ta'ma ble
par'ti cle	trag'ic al	whim'si cal	teach'a ble
spec'ta cle	op'tic al	prac'ti cal	ten'a ble
trea'cle	clas'sic al	phys'i cal	ca'pa ble
man'a cle	com'ic al	med'ic al	af'fa ble

114.

sing'er	preach'er	deaf'ness	soft'ness
wait'er	la'bor er	firm'ness	sweet'ness
start'er	lect'ur er	swift'ness	round'ness
catch'er	ex am'in er	fierce'ness	prompt'ness
think'er	com mand'er	hard'ness	wretch'ed ness

115.

ac crue'	a thwart'	con'quer	ath'lete
a chieve'	vict'uals	a'lien	fau'cet
a ghast'	cro chet'	al'oes	cal'lous
greas'y	bou quet'	anx'ious	bur'glar
ca det'	bru nette'	brig'and	dul'cet
con geal'	car toon'	drug'get	daunt'less

116.—HOMONYMS.

1. leaf, part of a tree or book
   lief, willingly
2. great, large; noble
   grate, to rub; iron frame for fire
3. heard, did hear
   herd, of cattle
4. lain, reclined
   lane, a narrow road
5. base, mean; foundation
   bass, a part in music
6. aught, anything
   ought, bound by duty
7. right, opposite of wrong
   rite, a form
   wright, a workman
   write, act of writing

Write the following sentences, putting the right word in the right place:

I see the (7), and I approve it too.—*Ovid.* It is a long (4) that has no turning.—*Prov.* We all do fade as a (1).—*Bible.* Do what you (6), come what may. Baptism is a religious (7). The (2) fishes eat up the little ones.—*Shakespeare.* I had as (1) sing (5). A ship (7) works in a shipyard. Have you (3) (6) against him? It is a (5) thing to betray a man who has trusted you.—*Prov.* A bright fire glowed in the (2). The lowing (3) winds slowly o'er the lea.—*Gray.* I never dare to (7) as funny as I can.—*Holmes.* The book has (4) on my table several days.

117.

ab′sence	fuzz	go′pher	la pel′
at′om	cleat	griz′zly	loi′ter
bar′gain	clev′er	haz′ard	lag′gard
but′ton	e clipse′	hu′mor	loz′enge
cat′kin	en cir′cle	jock′ey	ma chine′
glut′ton	fa tigue′	ker′o sene	med′i cine

118.

change	change′a ble	man′age	man′age a ble
charge	charge′a ble	mar′riage	mar′riage a ble
trace	trace′a ble	cour′age	cou ra′geous
no′tice	no′tice a ble	out′rage	out ra′geous
ser′vice	ser′vice a ble	ad vant′age	ad van ta′geous

119.

ey′ry	la′va	ma rine′	dis dain′
eye′let	leis′ure	mar′tyr	glob′ule
frag′ile	jour′nal	naph′tha	hos′tage
frag′ment	lan′guage	nui′sance	cres′cent
ghast′ly	laun′dry	nos′trils	tor′toise
gor′geous	mal treat′	cha rade′	tor′ture

120.—HOMONYMS.

1	rap, to knock wrap, to cover	4	strait, narrow straight, direct
2	peer, an equal; a nobleman pier, pillar of a bridge	5	vain, useless; proud vein, a blood vessel vane, a weathercock
3	colo′nel, an army officer ker′nel, central part of a nut	6	cel′lar, an underground room sell′er, one who sells

Write the following sentences, putting the right word in the right place:

(5) is the help of man.—*Bible.* In song he never had his (2).—*Dryden.* Strive to enter in at the (4) gate.—*Bible.* He cut a (5) in his hand. (1) on the door. The (2)s were of granite. A (3) commands a regiment. The (6) of wines keeps his stock in a (6). A (5) was on the church steeple. (1) up warmly, for the road is bleak. The (3) of the nut serves the squirrel for food.

121.—CHRISTMAS DINNER.

soup	tur′key	on′ions	ice-cream′
rolls	squash	sauce	cake
pick′les	po ta′toes	mince pie	fruit
cel′er y	gra′vy	plum pud′ding	con′serve

122.

bier	ag grieve′	ceil	rein
tier	re lief′	de ceit′	reign
mien	shield	de ceive′	weigh
wield	re trieve′	con ceive′	skein
lien	re prieve′	con ceit′	hei′nous
niece	mis′chief	re ceipt′	o bei′sance
siege	sor tie′	re ceive′	in veigh′
frieze	ker′chief	per ceive′	neigh′bor

123.

chas tise′	bap tize′	a pol′o gize	or′gan ize
crit′i cise	cap size′	har′mo nize	mag′net ize
cat′e chise	re′al ize	gal′va nize	sym′pa thize
ad′ver tise	i′dol ize	fer′til ize	sol′em nize
ex′er cise	civ′i lize	col′o nize	rec′og nize
mer′chan dise	cen′tral ize	tan′ta lize	pat′ron ize
en′ter prise	le′gal ize	dram′a tize	mem′o rize

124.—HOMONYMS.

Write the following sentences, using the right word:

Better alone than in (bad, bade) company. Don't give (to, two, too) much for the whistle. Everything comes in (thyme, time) to (hymn, him) who can (wait, weight). He who follows (two, too, to) (hares, hairs) is sure (two, too, to) catch neither. It never (reins, rains, reigns) but it (pores, pours). Men speak of the (fair, fare) as things went with them (there, their). Out of (site, sight, cite), out of (mind, mined). Time and (tied, tide) (wait, weight) for (know, no) man. Where (there's theirs) a will there's always a (weigh, way). (Faint, feint) (hart, heart) ne'er (won, one) (fare, fair) lady.

125.

val′leys	pur′chased	sal′a ry	ce′dar
cur′rent	a cross′	spruce	scen′er y
cap′i tal	glac′i ers	in hab′it ants	val′u a ble
cli′mate	va ri′e ty	re sem′ble	de spair′

126.

de clare′	can′cel	ap pease′	con sent′	af fair′
col lect′	a ver′	a bol′ish	re voke′	a gree′
al lay′	a mass′	as sert′	dis cuss′	an nul′
ap pall′	as suage′	con cede′	af firm′	ar′gue
at tach′	af fright′	zeal′ous	al low′	anx′ious
pac′i fy	ap pend′	con cern′	ar′dent	an′cient

127.

sluice	thyme	whoop	suite	spouse
shrift	type	wreathe	realm	manse
scribe	theme	writhe	myrrh	phrase

128.

nerve	un nerve′	ech′o	re ech′o
spell	mis spell′	e lect′	re e lect′
spend	mis spend′	le′gal	il le′gal
step	mis step′	sev′er	dis sev′er
en gage′	re en gage′	mor′tal	im mor′tal
sat′is fy	dis sat′is fy	sim′i lar	dis sim′i lar

129.—HOMONYMS.

Write the following sentences, selecting the right word:

The past is not (holy, wholly) (vane, vain, vein), if rising on its (wrecks, recks) to something nobler (we, wee) attain.—*Longfellow.* In the morning (sow, sew) thy (seed, cede).—*Bible.* Then he said, "Good (knight, night)," and with muffled (o'er, oar, ore), silently (road, rowed, rode) to the Charlestown shore.—*Longfellow.* The heaviest (dews, dues) fall on clear (nights, knights). The (pale, pail) light of the moon is the reflection of the (son's, sun's) light. "(There, Their) graves are green; they may be (scene, seen)," the little (made, maid) replied.—*Wordsworth.*

130.

leg′a cy	ur′gen cy	in′ti ma cy	em′bas sy
in′fan cy	cur′ren cy	ex′i gen cy	jeal′ous y
de′cen cy	con′stan cy	cel′i ba cy	min′strel sy
se′cre cy	fal′la cy	con spir′a cy	lep′ro sy
re′gen cy	clem′en cy	ac′cu ra cy	her′e sy
flu′en cy	bril′lian cy	e mer′gen cy	ec′sta sy
va′can cy	buoy′an cy	com′pe ten cy	hy poc′ri sy

131.

a bun′dance	tem′per ance	au′di ence	ig′no rant
venge′ance	ac quaint′ance	dil′i gence	as sist′ant

ig′no rance	an noy′ance	ab′sti nence	de pend′ent
sus′te nance	ve′he mence	rev′er ence	el′e gant
vig′i lance	el′o quence	dif′fi dence	dil′i gent

132.—DISEASES.

gout	ca tarrh′	jaun′dice	hem′or rhage
croup	asth′ma	quin′sy	rheu′ma tism
a′gue	mea′sles	scur′vy	neu ral′gi a
mumps	ul′cer	ty′phoid	pneu mo′ni a
fe′ver	ab′scess	scrof′u la	dys pep′si a
grippe	in flu en′za	nau se′a	pa ral′y sis
can′cer	chil′blain	pleu′ri sy	diph the′ri a
col′ic	ver′ti go	de lir′i um	hy dro pho′bi a

133.—HISTORY.

writs	bul′let	ef′fi gy	in de pend′ent
re sist′	pow′der	al li′ance	dec la ra′tion
mobbed	ram′part	as sem′bly	ca lam′i ty
war′rant	car′tridge	pro hib′it	vol un teer′
un′ion	pa′tri ot	tax a′tion	e vac′u ate
suf′frage	re pealed′	tyr′an ny	rev o lu′tion
of fi′cial	mi li′tia	op pres′sion	u ni ver′sal
re doubt′	lib′er ty	gov′ern ment	rep re sent′

134.—ARMY WORDS.

sol′dier	pla toon′	sen′try	siege
com′pa ny	bat tal′ion	pick′et	sut′ler
reg′i ment	sword	u′ni form	re cruit′
bri gade′	sa′ber	knap′sack	hos′pi tal
di vi′sion	bay′o net	can teen′	gar′ri son
corps	pis′tol	ep′au let	coun′ter sign
in′fan try	car′bine	bag′gage	court-mar′tial
cav′al ry	mus′ket	cais′son	am mu ni′tion
ar til′ler y	ri′fle	biv′ou ac	for ti fi ca′tion
en gi neers′	can′non	fur′lough	in trench′ment

135.—NEWSPAPER TERMS.

dai′ly	ed′i tor	ed i to′ri al
morn′ing	jour′nal ist	lead′er

eve′ning	con trib′u tor	ar′ti cle
sem i-week′ly	re port′er	i′tem
week′ly	cor re spond′ent	lo′cal
bi-month′ly	sub scrib′er	tel′e grams
month′ly	ad ver tis′er	no′tic es
ex chang′es	car′ri er	gos′sip
e di′tion	news′boy	fi nan′cial
pro pri′e tor	ex′tra	a muse′ments
pub′lish er	col′umn	mis cel la′ne ous

136.—REVIEW.

war′rior	cyn′ic al	fuch′si a	phlox
sa′chem	cler′ic al	a za′le a	cap′tain
skil′ful	proph′e cy	wretch′ed	trea′cle
pro′ceeds	proph′e sy	an′te ced ent	pre cede′
con sign′ee	mer′ri ment	par′ti ci ple	pa′tience
Mis sis sip′pi	prom′is so ry	de clar′a tive	de cease′

137.

pal i sade′	aux il′ia ry	mal′ice	scal′lop
cav a lier′	lin′e a ment	vi′cious	su′ture
nau′ti lus	Cin cin na′ti	ec′sta sy	si′phon
or′i fice	rheu′ma tism	jeal′ous y	fron tier′
se′cre cy	dys pep′si a	pen′sion	mus′cle
pes′ti lence	pneu mo′ni a	mis spell′	e′qual ly

138.—REVIEW.

league	bat′ter y	im ag′ine	niche
gui tar′	sal′a ble	fric as see′	sparse
cor′nice	tab′er na cle	lat′i tude	clique
fa çade′	syn′a gogue	mul′lein	na′dir
ac crue′	pre cen′tor	gor′geous	eye′let
pres′sure	cer′e mo ny	tor′toise	mi rage′

139.

coup′le	pau′ci ty	nui′sance	sues
fro′zen	venge′ance	mar′riage	route
vict′uals	griev′ance	wretch′ed	choked

bou quet′	ni′tro gen	ap′pe tite	ra vine′
bru nette′	cap′il la ry	ha rangue′	rou tine′
drug′gist	es pe′cial ly	phy sique′	pat′ent

140.—REVIEW.

nau′se a	dis ap pear′	sep′a rate	sphinx
mi li′tia	dis′si pate	re ceived′	can′cer
ver′ti cal	glyc′er ine	sur′geon	ca tarrh′
cal′en dar	ben′e fit ed	sol′dier	cap size′
ex′er cise	nec′es sa ry	bag′gage	bea′con
con′science	re bel′lion	fur′lough	bod′ice

141.

rec′i pe	in sur′er	fag′ot	dis cern′
def′i nite	ep′au let	for′feit	ex haust′
sac′ri lege	ma neu′ver	fer′rule	crip′ple
right′eous	ar′chi tect	for bade′	cau′tious
gra′cious	guar an tee′	sur′plice	symp′tom
en gi neer′	chrys′a lis	hic′cough	com′ment
chal′ice	con′fis cate	ton′sil	con′crete
chiv′al ry	con va les′cent	per spire′	poul′tice

142.—REVIEW.

cha rade′	bil′liards	al lege′	whey
po′rous	am′a teur	ax′iom	vig′or
plov′er	per se vere′	pur′ple	vic′ar
scep′ter	beau′te ous	ter′race	sol′der
lar′ynx	prej′u dice	spig′ot	bil′ious
tra′che a	ac com′plice	suc ceed′	su′mac

143.

pit′y	con′duit	bre vier′	siege
a byss′	col′umn	wrin′kle	o′gre
sol′ace	brick′kiln	tres′tle	trip′le
er′ror	ban dan′na	ten′ant	sher′iff
busi′ness	pam′phlet	shud′der	pal′ace
re scind′	cat′er pil lar	res er voir′	pi az′za
af′ghan	con ge′ni al	wor′shiped	qui′nine
bi′cy cle	chi rop′o dist	chan de lier′	fla′grant

144.

per suade′	per′fi dy	pur′ga to ry	purge
per form′	per se vere′	pur vey′or	pur′port
per′jure	per pet′u al	per′pe trate	pur′pose
per plex′	per son′i fy	per′ti nent	pur suit′

145.—RAILROAD.

mail	de′pot	lug′gage	tel′e graph
train	sig′nal	dis patch′	con duct′or
track	tick′et	sta′tion	post′al clerk
check	bun′dle	ex′press	pe ri od′ic als
freight	pack′age	brake′man	lo co mo′tive

146.

at tain′a ble	el′i gi ble	con tempt′i ble	beau′te ous
pit′i a ble	suit′a ble	ter′ri ble	be gin′ning
a gree′a ble	al′ma nac	vis′i ble	be ha′vior
sec′re ta ry	boun′da ry	cred′i ble	bil′ious
spec′i men	as cer tain′	cer tif′i cate	cu ri os′i ty
ex pi ra′tion	ex treme′ly	dif′fer ent	treach′er y
grat′i tude	sum′moned	co lo′ni al	pes′ti lence
ob serv′ance	re ceiv′a ble	ex cus′a ble	cer′e mony
av′er age	al read′y	al′pha bet	ac knowl′edge
ex pla na′tion	an′thra cite	ap pren′tice	res er voir′

147.—WORDS LIABLE TO BE CONFOUNDED.

sculp′tor, *a carver of stone*	light′en ing, *to make light*
sculp′ture, *carved work*	light′ning, *electric flash*
proph′e sy, *to foretell*	pa′tients, *sick people*
proph′e cy, *a prediction*	pa′tience, *endurance*
pre cede′, *to go before*	dis ease′, *illness*
pro ceed′, *to advance*	de cease′, *death*

Write the following sentences, selecting the right word:

1. His (prophesy, prophecy) was fulfilled. 2. Let us run with (patience, patients) the race that is set before us.—*Bible.* 3. (Lightening, lightning) must always (proceed, precede)

thunder. 4. Enjoy the kingdom after my (decease, disease). —*Shakespeare.* 5. Phidias was a famous (sculpture, sculptor) of ancient Greece. 6. The captain ordered the (lightning, lightening) of the vessel. 7. The remedy is worse than the (disease, decease).—*Shakespeare.* 8. The physician cured many (patience, patients). 9. He forth on his journey did (precede, proceed). 10. The Greeks ornamented their temples with (sculptor, sculpture).

148.—WORDS LIABLE TO BE CONFOUNDED.

1 { ac cept′, *to take*
ex cept′, *to leave out*

2 { ad vice′, *counsel*
ad vise′, *to give counsel*

3 { at tend′ance, *the persons who attend any service, etc.*
at tend′ants, *those who attend as servants, companions, etc.*

4 { for′mer ly, *time past*
form′al ly, *in a formal way*

5 { sta′tion a ry, *fixed*
sta′tion er y, *paper, pens, etc.*

6 { pop′u lous, *full of people*
pop′u lace, *the people*

Write the following sentences, using the right word from the list above:

China is a (6) country. Will you (1) my (2)? King Arthur had brave (3). The (6) swarmed in the streets. (4) buffaloes roamed over the Great Plains. A (5) engine drew a car to the top of the hill. The (3) was large at every lecture (1) the first. I shall no more (2) thee. A stationer sells (5). The meeting was (4) opened by the president.

149.—WORDS LIABLE TO BE CONFOUNDED.

ac cede′, *to agree to*
ex ceed′, *to go beyond*
af fect′, *to act upon*
ef fect′, *to accomplish*
ad di′tion, *process of adding*
e di′tion, *publication*
as say′, *to test metals*
es say′, *to try*
bal′lad, *a song*
bal′lot, *a vote*
close, *to shut*
clothes, *articles of dress*
cen′tu ry, *hundred years*
sen′try, *a sentinel*
cel′e ry, *a vegetable*
sal′a ry, *wages*
de scent′, *a going down*
dis sent′, *to disagree*

e lic′it, *to draw out*	ir rup′tion, *an invasion*
il lic′it, *unlawful*	em′i grate, *to leave*
em′i nent, *distinguished*	im′mi grate, *to move into*
im′mi nent, *threatening*	ex′er cise, *to use*
e lude′, *to escape from*	ex′or cise, *to drive away*
al lude′, *to refer to*	gla′cier, *an ice field*
e rup′tion, *a bursting forth*	gla′zier, *a glass setter*

150.—WORDS LIABLE TO BE CONFOUNDED.

in gen′ious, *skilful*	pe ti′tion, *a request*
in gen′u ous, *honest*	rel′ic, *a memorial*
jest′er, *one who jests*	rel′ict, *a widow*
ges′ture, *action*	stat′ue, *an image*
lin′i ment, *liquid ointment*	stat′ure, *height*
lin′e a ment, *a feature*	stat′ute, *a law*
lose, *to suffer loss*	sur′plus, *the remainder*
loose, *to untie*	sur′plice, *clergyman's robe*
pas′tor, *a minister*	se′ries, *a succession*
pas′ture, *a field for cattle*	se′ri ous, *solemn*
pres′ence, *nearness*	spe′cies, *a kind*
pres′ents, *gifts*	spe′cious, *plausible*
pop′lar, *kind of tree*	track, *a footstep*
pop′u lar, *agreeable*	tract, *a region*
prec′e dent, *an example*	ten′or, *part in music*
pre ced′ence, *superiority*	ten′ure, *a holder of land*
pres′i dent, *chief magistrate*	ve rac′i ty, *truthfulness*
par ti′tion, *division*	vo rac′i ty, *greediness*

STATES AND TERRITORIES WITH THEIR ABBREVIATIONS.

North Atlantic Division:		*South Atlantic Division:*	
Maine	Me.	Del′a ware	Del.
New Hamp′shire	N. H.	Ma′ry land	Md.
Ver mont′	Vt.	Dis′trict of Co lum′bi a	D. C.
Mas sa chu′setts	Mass.	Vir gin′i a	Va.
Rhode Is′land	R. I.	West Vir gin′i a	W. Va.
Con nect′i cut	Conn. or Ct.	North Car o li′na	N. C.

New York	N. Y.	South Car o li′na	S. C.
New Jer′sey	N. J.	Geor′gi a	Ga.
Penn syl va′ni a	Pa.	Flor′i da	Fla.

Northern Central Division:		*Western Division:*	
O hi′o	O.	Mon ta′na	Mont.
In di an′a	Ind.	Wy o′ming	Wyo.
Il li nois′	Ill.	Col o ra′do	Colo.
Mich′i gan	Mich.	New Mex′i co	N. Mex. Ty.
Wis con′sin	Wis.	Ar i zo′na	Ariz. Ty.
Min ne so′ta	Minn.	U′tah	U.
I′ow a	Ia.	Ne va′da	Nev.
Mis sou′ri	Mo.	I′da ho	Ida.
North Da ko′ta	N. Dak.	A las′ka	Alas. Ty.
South Da ko′ta	S. Dak.	Wash′ing ton	Wash.
Ne bras′ka	Neb.	Or′e gon	Or.
Kan′sas	Kan.	Cal i for′ni a	Cal.

Southern Central Division:

Ken tuck′y	Ky.	Lou i si a′na	La.
Ten nes see′	Tenn.	Tex′as	Tex.
Al a ba′ma	Ala.	In′di an Ter′ri to ry	Ind. Ty.
Mis sis sip′pi	Miss.	Ok la ho′ma	Okla. Ty.
Ar′kan sas	Ark.		

The population of the United States in 1890 was 62,622,250.

TABLE OF COMMON ABBREVIATIONS.

A. B. Bachelor of Arts.

acct. or a/c Account.

A. D. (*Anno Domini*) In the year of our Lord.

ad lib. (*ad libitum*) At pleasure.

Admr. Administrator.

adj. Adjective.

adv. Adverb.

aet. (*aetate*) Aged.

A. M. Master of Arts; before noon.

amt. Amount.

ans. Answer.

Anon. Anonymous.

Atty. Attorney.

Ave. or Av. Avenue.

bal. Balance.

bbl. Barrel.

B. C. Before Christ.

Bro. Brother.
bu. Bushel.
Capt. Captain.
Cap. Capital.
C. E. Civil engineer.
Co. Company, county.
Col. Colonel.
Cr. Creditor, credit.
cts. Cents.
cwt. Hundredweight.
D. D. Doctor of Divinity.
del. (*delineavit*) He drew it.
Dep. Deputy.
Dept. Department.
do. (*ditto*) The same.
doz. Dozen.
Dr. Doctor, debtor.
D. V. (*Deo volente*) God willing.
ed. Edition, editor.
e. g. (*exempli gratia*) For example.
Eng. England, English.
Esq. Esquire.
et al. (*et alii*) And others.
etc. (*et cetera*) And the rest.
Exr. Executor.
Fahr. or F. Fahrenheit.
fem. Feminine.
Fr. France, French.
ft. Foot, feet.
gal. Gallon, gallons.
Gen. General.
Ger. Germany, German.
Gov. Governor.
hhd. Hogshead.
H. M. His (or Her) Majesty.
Hon. Honorable.
H. R. H. His (or Her) Royal Highness.
ib. or ibid. (*ibidem*) In the same place.
id. (*idem*) The same.
i. e. (*id est*) That is.
in. Inch, inches.
inst. (instant) Of the present month.
Jr. or Jun. Junior.
£, lb., ℔, or lib. Pound.
LL. D. Doctor of Laws.
Lieut. Lieutenant.
M. Monsieur, midday.
Maj. Major.
mas. Masculine.
M. C. Member of Congress.
M. D. Doctor of Medicine.
mem. (*memento*) Remember.
min. Minute, minutes.
Mlle. Mademoiselle.
Mme. Madame.
M.P. Member of Parliament.
MS. Manuscript.
MSS. Manuscripts.
Mt. Mountain, mount.
Mus. D. Doctor of Music.
N. B. (*nota bene*) Mark well.
neut. Neuter.
N. G. National Guard.
No. (*numero*) Number.
ob. (*obiit*) Died.
p. Page; pp., pages.
per cent. or % (*per centum*) By the hundred.
Ph. D. Doctor of Philosophy.
P. M. (*post meridiem*) Afternoon, postmaster.

P. O. Post-Office.
pk. Peck.
pl. Plural.
pop. Population.
P. P. C. (*pour prendre congé*) To take leave.
Pres. President.
Prof. Professor.
pro tem. (*tempore*) For the time.
prox. (*proximo*) Of next month.
P. S. Postscript.
Ps. Psalm or Psalms.
pwt. Pennyweight.
q. e. (*quod est*) Which is.
qt. Quart.
q. v. (*quod vide*) Which see.
R. A. Royal Academy.
Rec'd. Received.
Rev. Reverend.
R. R. Railroad.
R. S. V. P. (*réspondez s'il vous plaît*) Answer, if you please.
Rt. Hon. Right Honorable.
Sec. Secretary.
sing. Singular.
St. Saint, street.
supp. Supplement.
Supt. Superintendent.
tr. Transpose, translator.
ult. (*ultimo*) Of last month.
U. S. A. United States Army.
U. S. M. United States Mail.
U. S. N. United States Navy.
viz. (*videlicet*) Namely.
vol. Volume, volumes.
vs. (*versus*) Against.
Xmas. Christmas.
yd. Yard, yards.
&c. And so forth.

PENMANSHIP.

PRELIMINARY INSTRUCTIONS.

GENERAL REMARKS.

1. Acquirement of Style.—The acquirement of a handsome, yet plain and practical, style of penmanship is the duty of every one. It is not necessary that all of us become professional penmen, but *it is necessary* that we all have a style of penmanship easily and rapidly written, and plain to read. It is thought by some that a handsome style of penmanship is a gift. This is not so; *it is an acquirement*, and any one who will *intelligently* study this course with that effort necessary to master any other subject, will acquire a style of penmanship that will be valuable both as an accomplishment and as a business qualification.

This course is designed to produce *practical results*, and includes everything needful for the acquirement of a plain and rapid handwriting.

The preliminary exercises in Lesson 1 enable the student to acquire the proper movement; several styles of capital letters are shown in the subsequent lessons; a complete set of business capitals is given in Lesson 7; and the following lessons treat upon notes, receipts, due bills, superscriptions, and signatures.

2. Specimen of Handwriting.—Before reading any further, we desire you to write two pages of specimens,

each containing one set of capitals, one set of small letters, one set of figures, and the following short letter:

....

(Your address and the current date.)

International Correspondence Schools,
Scranton, Pa.

Gentlemen:

Herewith I send you the specimens of capitals, small letters, and figures, as requested.

....

(Your class letter and number.)

Yours truly,

....

(Your signature.)

One page of these specimens you keep; the other send immediately to us. We do not acknowledge the receipt of the page you send us nor give the work any percentage mark, but we simply file it away so that we can compare it with your work on Lesson 12, to see how much you have improved.

METHOD OF PRACTICE.

3. There are twelve lessons in this course, and that you may get the most benefit from them it is desirable that you take them up in their order and closely follow the directions given.

Beginning with Art. **23,** of this section are given the copies for this course of lessons and the instructions for studying and practicing them. Lesson 1 is composed of copies 1 to 8 inclusive. Study these copies carefully and practice them as directed until you are fairly well satisfied with the results; then mail us a specimen of your work, showing at least three lines of each copy.

While waiting for the return of these specimens, you may

work on Lesson 2, copies 9 to 19. Practice on these copies until you have obtained good results; then send us specimens of your work as before. While waiting for the return of Lesson 2, take up Lesson 3. Unless otherwise directed, this will be the order of work throughout the course. *Never send us more than one lesson at a time.*

MATERIALS.

4. Penholders.—Before beginning the study and practice of these lessons, you must obtain suitable materials with which to work. For practical business writing, a common straight holder with a taper stem, like the one shown in the figures, made entirely of wood, or of wood with a cork or a rubber tip, is generally preferred, and is certainly the best. Avoid all holders with a polished metal piece at the bottom. The smooth, polished metal is very difficult to hold, and the student will acquire the habit of pinching the holder, which will contract the muscles and thus make the whole arm and hand rigid and entirely prevent the acquirement of that free and easy movement that is absolutely necessary for good writing.

Many "professional" penmen use the "oblique" holder. For their class of work it has many advantages, but for business writing it is not to be recommended.

5. Pens.—For practice and study it is best to use moderately fine steel pens, even if their use is not to be continued. No difficulty will be experienced in changing from a fine to a coarse pen later on, if a coarse one is preferred. With a fine pen, you will make the lines more accurately and be able to locate your errors with more certainty. Its use will also cultivate that lightness of touch that is actually necessary to good writing. Gillott's No. 404 and the Spencerian No. 1 are very good. You will also want a small piece of chamois skin, cotton cloth, or some other suitable article on which to wipe your pen. A dirty pen will not make a fine, neat line.

6. Ink.—Black ink is to be preferred to any other. Select a kind that flows freely and writes black. The pale green or blue inks that turn black are not good for practice. Carter's, Stafford's, and Caw's inks are reliable.

7. Paper.—A good quality of foolscap is the best for practice. Avoid that with too glossy a finish. A hard, firm surface will give the best results. The small extra cost of the best over the poor is not to be considered. For the *best* results, your materials must be of the *best. Always have a blotter under the right hand when writing, to keep the paper clean.* Having obtained the pens, paper, penholders, ink, and pen wiper, you are ready for study and practice.

Before you can profitably begin the use of these materials, you must learn how to take an easy and proper position at the table, and also how to hold the pen correctly. Your success in attaining a good handwriting depends very much on these things. You must, therefore, carefully study and practice the directions here given.

POSITIONS.

8. Position of the Body.—A correct position of the body and feet is quite as essential as that of the arms and hands. Carefully study and imitate the position shown in Fig. 1. This is called the "front" position. It is the position to be preferred, and is the one most used. In studying the "front" position, carefully observe the following points: *First*, that the body is nearly square with the table, quite near it but not touching it. Avoid putting any weight upon the right arm. *Second*, that the elbows project about two inches over the edge of the table. *Third*, that the arms cross the desk obliquely. *Fourth*, that the feet rest squarely on the floor, the left foot slightly in advance. This tends to give a firm support to the body. (The illustration does not show this.) *Fifth*, that the paper is nearly in front of the body, and turned slightly to the left; that is, with the long way of the paper in the same direction as the right forearm

and hand. *Sixth*, that the fingers of the left hand hold the paper in place. All these points having been carefully observed and learned, the next thing to study is the correct position of the right arm and hand, the manner of holding the pen, and the movements for producing good writing.

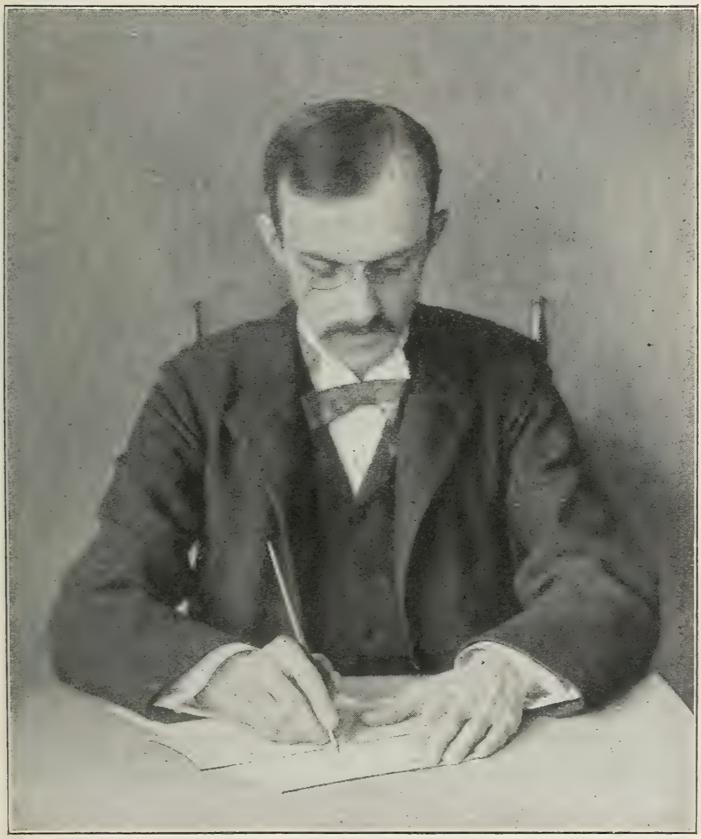


FIG. 1.

9. Position of the Arm.—By referring to Fig. 1, you will notice that the arm rests on the muscles of the forearm just below the elbow. This is the point from which the power of the arm is obtained. *Do not bear any weight on the right arm.*

10. Position of the Hand.—The position of the hand is one of the most important features in writing. By turning to Fig. 1, you will notice: *First*, that the hand rests on the nails of the third and fourth fingers, these fingers being

FIG. 2.

drawn back toward the palm of the hand. *Second*, that the wrist and the side of the hand are kept clear from the desk. *Third*, that the hand is turned well over to the left, so that the back of the hand is nearly parallel with the top of the table. This turning of the hand brings the arm on the

thickest and most muscular part of the forearm, and also points the top of the penholder in the proper direction; that is, directly over the right shoulder. Keeping the wrist and the side of the hand free and clear from the table, and the back of the hand facing the ceiling, are two of the hardest things you will have to acquire. You should, therefore, give them very close attention.

11. Holding the Pen.—The next thing for you to learn is the correct manner of holding the pen. Figs. 1 and 2 show the pen in the proper position, from two different views. The position is quite well described as follows: The holder is held between the first and second fingers and the thumb, crossing the second finger at the root of the nail on the side nearest the thumb, then running backwards and upwards under the first finger, crossing it either just in front of the knuckle joint or just behind it. Some prefer one way and some the other. If held below the joint, the pen is less apt to stick into the paper and spatter ink; it will also run over the paper more smoothly than if held in the more upright position in front of the joint. The thumb is placed against the left side of the holder, about opposite the first joint of the first finger. The pen held lightly in this manner, with the third and fourth fingers well drawn back and resting lightly on the tips of the nails, the hand well over to the left, the wrist and the hand clear of the table, the arm propelled with a light, yet strong and springy, movement from the muscles of the forearm, must, when combined with careful study and rightly directed practice, produce good results in the acquirement of a handsome and practical style of penmanship.

MOVEMENTS.

12. Names of Movements.—There are four methods of using the arm and the hand in producing writing. These are called *whole-arm*, *muscular* or *forearm*, *finger*, and *combined movement*.

13. Whole-Arm Movement.—This movement is produced by slightly raising the arm from the table and allowing it to swing freely from the shoulder. This movement is the one you would use if writing on a blackboard. It is much used by professional penmen in making off-hand capitals, but it is not desirable for our present use.

14. Muscular, or Forearm, Movement.—This movement is the foundation of all good writing, and you cannot acquire an easy, graceful, and rapid style of writing until you master this movement. Carefully study and practice every detail of it as given here, for without this movement your success as a penman will be small.

This movement is developed by resting the arm on the muscles of the forearm just below the elbow (see Fig. 1). These muscles act as a center of power, propelling the hand, which slides along on the third and fourth fingers (see Fig. 1). The thumb and the fingers must not be used in forming the letters, the whole work being done by the muscles of the forearm.

The acquirement of this movement is the first thing to be given attention to in learning to write a free, easy, business-like style of penmanship.

In studying and practicing for a good movement, be sure that the position of your body, arm, and hand is correct. Be sure, also, that the muscles of the arm, hand, and fingers are lax and free, as good writing cannot be done if the muscles are tense and hard. Writing must be done with a light touch and an elastic movement, or good results cannot be attained.

For those who find it difficult to get that easy, swinging motion of the hand so necessary in acquiring muscular movement, it is a good plan to practice as follows: Drop the arm and hand by the side, letting it hang listlessly and lifelessly; in this position the muscles will become lax, the fingers will curl up slightly, and naturally assume a position, which, if retained when the hand is raised to the table for writing, will be the correct position of the hand and fingers. With the arm resting lightly on the table, the back of the hand

facing the ceiling, close the hand, making a fist, and with the arm rolling easily on the muscles near the elbow, practice freely and with force the exercises of Lesson 1. When thus practicing these exercises, use no pen or pencil, and be sure that no part of the hand or wrist touches the table or paper. See that the arm rolls and slides freely in the sleeve but that the sleeve itself does not move.

In addition to the above, it is a good plan to practice retracing the copies, using a dry pen, held correctly, and writing at a good rate of speed.

See that neither the wrist nor the side of the hand touches the paper. Keep the muscles lax and free, steadying the hand by allowing it to rest lightly and slide easily on the tip of the fourth finger, or on the nails of the third and fourth fingers.

Success in writing with a purely muscular movement depends entirely on the command one has over the muscles of the forearm, and the method of practice here outlined will, if thoroughly persevered in and used in practicing all the exercises, do much to produce that complete command of the arm, hand, and pen so necessary to success in becoming a good writer.

15. Finger Movement.—This movement is produced by the action of the first and the second fingers, in connection with the thumb. You must guard against this movement, as the tendency at first is to use this movement entirely. Perhaps in your writing up to this time you have used the finger movement quite a good deal, if not entirely. If this is so, you may find it difficult to leave it off, but it must be done, as it is impossible to write a smooth, easy, and rapid style with the finger movement.

16. Combined Movement.—This movement is produced by the united action of the muscles of the forearm and the fingers, and is the one chiefly used by skilful penmen. The muscles, resting on their center below the elbow, propel the hand, which slides easily on the nails of the third and

fourth fingers; the first and second fingers and the thumb, acting together, assist the muscles of the arm in shaping the letters. Care must be taken not to use the fingers too much. They are used principally in forming the long upward and downward letters; the small letters should be made almost entirely with the muscular movement. This combination movement of muscles and fingers is the very best movement for both practical business writing and for fine penmanship.

Many fail to become good writers from lack of study. They realize that they make errors, but are not able to tell just where the fault is. This is because they do not have an accurate picture of the letter in their minds. To be successful in learning to write, you must be able to form a mind picture of the letter you wish to make; you must really see a correct picture of it on the paper before you make a mark with the pen. When you can do this and can readily name the strokes necessary to produce a letter, you will be able to see your errors and to correct them. The copies given in these lessons show the correct forms of letters. Study them closely and compare your work with them; this is the only way you can discover the points on which you fail.

DIMENSIONS.

17. The height of the small letter *i* is taken as the standard of measurement, and is called a *space*. The height and width of all the other letters are regulated by the height of the small letter *i*; the "space-lined" copies illustrate the height of the different parts of the capital letters. To get the letters the proper size for any width of ruling, imagine the space between the ruled lines to be divided into four equal parts, or spaces. Make the small letter *i* the height of one of these spaces, and make the other letters in the same proportion to this *i* as they are in the copies given.

PRINCIPLES.

18. Lines.—Before a successful study of penmanship can be made, you must become familiar with the principles by which letters are formed. There are three lines from which all the letters of the alphabet are formed: two curved lines and one straight stroke. They are named *right curve*, *left curve*, and *straight line.*

19. Curves.—A **right curve** is one that would appear at the right side of a straight line. A **left curve** is one that would appear at the left side of a straight line.

20. Straight Line.—A straight line is one that does not change its direction throughout its entire length. Nearly all the down strokes in small letters are straight lines on the main slant of writing.

21. Slant.—The slant of writing must also have attention, for without a uniform slant our writing will not have a good appearance. The degree of slant that is given writing is not of great importance, but all letters should slant alike. Watch your work closely in this respect.

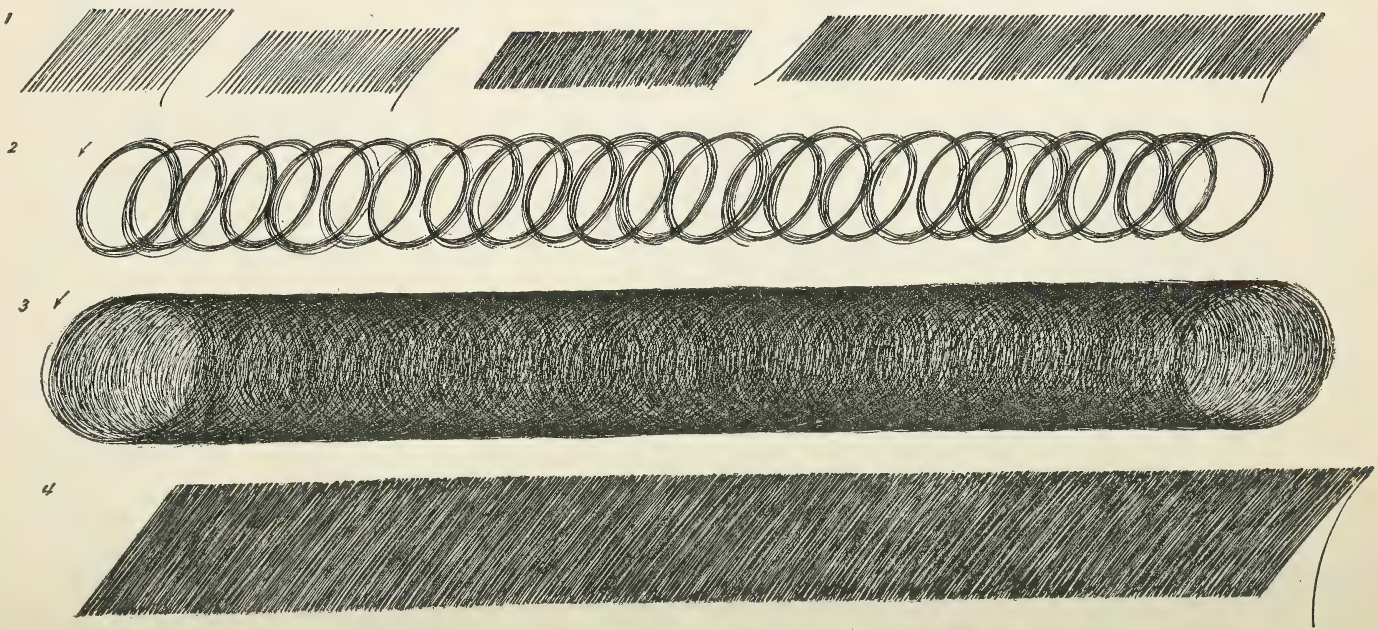
THE LESSONS.

22. The lessons in this course follow in their order. Each lesson consists of two pages of copies and about two pages of explanations and directions for practice. In preparing your work for examination, write at least three lines of each copy. *Never send more than one lesson at a time.*

LESSON 1.

23. Copy No. 1.—Correct position, proper holding of the pen, and a rapid movement must be maintained while practicing the exercises of this copy. Be sure that the wrist moves in and out of the sleeve, as no benefit will be

LESSON 1.



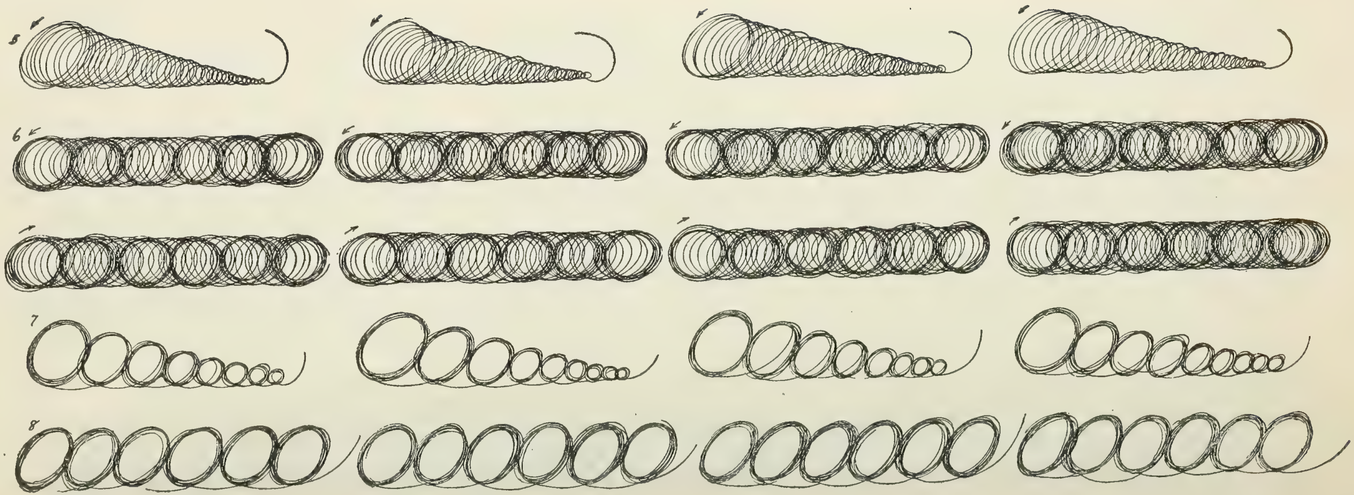
derived from making this exercise with a wrong movement. Study the instructions for all copies closely and do your best to follow them. Good writing depends greatly on the proper training of the muscles. In learning to write, movement is of first importance. Do not allow yourself to draw the letters and exercises slowly with finger movement. Aim to gain something with each copy. This, and all the other copies in the lesson, should be made rapidly and with a free movement. Let the hand glide slowly to the right, so that the exercises will not become too heavy. The weight of the hand should rest on the little finger. Make the exercises high enough to fill the space between two blue lines of your practice paper.

24. Copy No. 2.—Practice this copy at a good rate of speed and use only the pure muscular movement. Make the exercise cover three blue lines of your practice paper. Rest the arm lightly on the desk, and propel the hand in the direction indicated by the arrows in the copy. Let the movement be a quick, rolling motion, and make ten downward strokes in each oval. Notice the spacing of the ovals; be careful not to make them too round, and see that they are on the same slant as writing. Be careful not to use a finger movement. When you can make this exercise fairly well take up the next copy.

25. Copy No. 3.—This is called the compact oval exercise and should be made in the same manner as the preceding copy. Make it rapidly and be sure that you use the correct movement. Do not let the sleeve slip on the table, but make the arm roll freely in and out of it; the hand should glide to the right easily, thus giving an even tint to the written exercise. Do not bear heavily on the pen, nor allow the exercise to become blotted.

26. Copy No. 4.—This exercise is like copy No. 1, excepting that it is larger and darker. Put force into the movement. Do not bear on the pen to get the shade, but make the exercise dark with many lines.

LESSON 1.

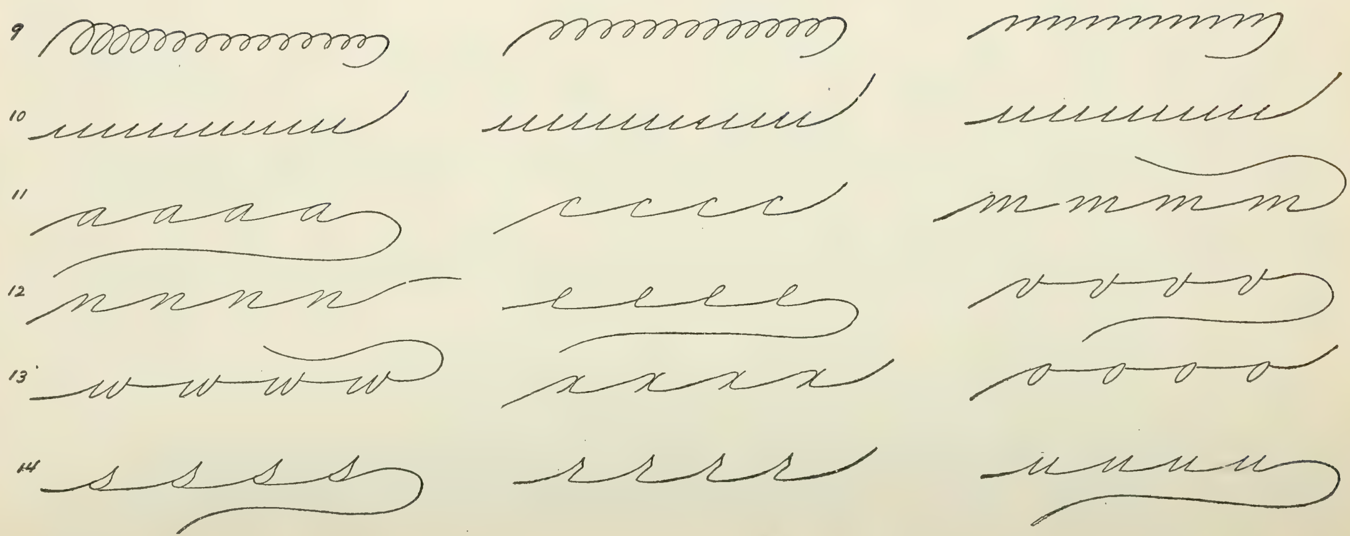


27. Copy No. 5.—This copy is designed to develop control of movement, and should train the hand to make forms of different sizes. The large part is for freedom and the small part is to show control and skill. Let the exercise taper gradually; the hand should move in the direction indicated by the arrows. This is called the direct oval movement. Be sure that you make ovals and not rings; watch the slant and the spacing carefully. There should be about forty downward strokes to each exercise. The exercise should be made rapidly; do not lift the pen until the exercise is finished. Aim for improvement in all your work. Careless practice is sure to bring unsatisfactory results.

28. Copy No. 6.—Make five downward strokes in the circle, then let the hand glide to the right while making the next few revolutions, and repeat until six heavy circles are made. It will take some practice to get this exercise just right. Notice that it is even at the top and base and that the retraced circles are close together. This exercise is for freedom and control of movement. The second line of this copy should be made with the reverse oval movement, as indicated by the arrows. When able to make these exercises well, take up the next copy.

29. Copy No. 7.—This should be made with the direct oval movement. The large ovals should be the full space between the blue lines. Make the ovals close together, so that the work will not appear to be scrawled. Make from seven to ten downward strokes in each oval. Notice that the ovals touch one another and that they taper gradually; also give careful attention to the width and the slant of the ovals. Each exercise should contain eight or nine ovals. We use these ovals as a means to an end; to train the muscles to the correct movement for making script forms. The ability to make good ovals will have much to do with your success in making good capitals. It is very important that a good movement be acquired, hence we devote the entire first lesson to exercises designed to develop freedom and speed.

LESSON 2.



30. Copy No. 8.—This exercise should be made in the same manner as the preceding one. See that all the ovals are of the same size. If you cannot make the ovals appear alike, you have not good control of your movement and should review all the exercises in the lesson. Make ten downward strokes in each oval, then swing to the next one, which should be close to the one finished. The upward strokes should be as plain as the downward strokes. If you have a poor pen, the upward strokes may not show plainly.

LESSON 2.

31. Copy No. 9.—Use a very light, quick movement for this copy and make twelve to fifteen downward strokes in each exercise. Notice that the downward strokes of the first two exercises are looped, while those of the third exercise are retraced.

32. Copy No. 10.—This and the following copies on this plate include all the letters that are one space in height. Be sure to curve the upward stroke of this exercise and retrace it with the downward stroke. The exercise should be made quite rapidly. Try to keep the downward strokes an equal distance apart.

33. Copy No. 11.—In making the letter *a*, see that it is very narrow at the base, also notice the slant of the last downward stroke. The top should be closed. Practice a free and easy movement; the curved ending stroke is used to illustrate that a free movement is used. For the letter *c*, make a small dot at the top, then make the downward stroke the same as the last downward stroke of letter *a*. In writing the letter *m*, students have trouble in making the three parts of the letter close enough together. There should be a point at the bottom of the first and the second downward strokes, but a round turn at the bottom of the third.

34. Copy No. 12.—See that the tops are rounded, that there is a point at the bottom of the first downward stroke

LESSON 2.

15 A A aaaa name case

16 A A A Annum Ammonia

17 B B. bbbbb bulk bulb

18 B B B Banner Business

19 C C cccc came case

and a round turn to the second. In writing the letter *e*, notice the direction of the downward stroke; a common error is to curve it too much. The loop of this letter must be open, otherwise the letter may be taken for *i* or *c*. Notice the last part of the *v* very carefully; see that the letter is narrow and that there is a round turn at the bottom.

35. Copy No. 13.—Make the letter *w* narrow; the last part should be the same as the last part of the *v*. Note the round turns at the bottom. The letter *x* can be made in three different ways, but we prefer the method given here, which is a combination of the first part of *n* and the last part of *c*. Lift the pen in making the letter. Be sure that the letter *o* is closed at the top. Both sides of this letter should be of the same curvature and the center of the letter should be on the main slant.

36. Copy No. 14.—The letters *s* and *r* are difficult and will require careful study. Examine the copy frequently as you practice. Both these letters are slightly taller than other small letters. See that the downward stroke in the *s* is heavily curved and that it connects with the upward stroke. The downward stroke in the *r* should be nearly parallel with the upward stroke. The top of this letter will need a great deal of practice; study it carefully. In the letter *u*, the downward strokes should be on the same slant and should have pointed tops and rounded bottoms.

37. Copy No. 15.—The *A* with horizontal dotted lines is for study and not for practice. We use these dotted lines to illustrate the height and proportion of letters. Practice the tracing exercise carefully; being sure to use a free movement and to make it as nearly like the copy as possible. If you have trouble to make the second downward stroke nearly straight, pause at the top of it; this will check the rolling motion and allow a fresh start. Next we have further practice on the *a* exercise and words containing that letter. Study movement and position as you practice.

38. Copy No. 16.—The style of the letter *A* is almost exactly like the small letter *a* without the first upward stroke. The downward stroke may be shaded the least bit if desired. Do not shade so that a blotter is necessary. The last downward, or finishing, stroke is made with a slight curve to give the letter a graceful appearance. Be sure to give the oval part enough slant; making it too straight is one of the most common errors. Be careful not to make the oval too wide. Do not shade the last downward stroke. Practice the words separately, unless preparing copy to be sent for criticism. Notice that the letters are compact and that the spacing is even. When you are able to write this line quite well, proceed with the next copy.

39. Copy No. 17.—After giving a few minutes to the practice of the tracing exercise, take up the letter *b*. Use the fingers slightly in forming the loop; the last part of the letter is similar to the last part of *v*. Make the letters *l*, *b*, and *k* the same height.

40. Copy No. 18.—In making the letter *B*, be sure that the capital stem has a full curve. The height of the letter is three spaces, the height of the capital stem two and one-half spaces. For a more detailed explanation of the capital stem, see *F*, copy 26. Make the ovals in the main part of this letter full and smooth. See that the small loop in the center of the letter points upwards; if necessary, the fingers may be used slightly in forming it. The shades should be slight and very low; the nearer the bottom of the letter they are, the better the letter will look. The style of *r* in the word *banner* is not used very much; if you prefer the other style, use it.

41. Copy No. 19.—Practice the tracing exercise first, 5 minutes' practice at a time should be sufficient for this, then take up the exercise and words in the order given. Notice that the first stroke in *c* is a left curve; do not make this letter too round, nor the hook at the top too large. Careful work will bring good results.

LESSON 3.

42. Copy No. 20.—Some prefer to make the letter *C* with a large loop, reaching down to within a space of the line; others prefer to have a very short loop, not more than half a space long. We think a loop one and one-half spaces long is about the proper size. In making this letter, be sure not to use the fingers, for, if you do, you will make a sharp angle in it and thus spoil the beautiful curves of the letter. Be careful to give the left side of this letter a full curve. Many fail in this, and make it straight. Make full turns at the top and bottom of the letter and you will be sure to have good results. Do not make the shade too heavy; little or no shade is best. Write the words with an even movement; all the small letters should be of the same height.

43. Copy No. 21.—Study the proportions of the letter as shown by the first figure, then practice the tracing exercise for 5 minutes. Now practice the *d* exercise and the words in their order, giving especial attention to the *d*. Learn to criticize your own work; choose the poorest parts and practice them separately.

44. Copy No. 22.—Notice that two parts of the letter rest on the base line, and that the ending stroke is at half the height of the letter. Practice the capital by itself and when able to make it well take up the words. When you practice, see that the little finger glides on the paper; do not let it rest in one place.

45. Copy No. 23.—The tracing exercise should be practiced for about 5 minutes. It may seem difficult at first, but it is surprising how rapidly the exercise can be made by good writers. Do not make the small center loop too low, and see that it points downwards. Practice the remainder of the copy in the order given. Be careful not to curve the downward stroke of *e* too much.

46. Copy No. 24.—Use only the muscular movement. The secret of making a good letter *E* is in making the

LESSON 3.

20 C C C Comma Commence

21 D D dddd deed deceive

22 D D D Dennis Dennison.

23 E E eeee even event

24 E E E Eunice Evidence

LESSON 3.

25 F F ffff fine fife f

26 F F F Fremont Fairman

27 G G gggg going adage

28 G G G Gaining Garrison

29 H H hhhh high heavy h

curves full and in having the small loop in the middle of the letter point across the letter at a right angle to the main slant. Be careful to put this loop in its proper place between the first and second parts of the letter. See that all the curves are full and broad. The bottom part of this letter is larger in every way than the upper part, and may be shaded a very little if desired.

47. Copy No. 25.—Study the form of the *F* carefully, then devote a few minutes to practicing the tracing exercise. In writing the *f* be sure not to make it too large. Let the upward line of the lower loop join the main part of letter at the base line. Because of its length the *f* is one of the most difficult of all the letters; watch the slant closely.

48. Copy No. 26.—The difficult part of the letter *F* is the capital stem, which should have a little more slant than the downward strokes of the other letters. Beginning at the top of the stem, two and one-half spaces above the line, descend about half way with a left curve, verge into a right curve, and continue to the line; carry it one space to the left, then upward one-half space, ending with a dot. Be sure to keep the shade, if any, below one-half the height of the stem. See that the dot is carried well to the left. The cross is made by a straight horizontal line in the middle of the letter, ending in a short downward stroke. One-half of the cross-line should be to the left of the letter and one-half to the right. In making the cap to this letter, make a loop about one space to the left of the capital stem and then carry a double curved line across the top of the stem, one space above it. Many spoil the cap by making it straight. Do not do this, but make it with full curves. The small *t* should be crossed at one and one-half spaces with a short horizontal stroke.

49. Copy No. 27.—Begin at the base line and use a right curve for the beginning stroke of the tracing exercise. Notice that the lower part is two spaces in height. In making the *g*, be careful not to make the loop too long; it

should not be more than two spaces below the line. The *a* part should be closed at the top. You will notice that three styles of the *g* are given; the second style should be used only when the *g* ends a word.

50. Copy No. 28.—To be successful with the letter *G*, you must make a full curve in the initial stroke; let the downward stroke of the upper loop come to the one-space line, then end with a full swing two spaces high. Finish the letter by lifting the pen while it is in motion.

51. Copy No. 29.—After studying the form of the letter *H*, practice the tracing exercise with free movement. Do not make the exercise too wide. The letter *h* is a combination of the *l* loop and the last three strokes of the letter *n*. Let the upward and downward strokes cross at the height of the letter *i*, or one space from the base line; a common error is to cross them too low. The downward stroke of the *n* part should have the same slant as the main downward stroke.

LESSON 4.

52. Copy No. 30.—The first part of the letter *H* is made by beginning at the top and forming an oval one space long on the main slant; descend with a full right curve to the base line, raising the pen while it is in motion. The lower part of the descending stroke may be slightly shaded. The last part of the letter is made by beginning at the extreme height and bringing the stroke downward with a full left curve. Finish the letter with two short, full, curved strokes. The ending stroke may be joined to the first small letter of a word. Practice the words very carefully. It is estimated that in general writing twenty small letters are used for every capital.

53. Copy No. 31.—Notice the relative proportion of the three parts of the tracing exercise. We do not believe in excessive practice on these tracing exercises, but we do believe a few minutes' practice to be helpful. The *i* is

LESSON 4.

30 H H H Hannum Hanover.

31 I I iiii issue inning

32 I I I Inman Immune

33 J J jjjj join joyous.

34 J J J Junior Jenness.

easily made; the first stroke is curved to insure making the letter sharp at the top. Place the dot directly over the letter in the direction in which it slants.

54. Copy No. 32.—The full height of the letter *I* is three spaces; the width of the top is one space. The width of the bottom loop is one and one-half spaces. The first upward and downward lines should cross one-fourth of a space above the base line. Be careful to get the top on the main slant; this is very important. The lower loop is carried well to the left, and is made full and without a break. Many writers finish the letter with a dot, one-half space above the base line at the same distance to the left as the extreme edge of the full curve.

55. Copy No. 33.—Make this tracing exercise different from that of the *I*. The lower part of this exercise extends below the base line and is very small. The arrow indicates the direction of motion. Make the top of the *j* similar to the *i* and dot it in like manner; the lower part should be short, much like the lower part of *g*.

56. Copy No. 34.—The difficult part of the letter *J* is in making the long downward stroke. It is made nearly straight, having a very slight curve to the right. Some prefer to make the lower part two spaces below the base line, and some only one and one-half. Some, also, prefer to make the top loop twice as wide as the bottom one, while others prefer to have them alike. We think that, if the top loop is a little larger than the bottom one, it gives a style to the letter not obtained by either of the above methods. See that all shade is below the base line and that both of the curved up strokes cross the main downward stroke at nearly the same place, one-fourth of a space above the base line, leaving a small three-cornered space in front of the main downward stroke.

57. Copy No. 35.—Give close attention to the proportions of the different parts of the *K*. Always compare your

LESSON 4.

35 K K kkkk kind kink

36 K K K Kenton. Kimmig.

37 L L llll line lull

38 L L L London Lexington.

39 M M mmmm m manner

work on the exercises with the copies and make frequent reviews. Use some finger movement in making the loop of the *k*; the last part of this letter should not be higher than one space. Devote considerable time to the words *kind* and *kink*.

58. Copy No. 36.—In making the first part of the letter *K*, see that the first stroke is a full right curve two and one-half spaces high; then descend with a straight line for about one space; then verge into a full right curve, continuing to the base line, then to the left and upward, finishing with a dot on the first stroke of the letter one-half a space above the base line. The second part of this letter is formed as follows: Begin at the full height of the letter, three spaces, and two spaces to the right of the first part; descend on nearly the connective slant, with first a left and then a right curve, to the center of the letter; then, forming a small loop pointing upwards and encircling the first downward stroke of the letter, descend with a graceful curve. Be sure to use both a left and a right curve in the first stroke of the second part of the letter. Many fail to do this and use a right curve all the way down to the small loop, thus completely spoiling the letter. Be sure that the little loop points across the first part of the letter at right angles, or you will not be able to get the last downward stroke in its proper place. The ending stroke extends one space below the base line.

59. Copy No. 37.—After a careful study of the form, practice the tracing exercise. The tendency at first will probably be to make the lower loop too large. You must guard against this by slackening the movement in making the downward stroke. For the *l*, use a light, quick movement. The fingers may be used slightly in curving the upward stroke; the downward stroke should be quite straight.

60. Copy No. 38.—The first part of the *L* is a full right curve that begins at two spaces in height. The downward stroke is a compound curve, much like the capital stem.

LESSON 5.

40 M M M Madison Marmora

41 N N nnnn none nine

42 N N N Norway Newman

43 O O oooo moon over

44 O O O Omnia Overton

The lower part is a small loop, which should point directly to the left. Be sure that the letter has the proper slant.

61. Copy No. 39.—This tracing exercise should be made without lifting the pen. The most common fault is to make the exercise too wide. End the last stroke one space below base line. In making the *m*, the downward strokes may be retraced a little. The top parts of the letter should be round, yet the letter should not occupy too much space; make it compact.

LESSON 5.

62. Copy No. 40.—The main points to observe in the letter *M* are that the slight shade is below half of the height of the letter, that the top of the second downward stroke is lower than the top of the first, and that the top of the third downward stroke is lower than the top of the second; also, that the downward strokes in these lines are on the same slant as the first downward stroke in the letter. They should be parallel, and a little less than one space apart. Practice the capitals separately from the words, except when preparing work for criticism.

63. Copy No. 41.—The *N* should be quite narrow. Make the first part three spaces and the second part two spaces in height; retrace the downward stroke of the *n* but very little, and end the letter with a full right curve. Give close attention to spacing and movement in writing the words.

64. Copy No. 42.—The letter *N* is the same as the first and the third part of the *M*, and the same things may be said regarding the special points to observe. The downward strokes in this letter should be about three-fourths of a space apart and parallel. The shade should be low, the turns should be short but full. Many fail on this letter by making it too wide. Do not do this.

65. Copy No. 43.—This tracing exercise is very easily made. Begin at the top and make ten downward strokes in

LESSON 5.

45 P P pppp apt appear

46 P P P Penman Panama

47 Q Q qqqq queen quince

48 Q Q Q Quince Quinine

49 R R rrrr roar rumor

each oval. Do not make circles. In writing the *o*, be sure to close the letter at the top. Do not let the connecting line curve much; it should be nearly straight. A line drawn lengthwise through the middle of the letter should be on the main slant. Practice the words very carefully.

66. Copy No. 44.—The two most common errors in making the letter *O* are in making the first downward stroke too straight and the turn at the base too short. Be sure to give the same degree of curvature to both sides of the letter. Make the loop at the top full and without a break; a sharp point in it destroys the appearance of the letter. The ending stroke should point to the right at one and one-half spaces in height.

67. Copy No. 45.—In making the first part of the tracing exercise, give attention to the proper slant. Make the last part nearly round and one and one-half spaces in height. For the *p*, curve the first stroke so that the top of the letter will be sharp, the downward stroke should be straight and on the main slant. Make the last part of this letter like the last part of *n*. The first stroke extends two spaces above the line and the downward stroke is one and one-half spaces below the line. Observe this proportion closely. Lift the pen between the parts of this letter. The words should be written carefully, yet at a good rate of speed.

68. Copy No. 46.—Two styles of the *P* are given in this copy. We prefer the style that has the straight downward stroke. The other style is, however, used a great deal and we therefore give instructions for making it. Begin at a height of one space and make the first stroke a full right curve to a height of two and one-half spaces, then bring the downward stroke under, in the same manner as the first part of *K*, but continue the stroke over the top of the first part and down to the one-space line. For the second style of letter, begin in same manner but bring the downward stroke straight, on the main slant, to the base line. The last part is made by placing the pen on this stroke at one and

LESSON 6.

50 R R R Runner Reserve

51 S S ssss scarce accuse

52 S S S Saranac Superior.

53 T T tttt tenant attend

54 T T T Trenton Terminus

one-half spaces in height and forming the loop, the end of the loop crossing the line at the place where the loop was begun.

69. Copy No. 47.—The *Q* is very much like a large figure 2. Give a few minutes' careful practice to the tracing exercise, then take up the small letter. This letter is composed of the first part of *a* and the last part of *f*. Make the lower part one and one-half spaces below the base line. Give especial attention to the *q* when writing the words.

70. Copy No. 48.—The first part of the letter *Q* is the same as the first part of *H*. Add the horizontal loop one space in length, cross the main downward stroke, touch the base line, and finish with a right curve one space in length. Study the points in the first stroke of the *H*. Be sure that the long diameter of the small loop is parallel with the ruling, and that the finishing line is an easy curve.

71. Copy No. 49.—Make this tracing exercise in the same manner as the first style of *P* in copy 46, with the addition of the last stroke of the *K*. Notice carefully the similarity of the *R* and *P*. The *r* is rather a difficult letter to make. The first stroke is a full curve one and one-fourth spaces in height. Make the short stroke at the top with care and finish with the last part of the *i*. This letter requires much practice.

LESSON 6.

72. Copy No. 50.—Two styles of the letter *R* are given in this copy. If you are able to make the letters *P* and *K* correctly, you should have no trouble with the *R*. Notice the instructions given for the *P* and form the *R* in like manner, adding the finishing stroke, which should extend one space below the base line, the same as the finishing stroke of *K*.

73. Copy No. 51.—Begin at the base line and use a right curve for the first part of the tracing exercise. Make

LESSON 6.

55 U U uuuu union uses

56 U U U Uneven Universe.

57 V V vvvv vivid voices

58 V V V Veronia Virginia

59 W W wwww wear award

the oval at the base one and one-half spaces in height. Make ten downward strokes in each exercise. Do not make the oval irregular. The *s*, like *r*, should be one and one-fourth spaces in height. To be successful with this letter, you must curve the first stroke a great deal, retrace it slightly at the top, and curve the lower part of the downward stroke very much. Notice that the finishing dot rests on the upward line of the letter a little above the base line.

74. Copy No. 52.—The usual difficulty with the letter *S* is to get enough curve to the upward stroke, also to get enough curve to the downward stroke. Make these curves very full and you will not have very much trouble with this letter. See that the upward and downward strokes cross at one and one-fourth spaces above the base line. We give two styles of finishing lines. Both styles are used a great deal and we advise you to learn to make both styles well.

75. Copy No. 53.—Study the form and proportion of the *T* as you practice the tracing exercise. Do not place the top, or cap, either too close or too far from the first part. Notice the double curve in both parts of the exercise. The full height of the letter *t* is two spaces. Up to one space high it is the letter *i*; above this it is a slightly shaded straight stroke, one space in length. Be sure to place the cross-stroke of the letter one and one-half spaces above the ruling, and make it only one space long. See that it crosses the letter, instead of placing it two or three spaces to the right, as is often done. Practice the words with care, giving especial attention to the *t*.

76. Copy No. 54.—The two parts of the letter *T* are the capital stem and the cap. Make the capital stem and the cap just as was described for the letter *F*. The horizontal part of the cap should be nearly parallel with the line of writing. Note the position of the shade and the ending of the stem. If you carefully follow all that has been said about the capital stem in *F*, you will not have much trouble with the letter *T*.

77. Copy No. 55.—Note the main points of the letter *U*. Retrace the upward stroke in the last part of the exercise; the ending stroke extends below the base line. Make the upward strokes of *u* full curves, and the downward strokes straight to the line. Have them parallel, one space high, and nearly one space apart. The tops of this letter should be pointed and the turns at the bottom should be round.

78. Copy No. 56.—The first part of the letters *U* and *V* are very much alike. The most difficult part of both of these letters is the second downward stroke. In the *U*, form the loop the same as in the *W*, giving the oval a full curve; descend one space with the curve and then, changing to the straight line, continue nearly to the base line, make a short turn, and ascend with a right curve two spaces and one space to the right of the first downward stroke; then descend with a slightly curved line to one space below the base line. See that the main downward stroke is straight for two full spaces. Confine any shade to this straight line; it is quite a common error to shade too high.

79. Copy No. 57.—Begin the tracing exercise the same as the *U*, make the letter narrow and end the upward stroke at two spaces in height. Be sure to make the top of the first part of the letter *v* a short turn and not an angle. Do not make the space between the first downward stroke and the second upward stroke too wide; remember, it is only one-half space. The bottom of the letter should be a round turn, not a sharp point.

80. Copy No. 58.—Form the first half of the letter *V* to the base line exactly like the *U*; then turn short and ascend with nearly a straight line one and one-fourth spaces; then, with a full left curve to the right, finish at two spaces above the base line. The width of this letter at one-half its height is one-half a space, and at the height of the last stroke it is two spaces. The most common error is to make the turn at the bottom too small and to ascend

with a gradually increasing width from the base to the top of the finishing line.

81. Copy No. 59.—The tracing exercise of the *W* is even more difficult than that of the *V*. The most common fault is to make the exercise too wide. See that the downward strokes are parallel, and that the top of the third part is only one-half a space from the last downward stroke. See that the points at the top of the letter are sharp; do not make the middle one a loop, as is often done.

LESSON 7.

82. Copy No. 60.—The *W* is a difficult letter to make correctly. Compare your work with the copy. Note the distance between different parts of the letter. The third downward stroke should be nearly straight. Make the letter narrower at the base than at the top. Give as much practice to the words as to the capitals.

83. Copy No. 61.—Make the first part of the tracing exercise like the first part of *W*; the last part should be made like a large figure 6. The *x* is not difficult to make. Use the first part of the *n* and the last part of the *c*. The letter may be made in three different ways, but from experience we find this style to be the best. This letter is seldom used but it pays to make it well.

84. Copy No. 62.—The first part of the letter *X* is exactly the same as the *W*. The downward stroke in the second part is a left curve, beginning at the full height of the letter and touching the main stroke at one and one-third spaces from its top, continuing with a left curve to the base line, touching it one space to the right of the preceding line, turning and finishing with a small oval. Use a free movement in making each part.

85. Copy No. 63.—The letter *Y*, to the bottom of the third downward stroke, is the letter *U*; then add the loop

LESSON 7.

60 W W W Winona Waywood

61 X X xxxx vexes vexed

62 X X X Xenia Xanthine

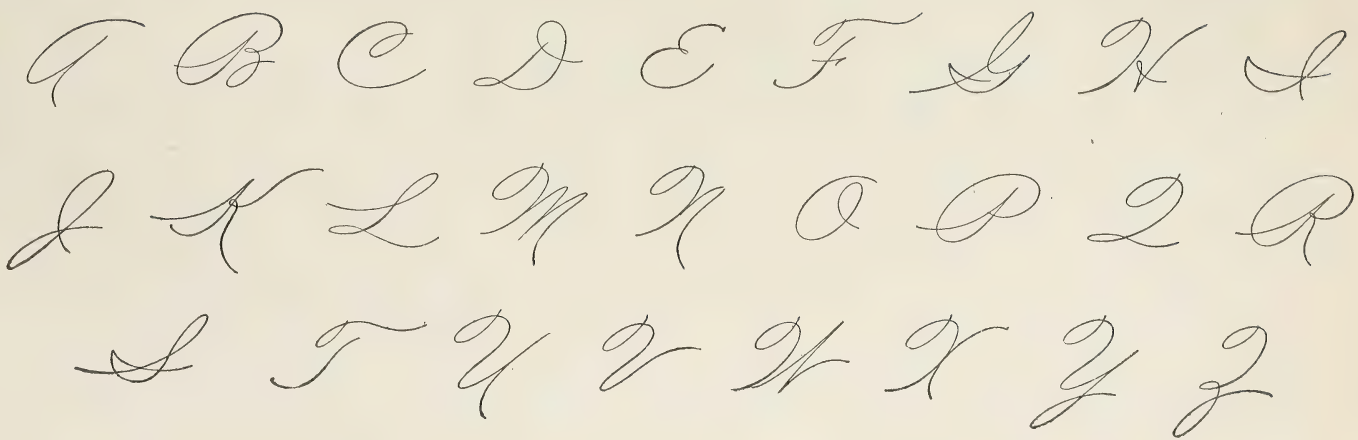
63 Y Y Y Yeoman. Yesterday

64 Z Z Z Zouave Zanzibar.

LESSON 7.

BUSINESS CAPITALS

65



as in the small *y*. See that both downward strokes are on the same slant. The last upward stroke should cross the long downward stroke at the base line. Watch the shade and the turn at the bottom of the second downward stroke. For the further points to observe in this letter, see the capital *U* and the small *y*.

86. Copy No. 64.—The first part of the letter *Z* is the same as the first part of *W*, excepting that, on account of the small loop, the shade is made a little higher. Form the loop, then turn to the right and descend, with a slight right curve, two spaces; then ascend with a full left curve, crossing the downward stroke at the base line, and finishing three-fourths of a space above the line.

87. Copy No. 65.—In this copy you have enough material for several days' practice. Not only should the student note each letter carefully, but he should study the relation of one capital to another. The size, slant, spacing, quality of line, all enter into the making of a model set of capitals. Do not allow yourself to make the letters slowly in your endeavor to obtain the correct form. Make them with a quick movement, cross out the poorest letters, then try again.

Business capitals should not be shaded. Of course, when using a fine, elastic pen some slight shade will sometimes be made by those who bear heavily on the pen, fine-pointed steel pens can be had that are stiff enough to hold up the heaviest hand. To those who have difficulty in this respect we suggest that they try Crawford's 901 pen.

It is quite probable that your faults are the result of a lack of control of your movement rather than a lack of knowledge of the correct forms. Some of the forms given here vary from those already studied by the student; the other styles may be used if preferred.

We have now reached a point where the pupil will be allowed to use his taste and judgment in the selection of styles of letters, yet we want him to adhere to the styles

LESSON 8.

66 St. Louis is the largest city of Missouri and has a population of 575000. Jefferson City is the capital of the state.

67 Newark is a city of New Jersey.

LESSON 8.

$900.00 Syracuse, N. Y., Jan. 4. 1900

On Feb. 4th next I promise to pay J. A. Leming Nine Hundred Dollars, value received.

No. 8. E. J. Foerster.

given in the course. Learn to severely criticize your writing. In these days, when proficiency in any line of work counts for so much, no one can afford not to write a good hand.

LESSON 8.

88. Copy No. 66 and No. 67.—These copies are given for special practice in what is called *body* writing. In the preceding copy we had practice on the large and free forms of capitals, now we have the smaller and more careful work of sentence writing. We know it is difficult work and not as fascinating as the former copy, but it is necessary work and it is best to do it and do it well.

The style of writing selected for these copies has a little more space between the letters than is given to *n* and *u*. This style has been selected because it has a tendency to induce the student to strike out more freely and will help him to break away from the common habit of using the fingers too much.

Because of lack of interest in this work one is liable to become careless and the writing will approach scribbling. The copy is so long that we do not hold ourselves down to it for the careful study that a shorter copy would receive. Here is where the student may make a mistake. Study each part well, then when the copy is finished, study the general appearance of the work. Michel Angelo, on being questioned as to why he gave so much attention to small things, said, "Trifles make perfection, and perfection is no trifle."

The style of *S* used as the first letter of this lesson, can be made well only by a good writer. The same may be said of the *C* in *City*, in the third line of the lesson. Use the old styles unless you can make these well. They are given for variety and to familiarize the student with forms that are sometimes used.

89. Copy No. 68.—In this copy we have a business form for practice. There are eleven different capitals in the copy and each should be made with care, yet easily and

LESSON 9.

69

$2000#

Denver, Colo., May 10. 1901.

Ninety days after date I promise to pay G. J. Andrews, Jr., or order, Two Thousand Dollars, value received.

Due Aug. 8-11, 1901

J. H. Parton.

LESSON 9.

$79⁰⁰ Lansing, Mich., Jan. 4. 1902.

Due Hiram Elverson Seventy-nine Dollars payable on the first day of June next, value received.

Walter F. Jones.

freely. Remember we want business writing, writing that is adapted to the dispatch of business. This copy is a promissory note. Those who have studied the matter know that credit is a very important factor in carrying on the commerce of the world, and that it adds greatly to the convenience and dispatch of business. It is true that the extreme use of credit has caused panic and disaster, but the custom of credit is growing with each year. Credit does not always consist of accounts, but may take the form of a written promise to pay, which is called a *promissory note.*

A **promissory note** is a written promise to pay a certain sum of money at a specified time, or on demand, to a person therein named, or to his order or assigns, or to the bearer. This form of credit covers the entire field of business activity. A note may pass from one man to another by endorsement, and in effect be the same as a bank note. It differs from a bank note only in this, that it is transferred by endorsement and matures at a stated subsequent time, and that the endorsers are liable to the holder in case the maker refuses payment. The person who signs the note is the **maker,** and the person to whom it is made payable is called the **payee.** The person who writes his name across the back of the note is an **endorser.** A note is negotiable, that is, transferable from one person to another by endorsement, when it reads "pay to the bearer," or "pay to the order of."

Copy No. 68 is a non-negotiable note as it is payable only to J. A. Leming. If the note were payable to the order of J. A. Leming, it would then be what is called negotiable. In this note E. J. Foerster is the maker and J. A. Leming is the payee.

LESSON 9.

90. Copy No. 69.—This is a form of a negotiable note that might pass between several persons before it is due. This form of note is payable to the person who holds it at maturity, while the one not negotiable is payable only to the person in whose favor it is drawn.

Of course in actual business most papers like notes, checks, receipts, etc. are made out on printed blanks. It is well, however, to know how those papers should read so that if ever required to draw one up on blank paper, you could do it correctly and without hesitation. It is said that the largest check ever drawn by Jay Gould was written in pencil on the back of an old envelope. Such copies as these are, therefore, valuable not only as penmanship lessons, but also as samples of business forms.

Write these notes many times. Aim for constant improvement and submit your best work for criticism and additional instructions.

91. Copy No. 70.—At an adjustment of claims between parties, a **due bill** may be issued, which is a written acknowledgment of debt and may be payable in merchandise or money. It may be made payable on demand or at a future date, and by the insertion of the words *or order* it becomes negotiable the same as a promissory note. If a due bill is payable in merchandise or property, it should be explicit; the exact quantity should be stated and any other information that would give a clear understanding of the matter to an outside party.

In writing this, the student should remember that he is at liberty to use styles of capitals that have been presented heretofore. The *L* and *E*, especially, vary somewhat from the forms already given. Such capitals as these require a very free movement and unless you have such a movement do not attempt them except as practice exercises to develop movement. Keep reviewing past lessons, always working for a light touch and good control of the pen. Business writing must be done quickly and should be dashy and clean cut in appearance.

The only way to learn to write well is through earnest and well directed practice. There is no royal road. We want the student's best work from this copy. All the copies in these lessons were photoengraved from pen and ink writing.

LESSON 10.

71

$500.00

Haverford, Pa., Jan. 2, 1901.

Received of F. J. Travers ____________

Five Hundred ____________ Dollars.

In full of all demands.

No. 17. H. J. Newcomer.

LESSON 10.

92. Copy No. 71.—This business paper is called a **receipt** and is a written or printed form acknowledging the receipt of value. The form of a receipt may vary according to the kind of value received and the reason why it is given; hence one may be given to apply on account; in full of all demands; for rent; to apply on a note; for a note; to executor for payment of a bequest; for instruction; and for many other causes. A receipt should state plainly and fully for what the payment was made. A receipt is not certain proof of payment; it may be invalid because of mistake or fraud and is open to explanation or contradiction.

Note the position that each part of this business form occupies, and study capitalization and punctuation closely. Do not make the capitals too large but be sure to make them with a quick, dashy movement so as to get smooth, clean-cut lines.

As you write this copy, notice the arrangement and general appearance of your work. Study to locate your errors and then do your best to eliminate them from your work. A constant striving for betterment will do much to improve your writing. You may receive ever so much instruction, but the real writing must be your own work produced by your own efforts. Some of our finest writers have had the most difficulties to overcome. The spider's web may break twenty times and yet he keeps on and eventually succeeds. The policy of indomitable pluck has carried many persons to success, and in these days of business competition it pays to write well. Do not allow yourself to become careless because of the poor writing of some famous men, or to consider that you are like them because you write in like manner.

93. Copy No. 72.—This copy is designed to give the student practice in writing a short business letter. Attention should be given to the arrangement as well as to the penmanship. Notice also the use of capitals and punctuation marks. Too much pains cannot be taken to have one's correspondence well written and each idea clearly stated.

LESSON 10.

Dayton, O., Jan. 7. 1901

Kingman & Howe,

New York.

Gentlemen:- We enclose New York Draft for Five Hundred Dollars to balance our account to date.

Yours respectfully,

Norman, Adams & Co.

The large insurance companies employ good penmen to fill in their policies. When they have any particularly formal correspondence it is quite customary to have the writing done by these penmen, as the letters are then more elegant and formal than if typewritten.

Poorly written letters are quite common in business. Many business men whose dealings are mostly with the poorer class of people of the large cities, often receive letters that are very difficult to decipher. When a package is to be mailed to such a customer, the dealer sometimes cuts the signature and address from the letter and trusts that the post-office authorities will be able to find the writer. A nicely written letter is appreciated even in the rush of business. Oftentimes a well paying and responsible position is secured through the applicant's ability to write a good letter. There are employers who judge of a young man's ability from his letter of application, in preference to a personal interview. A great many marks of character are expressed in a letter—neatness, arrangement, penmanship, expression of thought, and attention to details, all of which go to make up a model business letter. It is worthy of mention here that some of the best clerical positions in the world are for work requiring the best penmanship.

The style of capitals used varies somewhat in the different forms. In all other respects follow the copies closely, but as regards capitals always use that style of a letter that you can make the best. Work for movement, as that is the foundation of a good writing.

LESSON 11.

94. Copy No. 73.—This copy treats of envelope superscriptions. In arranging a superscription, begin the person's or firm name far enough to the left to allow of its being written in a free style and yet leave a fair amount of space at the right. Do not write it too low on the envelope, but be sure to leave plenty of room under it for the town and state. Some writers prefer to put a local address under

LESSON 11.

SUPERSCRIPTIONS

73

J. H. Perino, Jr.,
Havana,
Cuba.

Mr. C. R. Davidson,
Peoria,
Ill.

H. G. Jamison,
Box 475.
Riverside,
Cal.

84 West St.
Mr. Irwin Fanchiez,
Elmwood,
Pa.

the name above the town, instead of in the lower left-hand corner, and we believe that the post-office department favors this form. If the name of the person addressed is short, the names of the town and state can be written so that each ends a little to the right of the preceding line. If this style is not followed, let all the lines end at about the same distance from the right-hand end of the envelope; never write a superscription in the form of an inverted pyramid. There is enough material here for many hours of practice. Practice carefully and earnestly and then submit two copies of each superscription for criticism.

Note the styles of letters used in the copies and follow them unless there is a letter, say like *C*, that you cannot make well, then substitute another style. See that each small letter stands out clearly and plainly; the large space between the letters should give the writing a very clean-cut effect.

Pay close attention to your movement, there is no other place where an exceptionally free movement can be used to better advantage than in a superscription, for here there is plenty of room, the writing can be made a little larger than usual if desired, and more freedom in style is allowable in this work than would be desirable in a letter or in book work.

No attempt should be made to reproduce, on the work sent us for examination, the printed title that is at the top of the copy.

95. Copy No. 74.—A good signature is worth a great deal to a young man or woman engaged in business, but how often do we find signatures that are illegible, giving the impression that the writer was either very careless or wished to write something that could not possibly be read. It does not pay to write a signature that is a tangled mass of conglomerate lines. A signature that is neat, easily written, and *plain* is the best for all purposes. Capitals need not be joined to form a good signature; in fact, some of the best signatures we have ever seen have contained

LESSON 11.

C. R. Donald W. J. Keene A. R. Furner

A. T. Taylor C. E. Fraser

I. D. Parton. T. J. King F. B. Gainor

F. H. Ericson. L. E. Stacy.

capitals that were not joined. When an ending stroke is in a position from which a following capital should be made, then the capitals may be joined. In writing business signatures it is not a good plan to throw any line out of its course in order to connect capitals; this is fully illustrated by the signatures in this copy. There are some extra lines in this copy, but all are given for a reason and are of use. There is some difficult writing here also, and the student may need to work hard to get the desired results. That "we gain in proportion to the time and effort expended," seems to be especially true in the acquirement of a good handwriting. When you are satisfied that the work you have is the best that you can do at the present time, send it to us for criticism and proceed with the next lesson.

Send us three samples of your work on the copies, and kindly arrange them in the same order as here given. It would also be a good plan for you to send us samples of your own signature, using the styles of capitals you like best for it, and we will endeavor to offer such suggestions as will help you improve it. Do not write too large, and do not shade. Write at a good rate of speed, to get clean-cut lines, but do not rush ahead without seeing where your lines are going and how the work will look when done. You want movement and speed, but they must be under control.

LESSON 12.

96. Copy No. 75.—The figure *1* should be a downward stroke on the main slant and one space in height. Do not shade or curve the stroke. The *2* is not a difficult figure if given proper study. It is a small *Q*. Make a small dot or loop at the top and curve the downward stroke until the base line is reached, then end with a compound curve, or a straight line. For the *3*, begin as for the *2* and curve well the downward strokes. Aim to keep the figure plain and clear. Notice the direction of the ending stroke. See that the downward stroke of the *4* is higher than the first and somewhat curved, and that the horizontal stroke is a little

LESSON 12.

75 1 2 3 4 5 6 7 8 9 0 a/c a/c a/c % % % $ $ $ $

account — per cent.

76 $275.94 896.41 457^{40} c/o c/o c/o & & & & # # # #

care of — and — lbs.

77 A B C D E F G H I J K L M

N O P Q R S T U V W X Y Z

78 Small writing can sometimes be used to advantage.

LESSON 12.

(Your Residence and Date.)

,79

The Inter. Corres. Schools.

Gentlemen: This is a specimen of my writing after completing your course of lessons in penmanship

Yours truly,

(Your Class Letter and Number.)

(Your Name.)

above the base line and nearly horizontal with it. Give attention to slant. The *5* is much like figure *3* in formation. Be sure to connect the horizontal straight line to the top of the first part of the figure; make the ending stroke as in the *3*. For the *6*, begin a little above one space in height and curve the downward stroke slightly. Do not make the lower part too small. Make the first part of the *7* short, and follow with a straight line extending below the base line and on the main slant. The curved stroke of the *8* should be made first; give attention to the slant and direction of the second stroke. The *9* is an *a* and the last part of the *7*. This figure extends below the base line. The *0* is practically the same as the letter *O*. In book work this figure may be made perfectly round.

For the following copies, make the *a*, *c*, and *o* very small. The downward stroke should be made firmly and on the main slant; a slight shade at the base adds strength. Do not make the letters too far apart.

97. Copy No. 76.—Practice easily and freely; study every figure closely; and grasp every opportunity for improvement.

98. Copy No. 77 and No. 78.—Bookkeepers and others engaged in clerical work are often required to write a very small style. The style given is undoubtedly the best for all such work.

99. Copy No. 79.—This is the last lesson in the course, and we have included in it this copy, not only because of the excellent practice it affords, but also because it will give you a specimen, prepared under instruction, to compare with that written at the beginning of the course of lessons. Note carefully the arrangement of the different parts of the exercise, paying particular attention to those that you are to add, and endeavor to follow this arrangement in your work.

Only thoughtful, conscientious practice can produce the best results. Apply yourself earnestly, and you will be successful in making satisfactory improvement and acquiring a good handwriting.

VERTICAL PENMANSHIP

GENERAL INSTRUCTIONS

PRELIMINARY REMARKS

1. Advantages.—The advantages of a good handwriting are too well understood to require mention. Many competent authorities say that the position of the body assumed in vertical writing is less injurious to the health, that time and energy are saved, and that the writing looks better than slant writing.

2. Qualifications.—The only qualifications necessary are a good eye, normal hand, determination, and perseverance. Vertical writing is more easily learned, written, and read than slant writing, and it is reasonable to expect that the student who will pursue these lessons faithfully may become a good penman. The method given of presenting the subject is simple and direct, and experience has shown it to be the very best that has been devised for producing a practical handwriting. The results, however, depend as much on the student as on the plan of instruction.

3. Specimens.—Before proceeding with the lessons, write two pages of specimens, each containing one set of capitals, one set of small letters, one set of figures, and the following short letter:

Your address and the current date

International Correspondence Schools,
Scranton, Pa.

Gentlemen:

This is a good specimen of my writing before beginning your lessons in penmanship.

Yours truly,

Your signature

Keep one set of these specimens and send the other to us immediately.

MATERIALS

4. Quality.—No one can expect to do good work without good tools, and the student should be provided with the best. The difference between the price of a good and an inferior article for writing purposes is too slight to be considered, and the best materials are an incentive to do good work.

5. Pens.—Vertical writing requires a fairly coarse pen with a smooth point. Such a pen will not produce fine hair lines, but it will stand resistance and at the same time produce a smooth, firm line. Spencerian vertical pens graded "fine," "medium," and "coarse"; Esterbrook 556 and 570; and Gillott's vertical pens, are all good. The finer of these are recommended for the more advanced pupils.

6. Penholder.—There is nothing better than a plain, straight holder made of either wood or rubber, slightly tapered, and having a cork or rubber base on which to rest the thumb and fingers. A holder with a metal ferrule should be avoided. It is not only unpleasant to the touch but it is injurious to the nerves, on account of the metal being a conductor of electricity, which passes from the pen point to the hand.

7. Ink.—In order that one may write with confidence, the ink should flow freely, and it should be as black as

possible. Stafford's, Carter's, Arnold's, and Thomas's are all reliable.

8. Blotter and Pen Wiper.—A good blotter and pen wiper should always be at hand. A blotter is used not only to absorb the surplus ink but also to rest the hand on, so that the paper may not be soiled. The pen wiper should be used frequently to keep the pen clean, so that the ink will flow freely from it. A wet sponge is the best wiper. A small piece of chamois skin or a cotton cloth, however, will answer.

9. Paper.—Always provide a liberal quantity of good letter or foolscap paper; not necessarily the most expensive, but a paper of fair weight, with a hard, smooth finish, without gloss.

POSITION

10. Position of the Body.—There are three good reasons why a student should assume a correct position at the desk: First, it permits free play to the circulatory and respiratory organs; second, it allows control of the muscles of the arm and hand; third, the writer feels, appears, and acts better.

Sit squarely in front of the desk, leaning slightly forwards without bending the back. The right arm should be placed on the desk at right angles to the left, both elbows off the desk and quite near the body without touching it. The left hand should be used to hold or steady the paper. The feet should rest squarely on the floor, the left foot a little in advance of the right, a position that enables one to rise without changing the position of the feet. This will give necessary firmness as well as ease and comfort. (See Fig. 1.) While writing, every muscle of the hand, arm, and body should be in a relaxed condition.

11. Position of the Paper.—While the ideal position of the paper for children is parallel to the front edge of the desk, as shown in Figs. 1 and 2, the person accustomed to

FIG. 1

FIG. 2

Fig. 3

Fig. 4

slant writing will find it more natural and easy to slant the paper a little to the right, as shown in Fig. 3, so that the arm will have more room to rest on the desk, but not so much as in slant writing. The paper should be placed a little to the right of the middle of the body, as shown in Figs. 1 and 3, so that the pen will begin at a point opposite the breast bone. If the pen begins at a point to the left of the breast bone, the writing will slant to the left of the vertical, and will lead to backhand writing. It is very important that these points should be carefully observed and followed until they become a habit.

12. Position of the Hand.—One of the strongest claims made for the superiority of vertical over slant writing is the naturalness of the position of the hand in holding the pen. No turning or twisting of the hand or wrist into an abnormal position is necessary in vertical writing. Drop the hand to the side in a natural position; then raise and place it on the desk as you find it. All of the fingers will be in a slightly bent position, as shown in Figs. 2 and 3, the first finger being bent a little less than the second; the second, less than the third; the third, less than the fourth. Place the penholder between the thumb and first finger, both being in a slightly bent position, with the first finger on the top of the holder and about 1 inch from the point of the pen, as shown in Fig. 2; place the end of the thumb on the holder opposite the first joint of the second finger, as shown in Fig. 4. The second finger should drop a little to the right of the holder, so that the holder will pass opposite the root of the finger nail, as in Fig. 4. The third and fourth fingers should be drawn back so that they separate from the others at the first joint of the second finger, as in Figs. 1 and 2, resting them on the sides of the fingers, as in Fig. 4, or, better, on the nails, as in Fig. 3. This, with some shaped hands, is difficult. One of the best tests of the correctness of the position of the fingers holding the pen is that they can be moved easily and naturally and that the pen so held will move in a vertical line.

MOVEMENTS

13. There are four movements recognized in writing: the *whole-arm*, the *finger*, the *muscular* or *forearm*, and the *combined* or *mixed*.

14. The **whole-arm movement** consists of the independent motion of the whole arm from the shoulder, using all of the muscles that control the joints of the arm and hand, the only rest being on the pen and nails of the last two fingers. This movement is used in making large capitals and ornamental work, and also in writing on the blackboard, where there is no possible rest for the arm and hand.

15. The **finger movement** consists of simply moving the thumb and fingers by extending and contracting them. While this movement is not to be recommended for exclusive use, it can be used to advantage at the beginning, when learning the forms of the letters. The student will find he has a more sure control of the muscles of the joints of the hand than of the joints of the arm.

16. The **muscular,** or **forearm, movement** consists of resting the fleshy part of the forearm on the desk in a relaxed condition, the elbow just off the desk, and simply pushing and pulling the arm back and forth in the sleeve without moving the cuff of the sleeve. This brings into action the muscles of the shoulder joint, which run down from the shoulder to the spine. These are the muscles that very largely control the pull motion of the arm, and the muscles running down from the shoulder to the chest are those that very largely control the push motion. There are other muscles that control the smaller joints of the arm and hand, but they are of minor importance as compared with the shoulder muscles. To acquire a free, smooth, and rapid handwriting, these muscles must be brought under quick control, until their use has become a habit. This is possible with every one that is willing to give it thoughtful, painstaking practice.

17. The **combined, or mixed, movement** is just what its name indicates—a combination of the other movements—a simultaneous action of the arm and hand, one working in unison with the other. It is the ultimate result of the training that comes from the other movements. It becomes an ideal movement when all of the muscles controlling the joints of the shoulder, elbow, wrist, and fingers respond quickly to the mandates of the will.

DEFINITION OF TERMS

18. Before entering on the study of a subject, it is important to know the meanings of the terms used. The following are those with which the student of penmanship should be familiar:

Base Line.—The real or imaginary line on which the writing rests.

Circle.—A plane figure bounded by a curved line that is everywhere equally distant from a given center.

Connective Slant, or Line.—A curved line slanting 45° to the right of the vertical. The line may be a simple or a compound curve.

Headline.—The real or imaginary line to which the short letters extend.

Left Curve.—A curved line on the left-hand side of a circle.

Lower Turn.—The turn at the bottom of a letter.

Main Line.—A firm, straight, vertical line without shade.

Right Curve.—A curved line on the right-hand side of a circle.

Sharp Turn.—The point where two lines running in different directions meet and where the pen stops before changing direction.

Space.—The height of the small *i*, without the dot, is the unit of measurement for both height and width of letters, and is called a space.

Straight Line.—A line that does not change direction throughout its entire length.

Top Line.—The real or imaginary line to which the loop and capital letters extend.

Upper Turn.—The turn at the top of a letter.

PROPORTIONS OF LETTERS

19. The short letters are one-half the height of the loop and capital letters, the small *i* being the standard, or unit of measurement. The *t*, *d*, and *p* should not be made quite so high as the loop and capital letters; or, to be more definite, they should be one-fourth of a space shorter, as shown in Fig. 5. The *r* and *s* should be one-fourth higher than the short letters. The *p*, *f*, *y*, and *g* should extend one space below the base line, and all loop letters one and one-half spaces below, making the crossing on the base line. The crossing of the loop above the line should be a little less than one space above the base line.

The word *upright*, shown in Fig. 6, illustrates the different lengths of letters, both above and below the base line. The distances between the down strokes in letters, as well as those between letters in words, is shown in the word *nine*, Fig. 7.

It will be observed that the *o*, *a*, *c*, *d*, *g*, and *q*, shown in Fig. 8, are based very largely on the direct circle, and the *x*, *s*, and *p* on the reversed circle. All of the other small letters are based on the square.

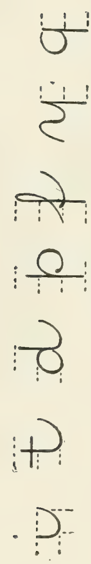


Fig. 5

PROPORTIONS

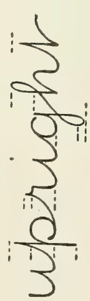


Fig. 6

SPACING

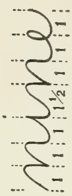


Fig. 7

LETTERS BASED UPON THE CIRCLE

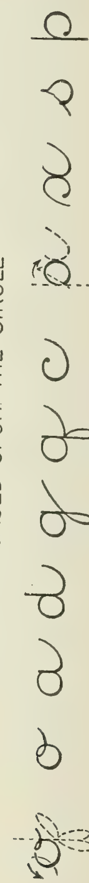


Fig. 8

THE LESSONS

20. The lessons follow in their regular order. Each consists of photoengravings of pen-written copies and the explanations and directions for practice. In preparing the work for examination, write each copy at least three times. *Never send in more than one lesson at a time.*

LESSON 1

Copy 1.—*Small o:* This letter begins and ends at the headline and is made with one motion of the pen; that is, the pen continues to move from the beginning to the finish of a letter, there being no sharp turn where it stops to make a change of direction. The down strokes are a little firmer, or stronger, than the up strokes. This is because the muscles that control the movement of the hand toward the body are stronger than those that push the hand from it. Practice the copy freely, using a strong, rolling motion, and maintaining a fair rate of speed.

Copy 2.—*Capital O:* This letter is made twice as high as the small letter. It is shown in several difficult combinations—joining the *n* and connecting with the top of the *w*. Maintain a correct position of the body, observe the directions for holding the pen, and be careful of the movements. Practice the copy until the muscles respond quickly to the will.

Copy 3.—*Small c:* This letter appears at the beginning of a word, between two other letters, and two are shown in combination. Study these forms and be careful to get a uniformity of the down strokes.

Copy 4.—*Capital C:* Begin with a short, vertical, straight line; then trace the circle a few times with a free, smooth motion, but not necessarily rapid. Finish the letter at the headline. Make the single *C*, and write the words with

LESSON 1

1 o o on one none moon

2 O O One Own Over Owen

3 c c cone come ocean occur

4 C C Cone Corner Concern

5 g g q gun gage gang gong

6 G G Great men are workers.

7 d d due dine dime dinner

8 D D Do not fail to improve.

9 a a an am crane cream

10 A A Are you in earnest?

11 e e ear are ever exceed

12 E E Eunice Emma Emerson

the same strong but smooth motion, observing carefully the uniformity of the small letters.

Copy 5.—*Small g:* Begin by tracing a circle; then add the loop or the straight, vertical line, according to the style of letter used in different parts of a word. Write the words with a strong, pulling, rolling motion, giving fulness to the round turns at the top and the bottom of the letters.

Copy 6.—*Capital G:* The first part of the letter is like *C*, and should be drilled on in the same way. Study carefully the up stroke of the loop, and give special attention to the liberal spacing between the words in the sentence.

Copy 7.—*Small d:* This letter is formed from the circle. Make and trace the circle as indicated in the copy, to gain a free control of the muscles as well as to get a clear idea of the form, and then apply it to the *d* in the words, keeping constantly in mind the smooth touch of the pen that will lead to quality of line in the letters.

Copy 8.—*Capital D:* The shape of this letter and the motion in making it should be studied with great care. Observe the small loop at the bottom of the letter and the oval at the top. Drill especially on the oval by tracing it as shown in the copy and apply it to the beginning of a sentence. Notice every new combination of small letters and try to show improvement in the general appearance of the sentence.

Copy 9.—*Small a:* This letter appears at the beginning of a word and between other letters. It should be treated practically the same as the small *d*. Inasmuch as this copy is composed of short and not difficult words, there should be freedom of action. This will result in a good quality of line.

Copy 10.—*Capital A:* This letter is similar in shape to the small *a*, the difference being largely in the proportion. The capital is based on an oval and the small *a* on the circle. Practice the letter in the same way as before, and then write the sentence as if you were asking the question. This will enable you to make your writing practical—thought and act will become one. Study carefully the shape of the interrogation point.

Copy 11.—*Small e:* This letter is shown as it appears alone, at the beginning of a word, and between other letters, giving its formation from the base and from its connection with the top of another letter.

Copy 12.—*Capital E:* Make a short, vertical, straight line and then trace the circle both at the top and at the bottom, finishing at the headline, the upper circle being a little smaller than the lower one. The loop joining the upper and the lower parts of the *E* should be horizontal. The *E* joined to the small *u* and another to the *m* gives different combinations. These should be practiced until the combinations can be made with ease.

LESSON 2

Copy 13.—*Small n:* Observe that this letter occupies three squares, as indicated by the dotted lines, and that it appears in the copy in different parts of a word. This copy is not difficult and should be practiced with the view of freedom of motion, keeping in mind the uniformity of the up and the down strokes.

Copy 14.—*Capital N:* This letter is one of the "reversed motion" class of letters, the motion being distinctly opposite to the *O*, *C*, *G*, *D*, *A*, and *E* in the last lesson. Begin by making a short, vertical, straight line, which gives a little momentum to the hand, and then trace the circle several times until you can make a full, round turn, and reverse the motion at the bottom. Following this, make the *N* and write the sentence, observing carefully the spacing and the difficult combinations.

Copy 15.—*Small m:* This letter occupies four spaces in width, or one more block than the *n*. The same instructions apply to the two letters.

Copy 16.—*Capital M:* Practice making the *M* in the same manner as the *N*, as indicated, and then write the rest of the copy, observing the punctuation marks. Study carefully the form of the figures and the general appearance of the whole line.

LESSON 2

13 n n nine run men man

14 N No time should be lost.

15 m m mum mine manner

16 M Monday, March 2, 190 .

17 x x z z ox mix zero zinc

18 X X Xerxes Z Z Zero Zone

19 w w win new renew review

20 W W Waste makes want.

21 q q qu quire quit quire

22 Q Q Quick come, quick go.

23 v v vim vine over oven

24 V V Vigorous work wins.

Copy 17.—*Small x:* Notice that this letter is based very largely on the direct and indirect circle, and that the first part of the *x* and of the *z* are very much alike.

Copy 18.—*Capital X:* It will be seen that the upper part of the *X* and also of the *Z* is based on a larger circle than *N* and *M*. Work for results in form and easy motion.

Copy 19.—*Small w:* This letter is the *u* doubled, and each part is practically the same width as a *u*. The connection between *w* and *i* should have attention and careful practice.

Copy 20.—*Capital W:* The instruction given for the practice of *N* and *M* applies equally well to the *W*, except that the point where the pen stops to make the sharp turn is at the bottom of the *N*, while in the *W* it is at the top.

Copy 21.—*Small q:* This letter is made with the same motions as the *g*, except that the down stroke of the loop turns to the right instead of to the left. The loop should be the same length in both letters, and the distance between the *q* and *u* should be practically the same as between the down strokes of the *u*.

Copy 22.—*Capital Q:* The instructions in regard to the drill on the circle in the capital *X* and *Z* applies equally well to the *Q*. The loop at the base should be horizontal.

Copy 23.—*Small v:* In writing a word containing a *v*, great care should be given to the connection with other letters.

Copy 24.—*Capital V:* Drill on the *V* according to the instructions given for the *W*, and write the sentence, thinking of what you write as well as of how you make the form of the letters and the combinations.

LESSON 3

Copy 25.—*Small u:* This letter is two spaces wide and is made the same as *i*. Different combinations are shown.

Copy 26.—*Capital U:* Drill on the circles in the *U* until confidence is felt in control of the hand, and then make the

single letter and follow it by writing the sentence, giving attention to both form and free control of the hand.

Copy 27.—*Small y:* Study carefully the shape of the first part of this letter, which is the same as the last part of an *n*. The loop crosses on the base line and finishes at the headline, and is one and one-half spaces long below the base line. The final *y* is finished with a straight, vertical line extending one space below the base.

Copy 28.—*Capital Y:* Practice tracing the circle; make the loop below the base the same length as that in the small *y*, and then write the sentence, observing the punctuation and uniformity of the up and the down strokes.

Copy 29.—*Small i:* This letter, without the dot, should fill one square block, as indicated by the dotted lines. Drill on the words in the copy to learn the different combinations.

Copy 30.—*Capital I:* Study carefully the lower part of the letter and observe that it is one-half of a circle. Drill by tracing the circle and follow this by making an *I* and writing the sentence.

Copy 31.—*Small j:* Observe that the loop is the same as in the *y*. Make the *j* and write the words with the different combinations.

Copy 32.—*Capital J:* Trace the letter as it appears in the copy, working for free control of the muscles as well as for shape of the letters. Make the single *J*, and write the name and date, observing the punctuation marks and shape of the figures.

Copy 33.—*Small h:* This letter is two spaces high and three wide. It is shown at the beginning and at the end of a word.

Copy 34.—*Capital H:* Trace the letter as indicated in the copy, and study the shape and the movement that produces it. Make the single *H*, and write the sentence as if you were giving advice.

Copy 35.—*Small k:* This letter is so nearly the shape of an *h* that the same instruction will apply except that the last part is a little higher and requires more care.

LESSON 3

25 u u urn run ruin union

26 U U Use the best materials.

27 y y y you year way say

28 Y Y Yours, with great hope.

29 i i in win vim aim rain

30 I I I am bound to succeed.

31 j j join joint rejoin ajar.

32 J J John I. James, June 17, 190 .

33 h h he hem hush high

34 H H He that is wise is honest.

35 k k kin kind ink kink

36 K K Keep your own counsel.

LESSON 4

37 s s sun sum cases miss

38 S S Scorn every injustice.

39 l l line will alone illness

40 L L Let wisdom be thy guide.

41 t t tin tint tent let letter

42 T T Think before you act.

43 f f fun fife of off offer

44 F F Few words are best.

45 p p pin pen pun up upper

46 P P Pay all of your debts.

47 b b ban able rub rubber

48 B B Be not vexed over trifles.

Copy 36.—*Capital K:* Observe that the tracing exercise of the *K* has both the direct and the reverse motion of the circle. The loop that joins the parts together should be horizontal. Write the sentence with a graceful movement.

LESSON 4

Copy 37.—*Small s:* Observe carefully the reversed motion of the circle that produces the main part of the *s*. Drill on the tracing. Make the single *s* a little higher than the other short letters and write with care the letter in the different combinations.

Copy 38.—*Capital S:* The lower part of the capital *S* is also based on the circle, and should be practiced until the muscles will produce it with ease. Make the single *S*, and write the sentence, which has some difficult combinations.

Copy 39.—*Small l:* The loop of the letter should cross three-fourths of a space above the base and be two spaces high, and two spaces wide. Make the *l* and write the words with *l* combinations.

Copy 40.—*Capital L:* Make a short vertical line at the beginning of the letter, and follow it with a small circle, tracing it several times; then descend with a vertical compound curve to the base line, forming a loop, and finishing at the headline with a horizontal compound curve. Write the sentence with a firm, strong motion.

Copy 41.—*Small t:* Observe that this letter is not quite so high as the loop letters, and that the crossing is one space long and equal on the two sides of the down stroke. The final *t* should be without the crossing. The final up stroke is used as a substitute for it, to make the letter more simple, so that it can be written rapidly.

Copy 42.—*Capital T:* This letter begins with a vertical straight line made very short, joined with a horizontal compound curve to a vertical compound curve, which should be made nearly straight. It is finished with a dot at a point a little to the left of the beginning. Practice the *T* with a free, graceful movement.

Copy 43.—*Small f:* This letter should extend two spaces above the base line and one below. The pen should be removed at the bottom of the final stroke. The width is two spaces. Observe that the *f* is used in different combinations.

Copy 44.—*Capital F:* The instructions given for *T* apply to the *F*, except that this letter is finished with a horizontal straight line about one space long and half the height of the letter. Drill as indicated in the copy.

Copy 45.—*Small p:* The first part of the *p* is a little less than two spaces above the base line and one below. Remove the pen at the bottom and finish the letter with a reversed circle, terminating with a dot. Practice as shown in the copy.

Copy 46.—*Capital P:* This letter begins with a slight curve, merging into a vertical straight line. Remove the pen at the bottom and finish with a reversed circle, terminating with a dot. The best results will be obtained by a systematic drill on the copy.

Copy 47.—*Small b:* Observe that the shape of the first part of the *b* is the same as the first part of the *l*, and the last the same as the final part of the *w* and the *v*. Practice with the view of mastering the difficult *b* combinations.

Copy 48.—*Capital B:* The first stroke of the *B* is the same as in the *P* and the rest is based on the two reversed circles of about equal size. Practice this with the same end in view as in the small *b*.

LESSON 5

Copy 49.—*Small r:* This letter is a little higher than the *i* and should be made with three simple curved lines and one straight down stroke. Do not make a compound curve in the down stroke. The words in the copy involve difficult combinations of the *r*, and should be practiced with the view of mastering them.

Copy 50.—*Capital R:* This letter is made in the same way as the *B*, except that the loop is a little shorter and that the final down stroke is a vertical compound curve. Practice according to the plan in the copy, and do not scatter your energy.

LESSON 5

49 r r run or error runner

50 R R Read only the best books.

51 O. H. King. E. G. Case. A. D. Hunter.

52 T. F. Loomis. N. Miner. X. Z. Quinn.

53 P. B. Rice. U. V. Wilson I. J. Sims.

54 L. W. Lewis L. V. New. V. U. Young.

55 1 2 3 4 5 6 7 8 9 0 # $ ¢ a/c c/o % & &c &c

56 # 9 8 7 6 5 4 3 2 1 0 $ 1.4,6 7 9,0 1 2,3 5 8.00

Work

57 "Work that is not finished is not work at all; it is merely a botch, a failure."

LESSON 6

Conquest

58 "The conditions of conquest are always easy. We have but to toil a while, endure a while, believe always, and never turn back."

Think for thyself.

59 "Think for thyself - one good idea,
But known to be thine own,
Is better than a thousand gleaned
From fields by others sown."

Copy 51-54.—*Review:* The purpose of these lines is to review different capitals, including all of the direct oval letters and also *H* and *K*. Another purpose is to show how these letters may be formed into a signature, which is a very important thing to know. There is nothing more interesting and useful for practice. Observe that some of the capitals are joined together, which will be suggestive to the student in joining together the initials in his name. It is advisable for every one to make a study of his own signature, and when he has found one that he likes, one that is perfectly legible, he should make it a practice to drill on it until the result becomes a habit. There is but one way by which this can be accomplished—by always trying to write the combination the same way.

Copy 55.—*Figures and Characters:* The figures should be made a very little higher than the small letters. Make a careful study of the shape of the figures and characters. The figures, with the exception of the *6* and the flourish on the *8*, are all of the same height; the *7* and *9* descend a little below the base line. This copy should receive much practice and should be made with a fair rate of speed.

Copy 56.—*Review:* Observe that this is a review of the figures and two of the characters. They should be written with greater speed than the former.

Copy 57.—*Paragraph:* The purpose of this copy is to teach the mechanical form of a paragraph, which begins about $\frac{3}{4}$ inch to the right of the beginning of the lines, and also to express thought in well-formed letters.

LESSON 6

Copy 58.—*Indention:* The purpose of this copy is not to teach how to write a word or a sentence, but to show the indention of a paragraph. A person may be able to repeat the same sentence and make a creditable page, but may not be able to write a pleasing page composed of several sentences. This copy is composed of two sentences, with a number of punctuation marks, which should be made with

care. First, read and study the copy carefully from the standpoint of the whole, not from that of a single letter or word; then write it with confidence and with a free, smooth motion, which will produce a pleasing effect. When the work is examined and criticized, it will be judged by its appearance as a whole.

Copy 59.—*Stanza:* This copy contains another difficulty of mechanical arrangement, every other line being indented from $\frac{1}{2}$ to $\frac{3}{4}$ inch. Study the copy carefully and write it with the same points in view as in writing the previous paragraph.

LESSON 7

Copy 60.—*Receipt:* Business forms are of such importance in every-day life that their use in connection with the subject of penmanship requires no explanation.

A *receipt* is a written or printed statement or acknowledgment of the receipt of money or other value. The most common kinds are on account and in full of account. *On account* simply means that a part of the indebtedness has been received; *in full of account* means that the whole indebtedness has been canceled. A receipt may be given for rent, for a note, to apply on a partial payment of a note, for tuition, and for various other causes.

The writing of a receipt involves mechanical arrangement, capitalization, punctuation, good writing, and the thought expressed. One of the secrets of success in this work is in learning how to look at the copy to gain the essential points, and how to sift them from the non-essential.

Copy 61.—*Due Bill:* This copy is also a common business paper. It should be studied with the same thoughtful care as the receipt.

A *due bill* is a written acknowledgment of debt, and is usually payable in merchandise, but it can be made payable in money. It is more common to make it payable on demand than at some stated time. If it is desired to make it transferable to another person, or negotiable, the words "or order" should be inserted the same as in a promissory note.

LESSON 7

60

$960.20 Albany, N.Y., April 1, 190

Received of Wilson E. Thomas
Nine hundred sixty $\frac{20}{100}$ Dollars,
in full of account.

V. W. Utley.

61 $47.50 Lowell, Mass., Nov. 3, 190

Due C. G. Quinton, on demand,

Forty-seven $\frac{50}{100}$ Dollars, in goods

from my store.

John H. King.

LESSON 8

62 $2699.10 Scranton, Pa., May 4, 190

On demand I promise to pay
L. L. Brown, or order, Twenty six
hundred ninety-nine $\frac{10}{100}$ Dollars.
value received.

Wm. J. Hunter.

'63

Atlanta, Ga., June 1, 190

Mr. U. V. Wilson,

Bought of P. B. Rounds.

May	26	1000 bu. Oats,	.55	550	—		
"	27	250 " Wheat,	.61	152	50	702	50
		Paid.					

P. B. Rounds.

LESSON 8

Copy 62.—*Note:* This copy is one of the most common business forms and is called a promissory note. It should be studied in every detail and practiced until its form is impressed on the memory and the writing of it is a physical equipment of the student.

A *promissory note* is a written promise to pay some designated person a specified amount of money at a certain time. It may be made negotiable by using the words "or order" following the name.

Notes are indorsed on the back in the following ways: In blank, in full, receipt, and without recourse. Indorsing a note *in blank* means simply that the person in whose favor the note is drawn writes his name on the back of the paper, about one-fourth the distance from the end, directly opposite the beginning of the note; *in full* means that the person in whose favor the note was originally drawn writes "Pay to the order of" a third person; a *receipt* indorsement is one in which the holder of the note writes the words "Received on the within note," stating the sum, and writes his name below it; *without recourse* is simply writing "Without recourse" and below it the signature.

Great care should be given to the writing as well as to the other essential points.

Copy 63.—*Bill:* This is another very important business form. The one shown in the copy is receipted. Study every detail of it with great care. This is an excellent copy on figures, writing, punctuation, and signature.

LESSON 9

Copy 64.—*Business Letter:* We have now reached a full-page unit composed of two paragraphs and arranged in the most approved manner on letter paper 8 inches by 10 inches. There is nothing of more importance than to know how to write a creditable business letter. Many a position has been lost because the person has not been able to come up

to a fair standard in this simple, every-day matter of writing a letter. Others have received their first recognition on account of methodical, painstaking work in this line, and have risen to positions of trust and influence.

There are three elements that constitute a good letter: First, the ability to express the thoughts in simple, concise language; second, in being able to arrange sentences and paragraphs in such a manner as will conform to custom and good taste; third, in having power over the muscles so as to be able to execute a good handwriting. The copy has been arranged with these points in view, and it remains with the student to master them.

As a rule, the name of the place should begin practically in the middle of the line. On letter paper, the name of the person should begin $\frac{3}{4}$ inch to the right of the edge of the paper, and a little less on note paper. This arrangement leaves a liberal margin. The address under the name should be so arranged as to break the space and avoid having two lines begin or end at the same point. The words "Dear Sir" and all paragraphs should begin at a point $\frac{3}{4}$ inch to the right of the beginning of the name. The margin at the right should not be as much as at the left. "Yours truly" should be practically in the middle of the line, and the name should finish at a point directly under the finishing of the full lines.

Write this letter several times with a free, strong movement.

Copy 65.—*Superscriptions:* While these copies seem very easy to write, they will be found quite difficult when they are to be placed on an envelope and arranged so that the space will be broken in a manner pleasing to a cultivated taste. Three different superscriptions are given, which are arranged to illustrate the most common kinds in use, and also a visiting card. The name should be located in the middle of the envelope, from left to right, and also from top to bottom. The space below the name is divided into as many equal parts as there are lines, and of two successive lines neither the beginning nor the ending of the second is in a vertical line with the beginning or ending of the first. Inasmuch as

LESSON 9

64 Columbus, O., Feb. 22, 190

Mr. E. V. Lyman,

307 Vermont Ave.,

Washington, D.C.

Dear Sir:- Your letter of the 18th inst. is received.

I hand you herewith check on the Union Trust Co. for $14.50, which please place to my credit, and oblige,

Yours truly,

A.V. Rice.

SUPERSCRIPTION

65

Miss Lucy L. Sims,
Hampton,
Washington Co. N.Y.

Hon. A. E. Bryant,
Weston,
Vt.

Mr. R. K. Zaner,
404 Main St.,
Buffalo,
N.Y.

W. H. Worthington.

LESSON 10

66

Dear William:

Will you be so kind as to loan me, for a few days, "Outlines for the Study of Art"? By so doing you will greatly oblige,

Your friend,

Jno. H. Case.

Wednesday,
Aug. 14, 190

LESSON 10—(Continued)

67

B

My dear John:
I take great pleasure in sending you the desired book, and hope you will enjoy its perusal.
It has been a very profitable book to me.

Yours very truly,
Wm K. Brown.

Thursday Morning,
Aug. 15, 190

LESSON 11

Review

68

A B C D E F G H I J K L M

N O P Q R S T U V W X Y Z

a b c d e f g h i j k l m n

o p q r s t u v w x y z &c.

A quick brown fox jumps over the lazy dog.

69

Rapid Printing

A B C D E F G H I J K L M N O P

Q R S T U V W X Y Z &

a b c d e f g h i j k l m n o p q r s t u

v w x y z 1 2 3 4 5 6 7 8 9 0 ½ ⅔ ¾ ⅘ ⅞

A quick brown fox jumps over the lazy dog.

there are no two superscriptions that are exactly alike, it requires a good eye and much practice to adapt the matter to the space.

Write these copies with great care.

LESSON 10

Copy 66 and 67.—*Friendly Notes:* The purpose of these copies is to teach how to write a friendly note, how to answer one, and also how to arrange them on commercial note paper. First, observe the appearance of the whole page of each and then carefully analyze the arrangement of the different lines; second, study the language that expresses the thought; third, study the letter forms and combinations.

Such notes are written on commercial note paper or on the note paper that comes in boxes, with envelopes to match. The ordinary note paper is 5 inches by $7\frac{1}{2}$ inches, and is folded twice to fit the envelope, while that which comes in boxes varies in size and is folded once to fit the envelope; a popular size is $5\frac{1}{2}$ inches by 7 inches.

Write the note of request several times.

It will not be necessary to make the monogram shown at the top of the note.

LESSON 11

Copy 68.—*Review:* This copy is a review of all the capital and small letters separately, and gives a sentence containing all of the small letters of the alphabet in nearly all of the different combinations. Drill on each letter separately before practicing the entire copy. These specimens should show a quality of work superior to the others, both in form and movement, and they will be a test of the student's perseverance. Every line will be a picture of the touch and motion that produced it, and each letter and combination will be closely examined and criticized in a spirit of justice and with a view of benefit to the student.

Copy 69.—*Rapid Printing:* With the exception of the specimen page, the final copy of the series has now been

LESSON 12

To

International
Correspondence Schools.
Scranton, Pa.

Gentlemen:- This is a specimen of
my writing after completing your
course of lessons in penmanship.
Yours truly,

reached, which is a few lines of rapid printing in a style well adapted to business use. The forms and proportions are the same as in writing. It is necessary, in printing, to acquire a firm, steady touch of the pen, keeping the lines uniform in width, spacing, and height, and as nearly as possible of the same strength.

LESSON 12

Copy 70.—*Specimen:* This copy is nearly like the letter the student was asked to write before beginning practice. The work on this specimen should show that the eye has been trained to see better and the muscles to act better, which means greater skill.

Study this letter carefully and observe the arrangement of the different parts, paying particular attention to those to be added. Give it conscientious practice, for it should be an improvement over the first specimen.

A SERIES

OF

QUESTIONS AND EXAMPLES

RELATING TO THE SUBJECTS
TREATED OF IN THIS VOLUME.

It will be noticed that the Examination Questions contained in the following pages are divided into sections corresponding to the sections of the text of the preceding pages, so that each section has a headline which is the same as the headline of the section to which the questions refer. No attempt should be made to answer any questions or to work any examples until the corresponding part of the text has been carefully studied.

ARITHMETIC.

EXAMINATION QUESTIONS.

(1) What is arithmetic?

(2) In what two ways may numbers be represented?

(3) What is a number? a unit?

(4) What is the difference between a concrete number and an abstract number?

(5) What is the reading of a number called?

(6) Express, in both Arabic notation and Roman notation, seven thousand five hundred three.

(7) For what purpose are ciphers used?

(8) Write each of the following numbers in words: (*a*) 980; (*b*) 605; (*c*) 28,284; (*d*) 9,006,042; (*e*) 850,317,002; (*f*) 700,004.

(9) Express in Roman notation the following numbers: (*a*) 76; (*b*) 353; (*c*) 1,732; (*d*) 1,496; (*e*) 1,888.

(10) Represent in figures the following expressions: (*a*) Seven thousand six hundred; (*b*) eighty-one thousand four hundred two; (*c*) five million four thousand seven; (*d*) one hundred eight million ten thousand one; (*e*) ten million six; (*f* thirty thousand ten.

(11) Find the sum of 83,027 + 46,928 + 4,769 + 81,987 + 46,729 + 479,897 + 627 + 14,896 + 987,649. Ans. 1,746,509.

(12) Multiply 29,800 by 390. Ans. 11,622,000.

(13) A man owes \$1,000; he pays \$329 the first year and \$438 the second year. How much does he still owe?

Ans. \$233.

(14) Find the difference between $23,896 + 4,982 + 96,875 + 59,674$ and $31,627 + 54,892 + 6,925 + 8,976$. Ans. 83,007.

(15) Multiply 8,765 by 987, and from the product subtract $4,695 \times 823$. Ans. 4,787,070.

(16) The greater of two numbers is 1,004, and their difference is 49. What is their sum? Ans. 1,959.

(17) A drover bought 36 oxen at \$24 each and 23 cows at \$96 each. How much did they all cost? Ans. \$3,072.

(18) Divide $2,937 \times 864$ by 923. Ans. $2,749\frac{241}{923}$.

(19) Multiply 8,976 by 4,298, and subtract 98,765 from the product. Ans. 38,480,083.

(20) An engine and boiler in a manufactory are worth \$3,246; the building is worth three times as much, plus \$1,200; and the tools are worth twice as much as the building, plus \$1,875. (*a*) What is the value of the building and tools? (*b*) What is the value of the whole plant?

Ans. { (*a*) \$34,689. (*b*) \$37,935.

(21) The divisor is 1,389, the quotient is 748, and the remainder is 1,263. What is the dividend? Ans. 1,040,235.

(22) The multiplicand is 4,896, and the product, 3,862,944. Find the multiplier. Ans. 789.

(23) A man left \$25,375 to his widow and \$12,450 to each of his seven children. How much did they all receive?

Ans. \$112,525.

(24) If I sell my farm for \$375 an acre, I would get \$31,734 more for it than if it sold for \$246 an acre. How many acres in the farm? Ans. 246 acres.

(25) Solve the following by cancelation:

(a) $\frac{231 \times 96 \times 192 \times 45}{11 \times 24 \times 48 \times 15 \times 7} = ?$

(b) $\frac{42 \times 64 \times 125 \times 98 \times 144}{14 \times 16 \times 25 \times 49 \times 36 \times 20} = ?$

Ans. $\begin{cases} (a)\ 144 \\ (b)\ 24. \end{cases}$

ARITHMETIC.

EXAMINATION QUESTIONS.

(1) Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$. Ans. $4\frac{1}{24}$.

(2) What is the value of $3\frac{3}{4}+6\frac{7}{8}-4\frac{5}{6}$? Ans. $5\frac{19}{24}$.

(3) A man bought $12\frac{3}{4}$ tons of coal. How much was left after he had burned $5\frac{4}{5}$ tons? Ans. $6\frac{19}{20}$ tons.

(4) How many dresses can be made of $53\frac{1}{3}$ yards of gingham, if each dress contains $6\frac{2}{3}$ yards? Ans. 8 dresses.

(5) Find the product of $83\frac{2}{3}\times 67\frac{4}{5}$. Ans. $5,672\frac{3}{5}$.

(6) Divide $4\frac{1}{2}\times 6\frac{3}{4}$ by $1\frac{4}{5}$. Ans. $16\frac{7}{8}$.

(7) At $\$3\frac{7}{8}$ a yard, what must be paid for 24 yards of velvet? Ans. \$93.

(8) Simplify $\frac{4\frac{7}{8}\times 3\frac{2}{3}\times 3\frac{1}{5}}{1\frac{5}{8}\times 4\frac{2}{5}}$. Ans. 8

(9) Multiply $8\frac{1}{20}$ by $\frac{1}{40}$. Ans. $\frac{161}{800}$.

(10) Subtract from 100 the sum of $8\frac{3}{4}$, $7\frac{5}{6}$, and $10\frac{1}{2}$. Ans. $72\frac{11}{12}$.

(11) A cask containing 40 gallons leaks at the rate of $3\frac{3}{4}$ gallons an hour. In how many hours will it be empty? Ans. $10\frac{2}{3}$ hr.

(12) Find the value of $\frac{65}{100}\times 4\frac{2}{3}\times 6\frac{1}{2}$. Ans. $19\frac{43}{60}$.

(13) The circumference of a wheel is $\frac{355}{113}$ times its diameter. What is the circumference of a wheel whose diameter is $8\frac{2}{3}$ feet? Ans. $27\frac{77}{339}$ ft.

(14) A company of soldiers consumes $165\frac{3}{4}$ pounds of meat, which is an allowance of $1\frac{5}{8}$ pounds each. How many soldiers are in the company? Ans. 102 soldiers.

(15) In a field of $4\frac{4}{5}$ acres there were raised 96 bushels of wheat. What was the average number of bushels per acre? Ans. 20 bu.

(16) How much is $4\frac{3}{4}$ times $\frac{2}{3}$ of $6\frac{1}{2}$? Ans. $20\frac{7}{12}$.

(17) Find the cost of $24\frac{3}{4}$ tons of hay at $\$23\frac{3}{4}$ a ton. Ans. $\$587\frac{13}{16}$.

(18) Three men in partnership gain \$3,696. In dividing the profits, one gets $\frac{1}{3}$ of the amount, the second gets $\frac{2}{5}$ of the remainder, and the third gets what still remains. Find the share of each. Ans. \$1,232; $\$985\frac{3}{5}$; $\$1,478\frac{2}{5}$.

(19) A man paid $\$357\frac{1}{2}$ for $9\frac{3}{4}$ acres of land. What did he pay per acre? Ans. $\$36\frac{2}{3}$.

(20) The sum of two numbers multiplied by $18\frac{2}{3}$ is $296\frac{2}{3}$. One of the numbers is $12\frac{3}{4}$. Find the other number. Ans. $3\frac{1}{7}$.

(21) If $12\frac{7}{8}$ times the difference between two numbers is $41\frac{1}{5}$, and one of the numbers is 21, what is the other number? Ans. $24\frac{1}{5}$ or $17\frac{4}{5}$.

(22) What is the circumference of a wheel that goes 108 feet in turning $5\frac{3}{4}$ times? Ans. $18\frac{18}{23}$ ft.

(23) From $67\frac{5}{8}$ take $48\frac{2}{3}$. Ans. $18\frac{23}{24}$.

(24) If a man travels $85\frac{5}{12}$ miles in one day, $78\frac{9}{15}$ miles the next day, and $125\frac{17}{35}$ miles the third day, how far did he travel in the three days? Ans. $289\frac{211}{420}$ mi.

(25) Bought $211\frac{1}{4}$ pounds of old lead for $1\frac{7}{8}$ cents per pound. Sold a part of it for $2\frac{1}{2}$ cents per pound, receiving for it the same amount as I paid for the whole. How many pounds did I have left? Ans. $52\frac{13}{16}$ lb.

ARITHMETIC.

EXAMINATION QUESTIONS.

(1) Change $2\frac{1}{2} \times 3\frac{1}{3} \div 8$ to a decimal. Ans. 1.0417–.

(2) Reduce $2.04\dot{1}\dot{8}$ to a mixed number. Ans. $2\frac{23}{550}$.

(3) By the method of aliquot parts, multiply 448 by $62\frac{1}{2}$. Ans. 28,000.

(4) How many tons of coal at 2.87\frac{1}{2}$ a ton should be received in exchange for 128 cords of wood at 1.62\frac{1}{2}$ a cord and $436 in money? Ans. 224 T.

(5) A dealer bought peaches at 1.87\frac{1}{2}$ a basket, and sold them so as to lose $12.50 on 100 baskets. How much did he receive per basket? Ans. $1.75.

(6) Reduce to decimals and add the following fractions: $\frac{3}{4}$, $\frac{7}{8}$, $\frac{11}{16}$, $\frac{17}{32}$. Ans. 2.84375.

(7) If the diameter of the earth is 7,912 miles and its circumference is 3.1416 times as much, how many miles are in its circumference? Ans. 24,856.3392 mi.

(8) Find the value of $\frac{3}{5}$ of $\frac{2}{3}$ of .0168. Ans. .00672.

(9) The sum of .37 of a number and .23 of the same number is 33.6. Find the number. Ans. 56.

(10) If $\frac{2}{3}$ of a number be subtracted from .8 of the number, the remainder is 87.8. What is the number? Ans. 658.5.

(11) If \$8.20 more is paid for $14\frac{5}{8}$ yards of cloth than for $9\frac{1}{2}$ yards, what is the price per yard? Ans. \$1.60.

(12) Change 4,620 feet to the decimal of a mile.
Ans. .875 mi.

(13) Find the cost of 6 barrels of flour at \$6.375 a barrel, 28 bushels of potatoes at \$.875 per bushel, and 120 pounds of sugar at $\$.03\frac{1}{8}$ a pound. Ans. \$66.50.

(14) A lot cost \$3,300, and this was .165 of the cost of the house erected upon it. Find what was paid for the house.
Ans. \$20,000.

(15) A man whose daily pay was $\$1.87\frac{1}{2}$ had his wages advanced to \$2.25. Counting 300 working days in a year, find the annual increase in his salary. Ans. \$112.50.

(16) A man paid .143 of his annual salary to his butcher, .347 of it to his grocer, .256 of it for clothing, and .154 for other expenses. How much did he save, if his butcher received \$327.47? Ans. \$229.

(17) A farm was bought at \$87.50 per acre and sold at \$112.50, by which there was a gain of \$3,670. How many acres were there? Ans. 146.8 A.

(18) If $.\dot{1}\dot{8}$ of my crop of wheat is 368.5 bushels, how many bushels are in the whole crop? Ans 2,026.75 bu.

(19) If a cubic inch of water weighs .03617 pound, what will be the weight of 231 cubic inches? Ans. 8.355+ lb.

(20) Using the method of aliquot parts, divide (*a*) 475 by $12\frac{1}{2}$; (*b*) 25 by $16\frac{2}{3}$.
Ans. $\begin{cases} (a) & 38. \\ (b) & 1\frac{1}{2}. \end{cases}$

(21) Reduce $\frac{5}{13}$ to a repeating decimal. Ans. $.\dot{3}8461\dot{5}$.

(22) Reduce $.\dot{3}5\dot{1}$ to a common fraction. Ans. $\frac{13}{37}$.

(23) Change $.30208\dot{3}$ to a complex fraction whose denominator is 24. Ans. $\frac{7\frac{1}{4}}{24}$.

(24) Express (approximately) .7292 as a common fraction whose denominator is 40. Ans. $\frac{29}{40}$.

(25) Find the value of the following expression when the result is carried to three decimal places:

$$\frac{74.26 \times 3.1416 \times 19.5 \times 19.5 \times 350}{33{,}000 \times 12 \times 4}.$$

Ans. 19.601+.

ARITHMETIC.

EXAMINATION QUESTIONS.

(1) Change .67 of a mile to integers of lower denomination. Ans. 214 rd. 2 yd. 7.2 in.

(2) How many miles apart are two points on the earth's equator, one being at 23° 28′ west longitude, and the other at 5° 22′ east longitude? (See example 9.) Ans. 1,994.11+ mi.

(3) How many yards of lining $\frac{7}{8}$ of a yard wide will be required to line a piece of goods 28 yards long and $\frac{3}{4}$ of a yard wide? Ans. 24 yd.

(4) What must be paid for a rectangular block of granite, its dimensions being 3 ft. $\times 4\frac{1}{2}$ ft. $\times$ 6 ft., at \$1.37$\frac{1}{2}$ per cubic foot? Ans. \$111.375.

(5) Find to four decimal places the number of times that 2 ft. 8 in. must be repeated to be equal to the circumference of a flywheel 12 feet in diameter. Ans. 14.1372.

(6) Change $\frac{5}{8}$ of a year (365 da. 5 hr. 48 min. 49.7 sec.) to integers of lower denomination.

Ans. 228 da. 6 hr. 38 min. 1.0625 sec.

(7) If 5 T. 875 lb. of coal cost \$9.13$\frac{1}{2}$, how much is it a ton? Use the long ton. Ans. \$1.69+.

(8) How many 8-inch furrows, each 40 rods long, are equivalent to an acre? Ans. 99.

(9) Assuming that a degree of latitude is 69.16 miles, how far from the equator is a place whose latitude is 42° 20′ 30″? Ans. 2,928.3497– mi.

(10) A cubical cistern is 8 ft. 6 in. in each of its three dimensions. How many gallons will it hold? Ans. 4,594– gal.

(11) Find the exact time in years and days between Jan. 14, 1898, and July 23, 1900. Ans. 2 yr. 190 da.

(12) What decimal fraction of an entire circle in 60° 40′ 30″? Ans. .1685+.

(13) How many bushels will a bin contain whose dimensions are 6 ft. $\times 6\frac{1}{2}$ ft. $\times$ 8 ft.? Ans. 250.7+ bu.

(14) A family used a quantity of sugar at the rate of $8\frac{3}{4}$ ounces per day and it lasted 80 days. How many pounds were there in all? Ans. $43\frac{3}{4}$ lb.

(15) Find the difference between 25 ft. square and 75 sq. ft. Ans. 550 sq. ft.

(16) A rectangular stick of lumber 36 ft. long and 10 in. thick contains 40 cu. ft. How wide is it? Ans. 16 in.

(17) A vessel having a capacity of 100 bushels will hold how many gallons? Ans. 930.92– gal.

(18) (*a*) How many square feet in a board 18.75 ft. long and 2 ft. 10 in. wide? (*b*) What will it cost at 8 cents a square foot? Ans. (*a*) 53.125 sq. ft. (*b*) $4.25.

(19) (*a*) What fractional part of 63 gallons is 18 gal. 3 qt. 1 pt.? (*b*) What decimal part? Ans. (*a*) $\frac{151}{504}$. (*b*) .2996+.

(20) A grocer bought 6 bushels of cranberries and sold them at 25 cents for 3 quarts. His gain was $4. What did they cost him per bushel? Ans. $2.

(21) A student spent 8 hours and 40 minutes daily in study for 4 years. How much time did he study in all, if school was in session 5 days each week for 40 weeks each year? Ans. $6{,}933\frac{1}{3}$ hr.

(22) From £21 5 s. 3 d. take £13 9 s. 6 d. Ans. £7 15 s. 9 d.

(23) If 16 square miles be equally divided into 62 farms, how much land will each farm contain?

Ans. 165 A. 25 sq. rd. 24 sq. yd. 3 sq. ft. 80+ sq. in.

(24) How many bushels of apples are contained in 9 barrels, if each barrel contains 2 bu. 3 pk. 6 qt.?

Ans. 26.4375 bu.

(25) Reduce 4,763,254 links to miles.

Ans. 595 mi. 32 ch. 2 rd. 4 li

ARITHMETIC.

EXAMINATION QUESTIONS.

(1) How many kilos of water will a tank contain that is 3 m. $\times$ 2 m. $\times$ 3 m.? Ans. 18,000 K.

(2) Find the cost of 1.5 tonnes of sugar at $2\frac{1}{2}$ cents a pound. Ans. \$82.67+.

(3) How much will 25.875 kilograms of opium cost at 10 cents a grain? Ans. \$39,930.30.

(4) At 25 cents a square yard, find how much must be paid to plaster a ceiling 12 meters long and 10 meters wide. Ans. \$35.88.

(5) What would be the value of the 5-cent nickel coins that, placed side by side, would make a row 50 meters long, the diameter of the 5-cent nickel coin being 2 centimeters? Ans. \$125.

(6) Find the cost, at 12 cents a quart, of 1,000 liters of cranberries. Ans. \$108.94–.

(7) Change 56.8 meters to yards. Ans. 62.1+ yd.

(8) Find the value of $\sqrt{32^2 + 255^2}$. Ans. 257.

(9) When $x = 10$, $y = 4$, $z = 6$, $a = 3$, $b = 5$, $c = 8$, what is the value of $\frac{5(xyz - abc)}{bc - ax}$? Ans. 60.

(10) What number must be squared and added to the square of 161 so that the square root of the sum shall be 289? Ans. 240.

(11) How many hektares of land in a field 40 rods square?
Ans. 4.0469+ Ha.

(12) Find, to five figures, the value of $\sqrt[3]{87.54} \div \sqrt{12.1}$.
Ans. 1.2765—.

(13) Find, in inches, the length of an edge of a cubical tank whose capacity is 4,913 liters. Ans. 66.929 in.

(14) What is the length in meters of a side of a square having an area of 40 acres? Ans. 402.34+ m.

(15) What is the value of $(43\frac{1}{2})^2 + (4\frac{2}{3})^3$?
Ans. 1,993.879+.

(16) When $a = 2$, $b = 6$, and $c = 3$, what is the value of $\sqrt[3]{72a^2bc^3}$? Ans. 36.

(17) Find the number of liters in 100 gallons of water.
Ans. 378.546— l.

(18) A gold eagle of the United States weighs 258 grains Troy; what would be the value of a kilo of these coins?
Ans. \$598.14—.

(19) If a cubic foot of marble weighs 168.75 lb., what will be the weight of a marble block, the dimensions of which are 2.5 m. × 2.8 m. × 3.75 m.?
Ans. 156,432 lb., nearly.

(20) What is the value of $\sqrt{7,921} \times \sqrt{9,604}$? Ans. 8,722.

(21) Find the cube root of $\frac{7}{8}$ to five decimal figures.
Ans. .95647—.

(22) What is the nearest whole number of city lots, 25 ft. by 100 ft., that are equivalent to a hektare of land?
Ans. 43 lots.

(23) A rectangular field 100 m. long contains 1 acre. How wide is it? Ans. 8.0488— rd.

(24) A fish weighing 50 Kg. was bought at 30 cents a Kg. and sold at 18 cents a pound. Find the gain or loss.

Ans. $4.84+ gain.

(25) Knowing that the longest side of a right triangle is equal to the square root of the sum of the squares of the other two sides, find the longest side of such a triangle of which the other sides are 52 and 675 feet. Ans. 677 ft.

ARITHMETIC.

EXAMINATION QUESTIONS.

(1) A square has the same area as a rectangle having a base of $56\frac{1}{3}$ rd. and an altitude of $16\frac{1}{3}$ rd. Find the side of the square. Ans. $30\frac{1}{3}$ rd.

(2) The perimeter of a field is 600 yd., and its length is 80 yd. more than its width. How many acres in the field? Ans. 4.32– A.

(3) The base of a right-angled triangle is 72 in. and its area is 2,340 sq. in. What are its other two sides? Ans. 65 in. and 97 in.

Suggestion.—Area $= \frac{1}{2} \times$ base $\times$ altitude; but in a right-angled triangle, the altitude is equal to the length of one of the short sides, the other side being the base. Hence, $2,340 = \frac{1}{2} \times 72 \times$ altitude $= 36 \times$ altitude, from which altitude $= 2,340 \div 36$. Knowing the length of the two short sides, find the hypotenuse.

(4) The diagonal of a square is 100 in.; find its sides. Ans. 70.71+ in.

Suggestion.—The diagonal divides a square into two right-angled triangles, the short sides of which are equal and the diagonal is the hypotenuse. Let l represent the length of one of the short sides. Then, according to Art. **58**, $l^2 + l^2 =$ (hypotenuse)^2; hence, $2l^2 = 100^2$, or $l = \sqrt{100^2 \div 2}$.

(5) What is the diameter in rods of a circle, the area of which is 2 A.? Ans. 20.185+ rd.

(6) How much greater is the perimeter of a square containing 5 A. than the circumference of a circle having the same area? Ans. 12.87+ rd.

(7) How many revolutions will a locomotive wheel 6 ft. in diameter make in going from New York to Philadelphia, a distance of 90 mi.? Ans. 25,210.08+ revolutions.

(8) Find the convex surface of a cylinder 24 in. long and 18 in. in diameter. Ans. 1,357.1712 sq. in.

(9) A regular hexagonal prism is 8 in. on each edge of its base, and 12 in. high. Find its convex surface.

Ans. 576 sq. in.

(10) Find the volume of a frustum of a cone, the diameter of its lower base being 20 in., its upper base 18 in., and its altitude 2 ft. Ans. 6,810.989– cu. in.

(11) What is the length of a side of a square equal in area to two squares, one being 7 ft. square, and the other 10 ft. square? Ans. 12.207– ft.

(12) A cylindrical tank 12 ft. high and 40 in. in diameter is full of oil worth 8 cents a gallon. What is the value of the oil? Ans. $62.67–.

(13) What is the capacity in heaped bushels of a bin 12 ft. by 8 ft. by 6 ft.? Ans. 362.24– bu.

(14) (*a*) Find the area of the largest square that can be cut from a circle of zinc 48 in. in diameter. (*b*) How much zinc is cut off? Notice that the diameter is the diagonal of the square.

Ans. $\begin{cases} (a) \ 1{,}152 \text{ sq. in.} \\ (b) \ 657.5616 \text{ sq. in.} \end{cases}$

(15) A circle having a radius of 24 ft. encloses another circle having a radius of 15 ft. How much of the larger circle lies without the inner circle? Ans. 1,102.7016 sq. ft.

(16) A cubical cask 8 ft. 6 in. each way is full of oil. A faucet capable of discharging an average of 5 gal. per minute is opened. In what time will the cask be emptied?

Ans. 15 hr. 18.795– min.

(17) A man wishes to make a cubical bin that will hold 1,000 bushels of wheat. What must be its dimensions?

Ans. Length of each edge, 10.75+ ft.

(18) A cask has a head diameter of 50 in., a bung diameter of 56 in., and a length of 6 ft. How many gallons will it hold?

Ans. 687.64+ gal.

(19) In a cubical room 14 ft. each way, how long is the line that will reach from a lower corner to the diagonally opposite upper corner?

Ans. 24.25– ft.

(20) What is the volume of the sphere that will exactly fit inside a cubical box having a capacity of 8 cubic feet?

Ans. 4.1888 cu. ft.

(21) How many tons of Schuylkill coal will fill a car 40 ft. long and $6\frac{1}{2}$ ft. wide to a depth of $4\frac{1}{2}$ ft.?

Ans. 33.4+ tons.

(22) What must be paid, at $4\frac{3}{4}$ cents per square yard, for kalsomining the walls and ceilings of 4 rooms, each having a height of 9 ft. 4 in., the dimensions of the rooms being as follows: $12' \times 14'$, $15' \times 15'$, $16' \times 18'$, and $15' \times 19'$?

Ans. \$17.31+.

(23) How many single rolls and how many double rolls of paper would be required to paper the walls of a room 16 ft. by 20 ft. 6 in., allowing for a baseboard 7 in. high, 5 doors 3 ft. by 7 ft., and 3 windows 3 ft. by 6 ft., the walls of the room being 11 ft. high? Also, how many double rolls of border 18 in. wide are required?

Ans. $\begin{cases} \text{7 double rolls.} \\ \text{17 single rolls.} \\ \text{2 double rolls border.} \end{cases}$

(24) At \$1.25 per yard, what will be the cost of carpeting a room 15 ft. by 17 ft., with Brussels carpet, allowing 1 foot on each strip for waste and matching?

Ans. \$52.50.

(25) What would be the cost of erecting the walls of a building 145 ft. by 75 ft., and 30 ft. high; the walls being 3 bricks (= 1 foot) thick, allowance to be made for 110 windows, each 7 ft. by 3 ft. 6 in.; for 4 doors, each 8 ft. by 10 ft.; and 2 doors, each 6 ft. by 8 ft.; the bricks to cost \$5.60 per M, and the laying to cost \$1.45 per M?

Ans. $\begin{cases} \$1,299.23+ \text{ for bricks.} \\ \$336.41- \text{ for laying.} \end{cases}$

ARITHMETIC.

EXAMINATION QUESTIONS.

(1) Divide: (*a*) the ratio of $\frac{3}{4} : \frac{4}{5}$ by 5; (*b*) the ratio of .2 : .05 by 4; (*c*) the inverse ratio of 12.5 : 125 by $\frac{3}{2}$.

Ans. $\begin{cases} (a) & \frac{3}{16}. \\ (b) & 1. \\ (c) & 6\frac{2}{3}. \end{cases}$

(2) Multiply: (*a*) the ratio of $\frac{4}{3} : \frac{3}{4}$ by 3; (*b*) the ratio of the volume of a gallon to a cubic foot by 27; (*c*) the inverse ratio of $33\frac{1}{3} : 187\frac{1}{2}$ by 4.

Ans. $\begin{cases} (a) & 5\frac{1}{3}. \\ (b) & 3.6+. \\ (c) & 22.5. \end{cases}$

(3) What is the square root of the inverse ratio of (*a*) 7.2 : 5? What is the cube root of the inverse ratio of (*b*) 16 : 54?

Ans. $\begin{cases} (a) & \frac{5}{6}. \\ (b) & 1.5. \end{cases}$

(4) Explain (*a*) why the following is not a correct proportion, 20 : 23 = 39 : 45; (*b*) how much must be added to 23 to make it correct? Ans. (*b*) $\frac{1}{13}$.

(5) Express in integers the ratio of 2 lb. 3 oz. to 5 lb. $1\frac{1}{2}$ oz. Ans. 70 : 163.

(6) Which is greater, and how much, 3 : 8 or 5 : 12?

Ans. The latter, $\frac{1}{24}$.

(7) Find the value of x in

$$\begin{array}{c|c|c} \frac{12}{15} & \frac{3}{5} & \\ \frac{2}{3} & \frac{5}{9} = x & 20. \\ 7 & 10 & \end{array}$$

Ans. $x = 22.4$.

(8) If the prices of grindstones vary directly as their weights and their weights vary as their thickness multiplied by the squares of their diameters, what must be paid for a grindstone 4 feet in diameter and 9 inches thick when $5 is paid for one that is 2 feet in diameter and 5 inches thick?

Ans. $36.

(9) The temperature being constant, the volume of a given weight of air varies inversely as the pressure of the air. If a pound of air has a volume of 10 cubic feet at a pressure of 17.8 pounds per square inch, what will be the volume if the pressure is 67.2 pounds per square inch, the temperature remaining the same? Ans. 2.65 cu. ft., nearly.

(10) If 5 be added to the antecedents of the proportion, $2 : 5 = 6 : 15$, what change of the first consequent will correct the proportion? What change of the second consequent?

Ans. $\begin{cases} 4\frac{6}{11} \text{ must be added.} \\ 7\frac{1}{7} \text{ must be subtracted.} \end{cases}$

(11) If a cylinder whose diameter is to its height as $3 : 5$ will hold 100 gallons, how many gallons will a cylinder hold whose diameter in the same units is to its height as $5 : 9$? (See Art. **50.**) Ans. 500 gal.

(12) Two cylinders have equal volumes. The diameter of the first is 15 in. and its length is 49 in.; what is the diameter of the second if its length is 36 in.? Ans. 17.5 in.

(13) What added to the first term, or what subtracted from the second term, will form from $3 : 7 = 8 : 12$ a correct proportion? Ans. $1\frac{2}{3}$; $2\frac{1}{2}$.

(14) If 5 men in 9 days of 10 hours each can build a wall 100 feet long, 6 feet high, and 18 inches thick, in how many days of 9 hours each will 8 men build a wall 120 feet long, 8 feet high, and 20 inches thick? Ans. 11 da. 1 hr.

(15) The time of oscillation of a pendulum varies directly as the square root of the length of the pendulum. If at a

certain place a pendulum 39 inches long oscillates once in a second, how long must a pendulum be to oscillate once in two seconds at the same place? Ans. 156 in.

(16) By the method of proportional parts, divide a gain of $4,750 among three men in the ratios of 2, 3, and 5.

$$\text{Ans.} \begin{cases} \$950, \\ \$1{,}425, \\ \$2{,}375. \end{cases}$$

(17) Arrange the fractions $\frac{2}{3}$, $\frac{5}{7}$, $\frac{15}{28}$, $\frac{1}{2}$, in a correct proportion and change to a form containing no fractions. Prove your work to be correct.

(18) Knowing that the amounts of water that can be discharged through pipes vary as the times, the velocities of flow, and the squares of the diameters of the pipes, find how much water will flow through a 5-inch pipe in 2 hours with a velocity of 44 feet per second, if a 4-inch pipe will discharge 5,385.6 gallons in 50 minutes, when the flow is at the rate of 33 feet per second. Ans. 26,928 gal.

(19) Divide 100 into parts proportional to $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.

$$\text{Ans.} \begin{cases} 26\frac{2}{23}, \\ 34\frac{18}{23}, \\ 39\frac{3}{23}. \end{cases}$$

(20) A fish 20 inches long weighs 10 pounds. Find the weight of a fish of similar shape 30 inches long, knowing that their weights vary as the cubes of their lengths.

Ans. $33\frac{3}{4}$ lb.

(21) Knowing that the interest for $250 for 2 years 6 months at 8% is $50, find by proportion the interest of $450 for 3 years 10 months at 6%. Ans. $103.50.

(22) The distances through which bodies will fall vary as the squares of the times during which they fall. If a body falls 144 feet in 3 seconds, how far will it fall in 5 seconds?

Ans. 400 ft.

(23) Knowing that the weights of spheres of similar material vary as the cubes of their radii, and that a sphere of lead 12 inches in diameter weighs 371.75 pounds, find the weight of a leaden sphere 30 inches in diameter.

Ans. 5,808.59+ lb.

(24) If a pendulum 39 inches long oscillates once per second, in what time will a pendulum oscillate that is 20 inches long? (See example 15.) Ans. .7161+ sec.

(25) The capacities of bins vary as the products of their three dimensions. Divide 3,700 bushels of grain among three bins in proportion to their capacities, their dimensions being as 2, 3, and 5; 3, 4, and 6; and 4, 5, and 6.

$$\text{Ans.} \begin{cases} 500 \text{ bu.} \\ 1{,}200 \text{ bu.} \\ 2{,}000 \text{ bu.} \end{cases}$$

ARITHMETIC.

EXAMINATION QUESTIONS.

(1) Express decimally: $\frac{1}{2}\%$, $\frac{3}{4}\%$, $1\frac{3}{8}\%$, $\frac{5}{8}$ of 1%, $\frac{2}{3}$ of $\frac{3}{4}\%$.

(2) A farmer planted $6\frac{1}{4}\%$ as many potatoes as his crop amounted to; this was 1,575 bushels more than he planted. How many bushels in his crop? Ans. 1,680 bu.

(3) I pay $37\frac{1}{2}\%$ of my month's salary to my grocer, $12\frac{1}{2}\%$ of it to my butcher, and $16\frac{2}{3}\%$ of it for other expenses, and then have $90 of it left. How much do I get per year?

Ans. $3,240.

(4) A merchant bought some goods at 20% below list price, and sold them so as to gain 30%. At what per cent. above list price did he sell them? Ans. 4%.

(5) Two horses were sold at $120 each. If by the transaction 25% was gained on one and 25% lost on the other, how much was gained or lost? Ans. $16 lost.

(6) A bought a horse and sold it to B at an advance of 20%; B sold it to C and gained 25%. What did each man pay for it if it cost C $60 more than it cost A?

Ans. A, $120. B, $144. C, $180.

(7) In a certain specimen of lead ore composed of lead and sulphur, the ratio of lead to sulphur is 206.95 : 32.06. Find the per cent. of each ingredient.

Ans. Lead, 86.5363+%. Sulphur, 13.4137−%.

(8) A speculator, after losing 35% of his money and 10% of the remainder, had \$17,550 left. How much did he have at first? Ans. \$30,000.

(9) A man earned 20% more during the second of three years than he did the first, and 25% more during the third year than he earned the second. In the three years he earned in all \$7,400. What sum did he earn each year?

Ans. { First year, \$2,000. Second year, \$2,400. Third year, \$3,000.

(10) If I should sell my horse for \$140, my gain would be 12%; for what must I sell him to gain 28%? Ans. \$160.

(11) What is the ad valorem duty at 22% on 18,600 yards of silk invoiced at 8 francs per yard? Ans. \$6,317.96.

(12) A commission merchant sold some goods for \$8,407.96. His commission was 2%, for guaranteeing payment he charged $1\frac{1}{2}$%, and his other charges were \$275; how much should he send his principal? Ans. \$7,838.68.

(13) Find A's tax, knowing that the tax rate is $12\frac{1}{2}$ mills on 100 cents, that his assessment on real estate is \$7,800, on personal property \$5,640, and that he pays for 3 polls at \$.75 each. Ans. \$170.25.

(14) How much better on a bill amounting to \$987.50 is a discount of 40% than discounts of 20% and 20%?

Ans. \$39.50.

(15) A boy buys oranges, and always receives for 3 of them as much as he pays for 5. What is his gain per cent.? Give reasons for your solution. Ans. $66\frac{2}{3}$%.

(16) If the premium for insuring a store is \$267 when the rate is $1\frac{1}{2}$%, what is the amount of insurance?

Ans. \$17,800.

(17) When the rate is $\frac{3}{4}$ of 1%, and the amount to be insured is \$75,375, what will be the premium?

Ans. \$565.31+.

(18) After deducting $2\frac{1}{2}\%$ commission and $\frac{1}{4}\%$ for insurance, a commission merchant remitted \$6,613 as the proceeds from a sale of peaches. For how much were the peaches sold? Ans. \$6,800.

(19) A commission merchant received \$520.45 to buy peaches after deducting expenses. Drayage cost \$18.75, storage, \$8.50, and his commission was $2\frac{3}{4}\%$. At \$.75 a crate, how many crates did he buy? Ans. 640.

(20) The value of a store is to that of its contents as 9 : 11. If both are insured for $\frac{4}{5}$ of their value when the rate of premium is $1\frac{1}{2}$ cents on a dollar, the premium is \$372. Find the value of each. Ans. { Contents, \$17,050. Store, \$13,950.

(21) A merchant purchased a bill of goods, on which he received a serial discount of 20%, 15%, and 5%. If, instead, he had been given a discount of 20%, 10%, and 10%, his bill would have been \$25.48 more. Find the amount of the bill without discount. Ans. \$12,740.

(22) A life-insurance company took a risk at $\frac{7}{8}\%$ annual premium for \$25,000. The company reinsured 28% of their risk at $\frac{1}{2}\%$ in a second company, and 48% of it at $\frac{3}{4}\%$ in a third company. What was the annual gain of the first company by reinsuring? Ans. \$41.25.

(23) A discount of 20% on $\frac{3}{4}$ of the amount of a bill, and 15% on the remainder is \$324.50 better than a discount of 16% on the whole bill. Find the amount of the bill.

Ans. \$11,800.

(24) A dealer receives \$33.90 more for a piano by allowing a serial discount of 20%, 10%, and 10% instead of one of 25%, 15%, and 5% from the list price. What is the list price?

Ans. \$800.

(25) A merchant sold 20% of his stock at a gain of 20%, 30% of the remainder at 1.25 of its cost, and what still remained he sold at $\frac{7}{8}$ of its cost. Did he gain or lose, and what per cent.? Ans. 3% gain.

ARITHMETIC.

EXAMINATION QUESTIONS.

(1) By the method of Art. **12**, find the interest of \$5,628.40 at $3\frac{4}{5}$% for 5 yr. 7 mo. 29 da. Ans. \$1,211.39.

(2) By the six-per-cent. method, find the interest of \$5,670.80 for 1 yr. 11mo. 27 da. at $4\frac{1}{2}$%. Ans. \$508.25.

(3) By the 60-day method, find the interest of \$4,689.50 at 6% for 87 days. Ans. \$68.00.

(4) Find, in the shortest way, how much more the interest of \$6,600 will be for 3 yr. 8 mo. 15 da. at $5\frac{1}{2}$% than at $4\frac{2}{3}$%.
Ans. \$203.96.

(5) If, in finding the interest at 6% of any sum of money, the multiplier is .2365, what is the time?
Ans. 3 yr. 11 mo. 9 da.

(6) How much more at 6% is the exact interest of a debt of \$100,000 for February, 1896, than for the same month in 1897? Ans. \$15.14.

(7) In 3 yr. 5 mo. 18 da. at $4\frac{1}{2}$%, what part of the principal equals the interest? Ans. .156.

(8) Find the exact interest at 7% of \$928.60 from April 8, 1899, to October 17, 1899. Ans. \$34.19.

(9) What is the compound interest of \$3,690 for 3 yr. 7 mo. 20 da. at 6%, interest being compounded semiannually?
Ans. \$886.05.

(10) What is the difference between the exact and the ordinary interest of $3,600 at 6% for 270 days of a common year? Ans. $2.22.

(11) The amount of a certain principal at 5% for 2 yr. 3 mo. 18 da. is $535.20; what is the principal? Ans. $480.

(12) What is the present worth of $1,260 due in 6 mo. 24 da., money being worth 5% a year? Ans. $1,225.28.

(13) Find the compound interest of $240 at 5% for 4 yr. 5 mo. 20 da. Ans. $58.61.

(14) What is the face of a 90-day note that will yield $1,200 proceeds when discounted at 6%? Ans. $1,218.89.

(15) Find the bank discount of a note for $2,400, due in 60 days, the rate of discount being 6%. Ans. $25.20.

(16) Write, in proper form, a 90-day note which, when discounted at a bank at 5%, will yield $6,000 proceeds. Give the work for finding the face. Ans. Face, $6,078.51.

(17) A 90-day note for $1,200, dated July 1, 1900, and bearing interest at 8%, was discounted at 6% at a bank, Sept. 1, 1900. Find the proceeds. Ans. $1,218.47.

(18) In what time will $860 give $139.32 interest, at 6%? Ans. 2 yr. 8 mo. 12 da.

(19) A note for $2,920, bearing interest at 5%, is dated April 1, 1900. At what date will the exact interest amount to $58.40? Ans. Aug. 25, 1900.

(20) A demand note for $12,000 with interest at 6% is dated May 10, 1894, and bears the following indorsements: Sept. 1, 1894, $200; Jan. 9, 1895, $960; Mar. 1, 1895, $150; May 1, 1895, $500. Settled, Sept. 1, 1895. Which is better for the lender, and by which will he receive the more interest —the merchants' rule or the United States rule, exact interest being computed? Ans. U. S. rule, $28.02.

(21) At what rate per cent. will $1,280, in 2 yr. 5 mo. 24 da., give $148.31 interest? Ans. $4\frac{2}{3}\%$, nearly.

(22) The bank discount on a 30-day note, bearing interest at 8%, is $66, the rate of discount being 6% per annum. Find the face of the note, if the note was discounted on the day it was made. Ans. $11,912.64.

(23) When interest is to be paid annually at 6%, but is not paid, what is due, in 5 yr. 7 mo. 24 da., on a note for $8,000? Ans. $11,093.60.

(24) I lend equal sums to each of two men, for which one pays me $5\frac{1}{2}\%$ and the other $4\frac{1}{3}\%$ per annum. At the end of 1 yr. 9 mo. 18 da., the interest due me from the first exceeds the interest due from the second by $42. What did I lend to each? Ans. $2,000.

(25) If the interest of $\frac{3}{4}$ of A's money added to that of $\frac{2}{3}$ of B's in 4 yr. at $5\frac{1}{2}\%$ is $1,320, how much has each, if $\frac{3}{4}$ of A's equals $\frac{2}{3}$ of B's? Ans. $\begin{cases} \text{A, \$4,000.} \\ \text{B, \$4,500.} \end{cases}$

ARITHMETIC.

EXAMINATION QUESTIONS.

(1) How much must be invested in stocks paying $3\frac{1}{2}\%$ semiannual dividends to yield a yearly income of \$5,600, if they are bought at $108\frac{1}{2}$, brokerage being $\frac{1}{8}\%$?

Ans. \$86,900.

(2) A broker at a commission of $\frac{1}{4}\%$ receives \$50 for buying stock at $110\frac{1}{2}$. (*a*) How many shares does he buy? (*b*) What do they all cost without brokerage?

Ans. $\begin{cases} (a) & 200. \\ (b) & \$22{,}100. \end{cases}$

(3) What will be the cost of 125 shares of bank stock bought at $114\frac{1}{8}$, brokerage being $\frac{1}{8}\%$? Ans. \$14,281.25.

(4) How much must be invested in stock paying a quarterly dividend of 2% to yield an annual income of \$4,800, their market price being $112\frac{3}{8}$ and brokerage $\frac{1}{8}\%$?

Ans. \$67,500.

(5) Which is better, and by how much per cent.—to invest in stock at 110 that pays an annual dividend of 10%, or in stock at 90 that pays 8% a year?

Ans. The former, by $\frac{20}{99}\%$.

(6) What must be paid for a sight draft for \$5,600 on New Orleans when exchange is at $1\frac{1}{8}\%$ premium?

Ans. \$5,663.

(7) What will be the cost of a draft for $8,000 payable in 60 days after sight, exchange being at $1\frac{1}{2}\%$ premium, and money being worth 6% interest? Ans. $8,036.

(8) Find the cost of a 90-day draft for $6,800 when money is worth 5% interest and exchange is at $1\frac{1}{4}\%$ discount.
Ans. $6,627.17.

(9) Exchange being at $\frac{3}{4}\%$ premium, find the cost of a sight draft for $2,800. Ans. $2,821.

(10) Find the face of a 60-day draft costing $12,000 when exchange is at $1\frac{1}{2}\%$ discount and money is worth 6% interest.
Ans. $12,314.01.

(11) What is the cost in New York of a draft on London for £400 15 s. when exchange is $4.855? Ans. $1,945.64.

(12) What must be paid in New York for a draft on Paris for 20,000 francs when $1 is equal in exchange to 5.22 francs? Ans. $3,831.42.

(13) How many bushels of oats are worth 30 bushels of wheat, when 3 bushels of corn are worth 5 bushels of oats, 7 bushels of corn are worth 4 bushels of rye, and 8 bushels of rye are worth 5 bushels of wheat? Ans. 140 bu.

(14) A in 4 days can earn as much as B in 5 days, B in 3 days as much as C in 7 days, C in 6 days as much as D in 11 days, and D in 5 days as much as E in 9 days. How much will A earn while E earns $100? Ans. $962.50.

(15) Four men, A, B, C, and D, engage in a bicycle race of 50 miles. A, who wins, goes 12 miles while B goes 11.5, B goes 10 miles while C goes $9\frac{1}{2}$, and C goes 9 miles while D goes $8\frac{1}{2}$. How far has D gone at the end of the race?
Ans. 42.99 mi.

(16) In what time will the interest of $1,000 be the same as the interest of $800 for 30 days, $500 for 20 days, and $400 for 60 days, the rate in each case being the same?
Ans. 58 da.

(17) A sold B a bill of goods as follows: silk for $1,200 on 30 days' credit, cloth for $3,600 on 60 days' credit, and cotton goods for $1,200 on 90 days' credit. In what time can the three debts be equitably paid at once?

Ans. 60 days.

(18) Find the equated time for paying in one payment the following debts: $800 due Sept. 1, $900 due Oct. 10, $1,000 due Oct. 30, and $1,200 due Dec. 1. Ans. Oct. 23.

(19) A bill was to be paid as follows: $\frac{1}{3}$ in 20 days, $\frac{1}{4}$ in 30 days, $\frac{1}{6}$ in 60 days, and the remainder in 90 days. Find the equated time for paying the whole bill in one payment.

Ans. 47 days.

(20) When may the following account be settled by cash payment without loss of interest to either party?

HENRY STELL.

1899.				1899.			
June	8	Mdse., 10 days,	1,200	July	1	Cash,	1,000
July	10	" 60 "	2,000		25	Draft, 30 days,	500
Aug.	20	" 30 "	3,000	Sept.	30	Cash,	800
Sept.	25	" 90 "	2,400	Oct.	10	Real Estate,	6,000
Dec.	1	" 30 "	5,000	Nov.	20	Cash,	900

Ans. Jan. 14, 1900.

(21) A mixture is composed of 12 lb. of tea worth 25 cents a pound, 12 lb. worth 35 cents a pound, and 48 lb. worth 45 cents a pound. Find the value of the mixture per pound. Ans. 40 cents.

(22) A speculator bought lots: 24 at $1,080 each, 36 at $2,160 each, and 48 at $3,240 each. He sold them all so as to average a gain of $100 a lot. Find the average price at which he sold them. Ans. $2,500.

(23) If certain multiples of 29, 37, 41, and 47 be added together, the sum is exactly divisible by 40. What is the least sum that will answer these conditions? Ans. 880.

(24) A coal merchant sold coal at $\$2\frac{1}{2}$, $\$3\frac{1}{2}$, $\$4\frac{1}{2}$, and $\$7\frac{1}{2}$ per ton. During a certain month his average selling price per ton was \$5. In what proportion were the different grades sold? Ans. 5, 5, 5, 9.

(25) A miller paid \$960 for 1,280 bu. of grain as follows: wheat at \$1.00, rye at \$.80, corn at \$.60, and oats at \$.40. How many bushels were there of each?

$$\text{Ans.}\begin{cases}560,\ 240,\ 80,\ 400, \text{ or}\\ 240,\ 560,\ 400,\ 80, \text{ etc.}\end{cases}$$

SPELLING.

LESSON 1.

INSTRUCTIONS FOR STUDYING.

The Instruction Paper in Spelling is apportioned into four divisions. The First Division contains thirty-four exercises and extends from page 1 to exercise 35 on page 7.

In studying the exercises in the Instruction Paper, look closely and carefully at each word, and form a correct mental picture of the word before attempting to write it. Form the habit of training the eye to *see* the word forms correctly before *writing* them.

Write all the words in each exercise as many times as it is necessary to write them before you can spell them correctly. If you can spell a word, it will not be necessary to write it more than once, but in case you cannot spell it, write it five, ten, or fifteen times, or until you *can* spell it. This may be done on paper or on a slate.

In order that the words in each exercise may be fully understood, we suggest that you consult a dictionary for the meaning of all words which are new.

Each exercise should be written in the order in which it occurs in the Instruction Paper. Number each exercise and write the words in columns, using capitals only where used in the Instruction Paper. Do not crowd your work.

In the exercises marked "Homonyms," write the definition after each word. Write, also, the sentences, putting in the right words, and underlining the words inserted.

INSTRUCTIONS FOR SENDING IN WORK.

After you have written the words in the exercises in a division, write correctly the list of words in the Question Paper for that division, and send both in at the same time. All words given in the Question Papers should be corrected without referring to the Instruction Paper.

For Lesson 1, send to the School for correction, the words in the First Division and the words in the following list, written correctly:

1. cloke,	26. libray,	51. basen,	76. minet,
2. aporn,	27. dence,	52. boul,	77. cousen,
3. coller,	28. city,	53. seive,	78. buzy,
4. forhead,	29. contries,	54. sausor,	79. waggen,
5. juce,	30. pigen,	55. portrate,	80. hearth,
6. cheek,	31. conder,	56. sissors,	81. schollars,
7. claus,	32. purch,	57. matress,	82. remidy,
8. chearful,	33. iland,	58. bureau,	83. ment,
9. pendilum,	34. crumb,	59. mirrer,	84. consert,
10. niece,	35. dresst,	60. bolester,	85. parrot,
11. nefew,	36. charcole,	61. curten,	86. eagel,
12. granfather,	37. serface,	62. musick,	87. fethers,
13. brekfast,	38. hoing,	63. climer,	88. pinyon,
14. shugar,	39. greatful,	64. croka,	89. channell,
15. sausage,	40. strate,	65. seder,	90. middel,
16. venzen,	41. prarie,	66. chestnut,	91. longe,
17. pudden,	42. plateau,	67. leaver,	92. severel,
18. practis,	43. presant,	68. pinsers,	93. hickry,
19. pallace,	44. shovles,	69. blosom,	94. adz,
20. travle,	45. ismus,	70. sipress,	95. mollases,
21. cubboard,	46. quarrel,	71. bruze,	96. carriges,
22. sellar,	47. mery,	72. neither,	97. shaul,
23. fernice,	48. reck,	73. necesary,	98. wrench,
24. atick,	49. speach,	74. gaters,	99. dumbly,
25. transom,	50. milatary,	75. parosol,	100. dettor.

LESSON 2.

INSTRUCTIONS FOR STUDYING.

The Second Division of the Instruction Paper contains forty-three exercises and extends from exercise 35, page 7, to exercise 78, page 17.

Read carefully the instructions in Question Paper No. 1, and follow them in all the Question Papers in Spelling.

Write all the words in the exercises in the Second Division and send them to the School, together with the following list of words written correctly.

Correct the following list of words:

1. skane,
2. habbit,
3. dungon,
4. comfert,
5. seive,
6. tong,
7. lettice,
8. Wensday,
9. orchard,
10. medow,
11. turnop,
12. pumkin,
13. sirpent,
14. radish,
15. transparrent,
16. gorrila,
17. cival,
18. obligeing,
19. courteous,
20. onest,
21. deceatful,
22. tobbaco,
23. rabit,
24. buffaloe,
25. patient,
26. hurrycane,
27. vessle,
28. frigget,
29. sereal,
30. rought,
31. pruen,
32. currant,
33. rasberry,
34. bannana,
35. mellon,
36. lilacks,
37. dasies,
38. geraneums,
39. atention,
40. secreat,
41. extreme,
42. discribe,
43. decent,
44. fredom,
45. treeson,
46. sceme,
47. sieze,
48. collum,
49. sourse,
50. circle,
51. daly,
52. savige,
53. paralel,
54. diviser,
55. chocalate,
56. vanila,
57. vinegar,
58. journy,
59. atorney,
60. commic,

61. galop,
62. guinea,
63. gorgous,
64. common,
65. biscut,
66. surloin,
67. crevacs,
68. promis,
69. croud,
70. scurge,
71. carpentar,
72. crokay,
73. nectar,
74. legue,
75. seeling,
76. surtan,
77. cemetary,
78. pedler,
79. engine,
80. compleat,
81. complection,
82. caben,
83. dessimal,
84. musketo,
85. favorite,
86. ancent,
87. fatel,
88. artic,
89. celary,
90. chrismas,
91. erand,
92. ernest,
93. comit,
94. jellous,
95. intrest,
96. silinder,
97. poisen,
98. sipher,
99. celibrate,
100. cinamon.

LESSON 3.

INSTRUCTIONS FOR STUDYING.

The Third Division of the Instruction Paper contains thirty-eight exercises and extends from exercise 78, page 17, to exercise 116, page 26.

When all the words in each exercise in the Third Division have been written, send them to the School, together with the following list of words written correctly.

Correct the following list of words:

1. migreat,
2. neather,
3. prarie,
4. omolet,
5. endeaver,
6. ridicule,
7. freckel,
8. liquer,
9. decied,
10. determin,
11. secret,
12. temperit,
13. remander,
14. batchilor,
15. wate (heaviness),
16. peice (a part),
17. spicey,
18. crasy,
19. axcede,
20. adieu,
21. councel,
22. disperce,
23. recieve,
24. present,
25. emmigrate,
26. resite,
27. isical,
28. comprize,
29. gilty,
30. excursion,
31. grannet,
32. metalic,
33. sythe,
34. sterril,
35. crulety,
36. dubble,
37. ceder,
38. sphear,
39. tumer,
40. seenery,
41. colander,
42. elipse,

43. dispair,
44. commerce,
45. deposite,
46. courtesey,
47. villan,
48. occured,
49. truly,
50. cisturn,
51. transum,
52. ludicrous,
53. corse (rough),
54. creture,
55. rane (rule),
56. linen,
57. callico,
58. chints,
59. cashmear,
60. alpacca,
61. parsle,
62. solem,
63. siringe,
64. calandar,
65. splender,
66. eraser,
67. musilage,
68. mackeral,
69. anualy,
70. porus,
71. studyous,
72. capatel (building),
73. thear (in that place),
74. dilicious,
75. noisey,
76. hight,
77. busely,
78. usualy,
79. mussles,
80. hesitate,
81. surprize,
82. campain,
83. vegitable,
84. salery,
85. seperate,
86. keresene,
87. meddicine,
88. believe,
89. warior,
90. crule (unkind),
91. steake (a slice of meat),
92. miricle,
93. sergicle,
94. phisicile,
95. feirceness,
96. fausett,
97. burgler,
98. victals,
99. callous,
100. boquet.

LESSON 4.

INSTRUCTIONS FOR STUDYING.

The Fourth Division of the Instruction Paper contains thirty-five exercises; also, the names of the states and territories, and a table of common abbreviations. This division extends from exercise 116, page 26, to the end, on page 39.

Learn to spell the names of the states and territories and their abbreviations; study, also, the table of common abbreviations.

Send to the School the words in each exercise in the Fourth Division, together with the following list of words written correctly.

Correct the following list of words:

1. absense,
2. bargin,
3. fatige,
4. machien,
5. notise,
6. courige,
7. marridge,
8. leisure,
9. nusance,
10. nostrels,
11. tortise,
12. kurnel (an army officer),
13. shariad,
14. celler (underground room),
15. gravey,
16. mischief,
17. conciet,
18. neibor,
19. exorcise,
20. idleize,
21. civalise,
22. varitey,
23. rezemble,
24. pacify,
25. discuss,
26. reathe,
27. mispend,
28. similiar,
29. imortal,
30. legicy,
31. herecy,
32. secrecy,

33. abundence,
34. eligant,
35. temperence,
36. quincy,
37. dilagent,
38. feaver,
39. typhiod,
40. canser,
41. collick,
42. naushea,
43. dispepsey.
44. neuralgia,
45. rhumatism,
46. parylasis,
47. pleurisy,
48. dilireum,
49. absess,
50. prohibbit,
51. independant,
52. unyun,
53. decliration,
54. cartrige,
55. oficial,
56. redout,
57. tirany,
58. reprisent,
59. goverment,
60. cavelry,
61. artilery,
62. bayonet,
63. recrute,
64. hospitle,
65. furlow,
66. pistle,
67. propriator,
68. editer,
69. corespondent
70. entry,
71. teligrams,
72. amuzements,
73. miscelaneous,
74. warior,
75. procedes,
76. sinical,
77. promisary,
78. anticedant,
79. partecipal,
80. captain,
81. precede,
82. decease,
83. pallisade,
84. linement,
85. Cincinatti,
86. penshion,
87. scollop,
88. gitar,
89. sinnegogue,
90. glicerin,
91. nesesary,
92. garantee,
93. simton,
94. buisness,
95. catipiler,
96. perpus,
97. agreable,
98. explanation,
99. suitible,
100. curiosity.

Write the abbreviations for each of the following words:

1. Louisiana,
2. Missouri.
3. Maryland,
4. Colorado.

5. Nebraska,
6. Maine,
7. Indian Territory,
8. District of Columbia,
9. Georgia,
10. Kentucky,
11. Oklahoma,
12. Indiana,
13. Virginia,
14. Florida,
15. Oregon,
16. Utah,
17. California,
18. Wyoming,
19. Vermont,
20. Illinois,
21. Iowa,
22. Idaho,
23. Alabama,
24. Alaska,
25. Connecticut.

Write the meaning of each of the following abbreviations:

1. Col.,
2. inst.,
3. Esq.,
4. MS.,
5. acct. or a/c,
6. D. D.,
7. Admr.,
8. Dec.,
9. A. D.,
10. ult.,
11. i. e.,
12. viz.,
13. P. M.,
14. pro tem.,
15. Gen.,
16. doz.,
17. vs.,
18. Atty.,
19. e. g.,
20. etc.,
21. N. B.,
22. P. S.,
23. R. R.,
24. prox.,
25. U. S. M.

A KEY

TO ALL THE

QUESTIONS AND EXAMPLES

INCLUDED IN THE

Examination Questions on Arithmetic.

It will be noticed that the Key is divided into sections which correspond to the sections of the Examination Questions on Arithmetic. The answers and solutions are so numbered as to be similar to the numbers before the questions to which they refer.

To be of the greatest benefit, the Keys should be used sparingly. They should be used much in the same manner as a pupil would go to a teacher for instruction with regard to answering some example he was unable to solve. If used in this manner, the Keys will be of great help and assistance to the student, and will be a source of encouragement to him in studying the various papers composing the Course.

ARITHMETIC.

(1) See Art. **1**.

(2) See Art. **9**.

(3) See Arts. **2** and **3**.

(4) See Arts. **5** and **6**.

(5) See Art. **21**.

(6) Seven thousand five hundred three is expressed in Arabic notation by 7,503; in Roman notation by $\overline{\text{V}}$MMDIII (see Arts. **12** and **22**)

(7) See Art. **19**.

(8) (*a*) Nine hundred eighty.

(*b*) Six hundred five.

(*c*) Twenty-eight thousand two hundred eighty-four.

(*d*) Nine million six thousand forty-two.

(*e*) Eight hundred fifty million three hundred seventeen thousand two.

(*f*) Seven hundred thousand four.

(9) In the Roman notation,

(*a*)	76	is written	LXXVI.
(*b*)	353	" "	CCCLIII.
(*c*)	1,732	" "	MDCCXXXII.
(*d*)	1,496	" "	MCCCCXCVI.
(*e*)	1,888	" "	MDCCCLXXXVIII.

(10) (*a*) Seven thousand six hundred is written 7,600.

(*b*) Eighty-one thousand four hundred two is written 81,402.

(*c*) Five million four thousand seven is written 5,004,007.

(*d*) One hundred eight million ten thousand one is written 108,010,001.

(*e*) Ten million six is written 10,000,006.

(*f*) Thirty thousand ten is written 30,010.

(11)

$$\begin{array}{r} 83027 \\ 46928 \\ 4769 \\ 81987 \\ 46729 \\ 479897 \\ 627 \\ 14896 \\ 987649 \\ \hline 1746509 \end{array} \text{ Ans.}$$

(12) As both multiplicand and multiplier end in ciphers, place the right-hand digits under each other, as shown. Multiplying the first digit at the right of the multiplicand, or 8, by the multiplier 9, the result is 72 units, 7 tens and 2 units. Write the two units in units place in the product, and reserve the tens to add to the product of tens. Multiplying the second digit of the multiplicand by the multiplier 9, we have 81 tens; 81 tens + 7 tens reserved = 88 tens, or 8 hundreds and 8 tens. Write the 8 tens and reserve the 8 hundreds. 9×2 hundreds = 18 hundreds; 18 hundreds + 8 hundreds = 26 hundreds, or 2 thousands and 6 hundreds. Then multiply 298 by 3 and write the product, 894, under the first partial product, as shown, with the right-hand digit, 4, under the multiplier 3. Add the two partial products; their sum gives the entire product of the numbers 298 and 39. Now annex the three ciphers (the number of ciphers to the right of the right-hand digits of both multiplier and multiplicand) to the product, as shown, and we have the entire product, 11,622,000.

$$\begin{array}{r} 29800 \\ 390 \\ \hline 2682 \\ 894 \\ \hline 11622000 \end{array} \text{ Ans.}$$

(13)

$$\begin{array}{rl} \$329 & = \text{amount paid the 1st yr.} \\ \$438 & = \text{amount paid the 2d yr.} \\ \hline \$767 & = \text{whole amount paid.} \end{array}$$

$$\begin{array}{rl} \$1000 & = \text{amount of debt.} \\ \$\ \ 767 & = \text{amount paid.} \\ \hline \$\ \ 233 & = \text{amount he still owes.}\quad \text{Ans.} \end{array}$$

(14) The numbers connected by the plus (+) sign must first be added. Performing these operations, we have

$$\begin{array}{rlrl}
23896 & & 31627 & \\
4982 & & 54892 & \\
96875 & & 6925 & \\
59674 & & 8976 & \\
\hline
185427 & \text{sum.} & 102420 & \text{sum.}
\end{array}$$

Subtracting the less number (102,420) from the greater (185,427), we have

$$\begin{array}{rl}
185427 & \\
102420 & \\
\hline
83007 & \text{difference. } \textbf{Ans.}
\end{array}$$

(15)

$$\begin{array}{r}
8765 \\
987 \\
\hline
61355 \\
70120 \\
78885 \\
\hline
8651055
\end{array}
\qquad
\begin{array}{r}
4695 \\
823 \\
\hline
14085 \\
9390 \\
37560 \\
\hline
3863985
\end{array}$$

Subtracting the second product from the first, we have

$$\begin{array}{rl}
8651055 & \\
3863985 & \\
\hline
4787070 & \text{Ans.}
\end{array}$$

(16) We have given the minuend, or greater number (1,004), and the difference, or remainder (49). Placing these in the usual form of subtraction, we have

$$\begin{array}{r}
1004 \\
\text{——} \\
\hline
49
\end{array}$$

in which the dash (——) represents the number sought. This number is evidently less than 1,004 by the difference 49; hence the smaller number is 1,004 − 49 = 955. The sum of the two numbers is then

$$\begin{array}{rl}
1004 & \text{larger.} \\
955 & \text{smaller.} \\
\hline
1959 & \text{sum. Ans.}
\end{array}$$

(17) 36 oxen at \$24 each would cost 36 × \$24 = \$864.

$$\begin{array}{r}
24 \\
36 \\
\hline
144 \\
72 \\
\hline
864
\end{array}$$

23 cows at \$96 each would cost $23 \times \$96 = \$2,208$.

$$\begin{array}{r} 96 \\ 23 \\ \hline 288 \\ 192 \\ \hline 2208 \end{array}$$

$$\begin{array}{rl} \$2208 &= \text{cost of cows.} \\ \$\ \ 864 &= \text{cost of oxen.} \\ \hline \$3072 &= \text{total cost.} \quad \text{Ans.} \end{array}$$

(18) $2,937 \times 864 \div 923 = 2,749\frac{241}{923}$. Ans.

$$\begin{array}{r} 2937 \\ 864 \\ \hline 11748 \\ 17622 \\ 23496 \\ \hline 2537568 \end{array}$$

$$\begin{array}{l} 923)2537568(2749\frac{241}{923} \\ \quad\ 1846 \\ \quad\ \overline{\ 6915} \\ \quad\ \ 6461 \\ \quad\ \ \overline{\ 4546} \\ \quad\ \ \ 3692 \\ \quad\ \ \ \overline{\ 8548} \\ \quad\ \ \ \ 8307 \\ \quad\ \ \ \ \overline{\ \ 241} \end{array}$$

(19) $8,976 \times 4,298 = 38,578,848$.

$$\begin{array}{r} 8976 \\ 4298 \\ \hline 71808 \\ 80784 \\ 17952 \\ 35904 \\ \hline 38578848 \end{array} \text{ product.}$$

$$\begin{array}{rl} 38578848 & \text{minuend.} \\ 98765 & \text{subtrahend.} \\ \hline 38480083 & \text{remainder.} \quad \text{Ans.} \end{array}$$

(20) (*a*) If an engine and boiler are worth \$3,246, and the building is worth 3 times as much, plus \$1,200, then the building is worth $\$3,246 \times 3 + \$1,200 = \$10,938$. If the tools are worth twice as much as the building, plus \$1,875, then the tools are worth $\$10,938 \times 2 + \$1,875 = \$23,751$.

$$\begin{array}{r} \$3246 \\ 3 \\ \hline \$9738 \\ \$1200 \\ \hline \$10938 \end{array} \qquad \begin{array}{r} \$10938 \\ 2 \\ \hline \$21876 \\ \$\ \ 1875 \\ \hline \$23751 \end{array}$$

$$\begin{array}{rl} \$10938 &= \text{value of building.} \\ \$23751 &= \text{value of tools.} \\ \hline \$34689 &= \text{value of building and tools.} \quad \text{Ans.} \end{array}$$

(*b*)

$$\begin{array}{rl} \$\ \ 3246 &= \text{value of engine and boiler.} \\ \$34689 &= \text{value of building and tools.} \\ \hline \$37935 &= \text{value of whole plant.} \quad \text{Ans.} \end{array}$$

(21) We have given the divisor (1,389) and the quotient (748), and are required to find the dividend. Placing these in the usual form of division, we have 1,389) —— (748, in which the dash (——) represents the dividend, or number sought. Leaving the remainder out of account, the dividend is evidently the product of 1,389 and 748, which is 1,038,972. But there is a remainder of 1,263, so that the actual dividend or number is

$$
\begin{array}{r}
1389 \\
748 \\
\hline
11112 \\
5556 \\
9723 \\
\hline
1038972
\end{array}
\qquad
\begin{array}{r}
1038972 \\
1263 \\
\hline
1040235
\end{array}
$$

$$1,038,972 + 1,263 = 1,040,235. \quad \text{Ans.}$$

(22) Placing these numbers in the usual form of multiplication, we have

$$
\begin{array}{r}
4896 \\
\text{——} \\
\hline
3862944
\end{array}
$$

in which the dash (——) represents the multiplier or number we wish to find. 4,896 multiplied by this number gives the product, or 3,862,944. Then, if we divide 3,862,944 by 4,896, the quotient will be the multiplier required.

$$3,862,944 \div 4,896 = 789. \quad \text{Ans.}$$

$$
\begin{array}{r}
4896)3862944(789 \\
34272 \\
\hline
43574 \\
39168 \\
\hline
44064 \\
44064 \\
\hline
\end{array}
$$

(23)

$$
\begin{array}{rl}
\$25375 & = \text{amount left to his widow.} \\
\$12450 & = \text{amount left to one child.} \\
7 & \\
\hline
\$87150 & = \text{amount left to 7 children.} \\
\$25375 & \\
\hline
\$112525 & = \text{amount all received. Ans.}
\end{array}
$$

(24) If the farm is sold for \$375 an acre instead of \$246 an acre, I gain \$375 − \$246 = \$129 on 1 acre. If the whole amount gained is \$31,734, then dividing the total gain on the farm by the amount gained on 1 acre gives the number of acres. 31,734 ÷ 129 = 246. Hence the number of acres is 246. Ans.

$$
\begin{array}{r}
129)31734(246 \\
258 \\
\hline
593 \\
516 \\
\hline
774 \\
774 \\
\hline
\end{array}
$$

(25) See Art. **94.**

(*a*) $$\frac{\overset{\overset{3}{\cancel{21}}}{\cancel{231}} \times \overset{\cancel{8}}{\cancel{96}} \times \overset{\overset{16}{\cancel{32}}}{\cancel{192}} \times \overset{3}{\cancel{45}}}{\cancel{11} \times \underset{\cancel{2}}{\cancel{24}} \times \underset{\cancel{6}}{\cancel{48}} \times \cancel{15} \times \cancel{7}} = \frac{3 \times 16 \times 3}{1} = 144. \quad \text{Ans.}$$

(*b*) $$\frac{\overset{3}{\cancel{42}} \times \overset{4}{\cancel{64}} \times \overset{\cancel{5}}{\cancel{125}} \times \overset{2}{\cancel{98}} \times \overset{\cancel{4}}{\cancel{144}}}{\cancel{14} \times \cancel{16} \times \cancel{25} \times \cancel{49} \times \cancel{36} \times \underset{\cancel{5}}{\cancel{20}}} = \frac{3 \times 4 \times 2}{1} = 24. \quad \text{Ans.}$$

ARITHMETIC.

(1) The denominators are not alike, so before adding them we must find a common denominator for the denominators of all the fractions. Find the least common denominator as in Art. **26.** Place the denominators in a row, separated by commas.

$$\begin{array}{r|l} 2 & 3,\ 4,\ 6,\ 8,\ 12 \\ \hline 3 & 3,\ 2,\ 3,\ 4,\ \ 6 \\ \hline 2 & 1,\ 2,\ 1,\ 4,\ \ 2 \\ \hline & 1,\ 1,\ 1,\ 2,\ \ 1 \end{array}$$

Divide by some prime number that will divide at least two of the denominators without a remainder, bringing down to the row below those denominators that will not contain the divisor without a remainder. Thus using 2 as a divisor the second row becomes 3, 2, 3, 4, 6, since 2 will not divide 3 without a remainder. Using 3 as a divisor the result is 1, 2, 1, 4, 2. Again using 2 as a divisor for the third row, the result is 1, 1, 1, 2, 1. The product of these numbers and the divisors is $2 \times 3 \times 2 \times 2 = 24$, the least common denominator. We must reduce all the fractions to 24ths, and then add their numerators. $\frac{2}{3} = \frac{16}{24}$, $\frac{3}{4} = \frac{18}{24}$, $\frac{5}{6} = \frac{20}{24}$, $\frac{7}{8} = \frac{21}{24}$, $\frac{11}{12} = \frac{22}{24}$.

To reduce a fraction to a fraction having 24 for the denominator, multiply both terms of the fraction by some number that will make the denominator 24. Thus, $\frac{2 \times 8}{3 \times 8} = \frac{16}{24}$; $\frac{3 \times 6}{4 \times 6} = \frac{18}{24}$; $\frac{5 \times 4}{6 \times 4} = \frac{20}{24}$; $\frac{7 \times 3}{8 \times 3} = \frac{21}{24}$; $\frac{11 \times 2}{12 \times 2} = \frac{22}{24}$.

Or divide the least common denominator by the denominator of the given fraction, and multiply both terms by the quotient.

The fractions now have a common denominator, 24; hence, we find their sum by adding the numerators and placing their sum over the common denominator; thus,

$$\frac{16}{24} + \frac{18}{24} + \frac{20}{24} + \frac{21}{24} + \frac{22}{24} = \frac{16 + 18 + 20 + 21 + 22}{24} = \frac{97}{24} = 4\tfrac{1}{24}. \quad \text{Ans.}$$

(2) The common denominator is readily seen to be 24.

$$3\tfrac{3}{4} = 3\tfrac{18}{24}$$
$$6\tfrac{7}{8} = 6\tfrac{21}{24}$$
$$9 + \tfrac{39}{24} = 9 + 1\tfrac{15}{24} = 10\tfrac{15}{24}.$$

$4\frac{5}{6} = 4\frac{20}{24}$; $10\frac{15}{24} - 4\frac{20}{24} = 5\frac{19}{24}$. Ans. See Art. **42**.

(3)

Bought	$12\frac{3}{4} =$	$12\frac{15}{20}$
Burned	$5\frac{4}{5} =$	$5\frac{16}{20}$
Remainder	$=$	$6\frac{19}{20}$.

Reducing the fractions to a common denominator, we find that $\frac{16}{20}$ in the subtrahend is greater than $\frac{15}{20}$. Borrow 1 from 12 (as in subtraction of whole numbers) calling the 1, $\frac{20}{20}$. $\frac{20}{20} + \frac{15}{20} = \frac{35}{20}$. $\frac{35}{20} - \frac{16}{20} = \frac{19}{20}$. We borrowed 1 from 12; hence, 5 from 11 is 6. The remainder is $6\frac{19}{20}$ tons. Ans.

(4) If 1 dress contains $6\frac{2}{3}$ yards, or $\frac{20}{3}$ yards, from $53\frac{1}{3}$ yards, or $\frac{160}{3}$ yards, you can make as many dresses as $\frac{20}{3}$ is contained in $\frac{160}{3}$. The fraction $\frac{20}{3}$ is contained in $\frac{160}{3}$, 8 times, for the denominators are the same and $160 \div 20 = 8$. Therefore, 8 dresses can be made. Ans.

(5) $83\frac{2}{3} \times 67\frac{4}{5} = \frac{251}{3} \times \frac{339}{5} = \frac{85089}{15} = 5{,}672\frac{3}{5}$. Ans.

First reduce the mixed numbers (see Art. **11**) to improper fractions (see Art. **10**), as, $83\frac{2}{3} = \frac{251}{3}$ and $67\frac{4}{5} = \frac{339}{5}$ (see Art. **19**). Then divide the product of the numerators by the product of the denominators, canceling if possible.

(6) $4\frac{1}{2} \times 6\frac{3}{4} \div 1\frac{4}{5} = \frac{9}{2} \times \frac{27}{4} \div \frac{9}{5} = \frac{9}{2} \times \frac{\overset{3}{\cancel{27}}}{4} \times \frac{5}{\cancel{9}} = \frac{135}{8} = 16\frac{7}{8}$. Ans.

Reducing the mixed numbers to improper fractions, inverting the divisor (see Art. **67**), and multiplying, the result is $16\frac{7}{8}$.

(7) If 1 yard of velvet costs $\$3\frac{7}{8}$, 24 yards will cost $\$3\frac{7}{8} \times 24 = \$93$. Ans.

First multiply $\frac{7}{8}$ by 24; thus, $\frac{7}{\cancel{8}} \times \frac{\overset{3}{\cancel{24}}}{1} = 21$. Multiply 3 by 24 in the usual way.

$$\begin{array}{r} 3\tfrac{7}{8} \\ 24 \\ \hline 21 \\ 72 \\ \hline 93 \end{array}$$

(8) $$\frac{4\frac{7}{8} \times 3\frac{2}{3} \times 3\frac{1}{5}}{1\frac{5}{8} \times 4\frac{2}{5}} = ?$$

Reducing the mixed numbers to fractions, the expression becomes

$$\frac{\frac{39}{8} \times \frac{11}{3} \times \frac{16}{5}}{\frac{13}{8} \times \frac{22}{5}}$$

Regard the fractions above the long line as the numerator of a fraction whose denominator are the fractions below the line. Transferring

the denominators of the fractions, or in reality inverting the divisor and multiplying (see Art. **67**),

$$\frac{\overset{13}{\cancel{39}} \times \cancel{11} \times \overset{2}{\cancel{16}} \times 8 \times \cancel{5}}{\cancel{8} \times \cancel{3} \times \cancel{5} \times \cancel{13} \times \underset{11}{\cancel{22}}} = 8. \quad \text{Ans.}$$

(9) Reducing $8\frac{1}{20}$ to an improper fraction,

$8\frac{1}{20} = \frac{161}{20}$. Multiplying, $\frac{161}{20} \times \frac{1}{40} = \frac{161}{800}$. Ans.

(10) First add $8\frac{3}{4}$, $7\frac{5}{6}$, and $10\frac{1}{2}$. The common denominator is 12. Then $8\frac{3}{4} = 8\frac{9}{12}$; $7\frac{5}{6} = 7\frac{10}{12}$; $10\frac{1}{2} = 10\frac{6}{12}$.

$$\begin{array}{r} 8\frac{9}{12} \\ 7\frac{10}{12} \\ 10\frac{6}{12} \\ \hline 25\frac{25}{12} \end{array} = 25 + 2\frac{1}{12} = 27\frac{1}{12}. \qquad \begin{array}{r} 100 \\ 27\frac{1}{12} \\ \hline 72\frac{11}{12} \end{array}$$

Subtracting $27\frac{1}{12}$ from 100, the result is $72\frac{11}{12}$. Ans.

(11) If in 1 hour $3\frac{3}{4}$ gallons leak away, it will take as many hours for 40 gallons to leak away as $3\frac{3}{4}$ is contained in 40.

$$3\frac{3}{4} = \frac{15}{4}. \quad \frac{40}{1} \div \frac{15}{4} = \frac{\overset{8}{\cancel{40}}}{1} \times \frac{4}{\underset{3}{\cancel{15}}} = \frac{32}{3} = 10\frac{2}{3}.$$

Hence, the time required is $10\frac{2}{3}$ hr. Ans.

$$(12) \quad \frac{65}{100} \times 4\frac{2}{3} \times 6\frac{1}{2} = \frac{\overset{13}{\cancel{65}}}{\underset{20}{\cancel{100}}} \times \frac{\overset{7}{\cancel{14}}}{3} \times \frac{13}{\cancel{2}} = \frac{13 \times 7 \times 13}{20 \times 3} = \frac{1,183}{60} = 19\frac{43}{60}.$$

Ans.

(13) If the diameter of a wheel is $8\frac{2}{3}$ feet and the circumference is $\frac{355}{113}$ times the diameter, the circumference is

$$\frac{26}{3} \times \frac{355}{113} = \frac{9,230}{339} = 27\frac{77}{339} \text{ feet.} \quad \text{Ans.}$$

(14) If 1 soldier consumes $1\frac{5}{8}$ pounds, as many soldiers will be required to consume $165\frac{3}{4}$ pounds as $1\frac{5}{8}$ is contained in $165\frac{3}{4}$. $165\frac{3}{4} = \frac{663}{4}$; $1\frac{5}{8} = \frac{13}{8}$. Dividing,

$$\frac{663}{4} \div \frac{13}{8} = \frac{\overset{51}{\cancel{663}}}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{\cancel{13}} = 102.$$

Hence, there are 102 soldiers in the company. Ans.

(15) If $4\frac{4}{5}$ acres produce 96 bushels of wheat, 1 acre will produce as many bushels as $4\frac{4}{5}$, or $\frac{24}{5}$, is contained in 96.

$$\frac{96}{1} \div \frac{24}{5} = \frac{\overset{4}{\cancel{96}}}{1} \times \frac{5}{\cancel{24}} = 20.$$

The yield is therefore 20 bushels per acre. Ans.

(16) $\frac{2}{3}$ of $6\frac{1}{2} = \frac{\cancel{2}}{3} \times \frac{13}{\cancel{2}} = \frac{13}{3}$; $4\frac{3}{4} \times \frac{13}{3} = \frac{19}{4} \times \frac{13}{3} = \frac{247}{12} = 20\frac{7}{12}$. Ans.

(17) One ton cost $\$23\frac{3}{4}$, $24\frac{3}{4}$ tons will cost $24\frac{3}{4} \times \$23\frac{3}{4} = \$587\frac{13}{16}$. Ans. This may be solved in two ways: either by reducing the mixed numbers to improper fractions or by multiplying the mixed numbers as follows:

$$\begin{array}{rl} & 23\frac{3}{4} \\ & 24\frac{3}{4} \\ \hline 4)\,69 & \\ 4)\,72 & \\ \hline 17\frac{1}{4} & \\ 18 & \\ 92 & \\ 46 & \frac{9}{16} \\ \hline 587\frac{13}{16} & \end{array}$$

First multiply 23 by $\frac{3}{4}$; multiply 23 by 3 and divide by 4, but in order to save space the division is merely indicated for the present. $\frac{3}{4}$ is multiplied by 24 in the same manner, multiplying 24 by 3, and indicating the division by 4. 69 (i. e., 23×3) is now divided by 4, obtaining $17\frac{1}{4}$; 72 (i. e., 24×3) is divided by 4, obtaining 18, which is written under the $17\frac{1}{4}$. Then multiply 23 by 24 in the usual manner, placing the unit figure under the unit figures of the two mixed numbers. Finally multiply $\frac{3}{4}$ by $\frac{3}{4}$. Adding the separate products, the result is $587\frac{13}{16}$.

(18) The first one gets $\frac{1}{3}$ of the amount or $\$3,696 \times \frac{1}{3} = \$1,232$. Ans. The second gets $\frac{2}{5}$ of the remainder or $(\$3,696 - \$1,232) \times \frac{2}{5} = \$2,464 \times \frac{2}{5} = \$985\frac{3}{5}$. Ans. The third gets what still remains, or $\$3,696 - (\$1,232 + \$985\frac{3}{5}) = \$3,696 - \$2,217\frac{3}{5} = \$1,478\frac{2}{5}$. Ans.

(19) If $9\frac{3}{4}$ acres cost $\$357\frac{1}{2}$, 1 acre will cost as many dollars as $9\frac{3}{4}$ is contained in $\$357\frac{1}{2}$. $357\frac{1}{2} = \frac{715}{2}$; $9\frac{3}{4} = \frac{39}{4}$;

$$\frac{715}{2} \div \frac{39}{4} = \frac{715}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{39} = \frac{1,430}{39} = 36\frac{26}{39} = 36\frac{2}{3}.$$

The price per acre is therefore $\$36\frac{2}{3}$. Ans.

(20) If the sum of two numbers multiplied by $18\frac{2}{3}$ is $296\frac{2}{3}$, then $296\frac{2}{3}$, or $\frac{890}{3}$, divided by $18\frac{2}{3}$, or $\frac{56}{3}$, is equal to the sum of the numbers.

$$\frac{890}{3} \div \frac{56}{3} = \frac{\overset{445}{\cancel{890}}}{\cancel{3}} \times \frac{\cancel{3}}{\underset{28}{\cancel{56}}} = \frac{445}{28} = 15\frac{25}{28}.$$

$15\frac{25}{28}$ is the sum of the numbers; if one of the numbers is $12\frac{3}{4}$, then the other is $15\frac{25}{28} - 12\frac{3}{4} = 15\frac{25}{28} - 12\frac{21}{28} = 3\frac{4}{28} = 3\frac{1}{7}$. Ans.

(21) The difference between the numbers is the quotient of $41\frac{1}{5}$ divided by $12\frac{7}{8}$.

$$41\frac{1}{5} = \frac{206}{5};\ 12\frac{7}{8} = \frac{103}{8};$$

$$\frac{206}{5} \div \frac{103}{8} = \frac{\overset{2}{\cancel{206}}}{5} \times \frac{8}{\cancel{103}} = \frac{16}{5} = 3\frac{1}{5}.$$

Since $3\frac{1}{5}$ is the difference between two numbers and 21 is one of the numbers, the other number must be either $21 - 3\frac{1}{5} = 17\frac{4}{5}$, or $21 + 3\frac{1}{5} = 24\frac{1}{5}$.

Therefore the other number is $17\frac{4}{5}$ or $24\frac{1}{5}$. Ans.

(22) If the wheel goes 108 feet in turning $5\frac{3}{4}$ times, in turning once it will go as many feet as $5\frac{3}{4}$, or $\frac{23}{4}$, is contained in 108.

$$\frac{108}{1} \div \frac{23}{4} = \frac{108}{1} \times \frac{4}{23} = \frac{432}{23} = 18\frac{18}{23}.$$

Hence, the circumference is $18\frac{18}{23}$ feet. Ans.

(23)
$$\begin{array}{rcl} 67\frac{5}{8} &=& 67\frac{15}{24} \\ 48\frac{2}{3} &=& \underline{48\frac{16}{24}} \\ && 18\frac{23}{24} \text{ difference. Ans.} \end{array}$$

Reduce $\frac{5}{8}$ and $\frac{2}{3}$ to a common denominator, 24. $\frac{16}{24}$ is greater than $\frac{15}{24}$, so we borrow 1 from 7, calling it $\frac{24}{24}$. $\frac{15}{24}+\frac{24}{24} = \frac{39}{24}$; $\frac{39}{24}-\frac{16}{24} = \frac{23}{24}$. Then 48 from 66 leaves 18.

(24) If a man travels $85\frac{5}{12}$ miles in one day, $78\frac{9}{15}$ miles the next day, and $125\frac{17}{35}$ miles the third day, in the three days he will travel the sum of the three distances.

$$\begin{array}{rcl} 85\frac{5}{12} &=& 85\frac{175}{420} \\ 78\frac{9}{15} &=& 78\frac{252}{420} \\ 125\frac{17}{35} &=& \underline{125\frac{204}{420}} \\ \text{Sum} &=& 288\frac{631}{420} = 288 + 1\frac{211}{420} = 289\frac{211}{420}. \end{array}$$

Hence, the distance traveled is $289\frac{211}{420}$ miles. Ans.

(25) $211\frac{1}{4} = \frac{845}{4}$; $1\frac{7}{8} = \frac{15}{8}$; $\frac{845}{4} \times \frac{15}{8} = \frac{12675}{32}$; hence, the cost of the lead was $\frac{12675}{32}$ cents.

If by selling it at $2\frac{1}{2}$ cents per pound I received the same amount as I paid for the whole, or $\frac{12675}{32}$ cents, I sold as many pounds as $2\frac{1}{2}$, or $\frac{5}{2}$, is contained in $\frac{12675}{32}$.

$$\frac{12{,}675}{32} \div \frac{5}{2} = \frac{\overset{2{,}535}{\cancel{12{,}675}}}{\underset{16}{\cancel{32}}} \times \frac{\cancel{2}}{\cancel{5}} = \frac{2{,}535}{16} = 158\frac{7}{16}.$$

I therefore sold $158\frac{7}{16}$ pounds and have left $211\frac{1}{4} - 158\frac{7}{16} = 52\frac{13}{16}$ pounds. Ans.

$$\begin{array}{rcccl} 211\frac{1}{4} &=& 211\frac{4}{16} &=& 210\frac{20}{16} \\ 158\frac{7}{16} &=& 158\frac{7}{16} &=& \underline{158\frac{7}{16}} \\ &&&& 52\frac{13}{16}. \end{array}$$

ARITHMETIC.

(1) $$2\tfrac{1}{2}\times 3\tfrac{1}{3}\div 8 = \frac{5}{\cancel{2}}\times\frac{\cancelto{5}{10}}{3}\times\frac{1}{8} = \tfrac{25}{24}.$$

Reduce $\frac{25}{24}$ to a decimal as in Art. **33.**

Since this is an unending decimal (see Art. **30**) and the fifth figure is 6, we increase the fourth figure by 1, obtaining as the result 1.0417–. Ans.

(2) $2.04\dot{1}\dot{8}$ is a mixed repetend, that is, only part of the decimal $(\dot{1}\dot{8})$ repeats, as shown by the dots over the 1 and 8. Applying the rule in Art. **52** to the part that repeats,

$$2.04\dot{1}\dot{8} = 2.04\tfrac{18}{99} = 2\frac{4\frac{18}{99}}{100} = 2\frac{\frac{414}{99}}{100} = 2\tfrac{414}{9900} = 2\tfrac{207}{4950} = 2\tfrac{23}{550}.\quad \text{Ans.}$$

(3) Applying the rule in Art. **62,** the fraction opposite $62\frac{1}{2}$ is $\frac{5}{8}$. Hence, $448\times 62\frac{1}{2} = (44{,}800\div 8)\times 5 = 28{,}000$. Ans.

(4) Since $.62\frac{1}{2} = \frac{5}{8}$, $\$1.62\frac{1}{2} = \$1\frac{5}{8}$. $\$1\frac{5}{8}\times 128 = \$208$, the value of the wood. $\$208 + \$436 = \$644$. As many tons of coal at $\$2.87\frac{1}{2}$, or $\$2\frac{7}{8}$, per ton should be received as $2\frac{7}{8}$ is contained in 644.

$$\begin{array}{r} 128 \\ 1\frac{5}{8} \\ \hline 8\,)\,640 \\ \hline 80 \\ 128 \\ \hline 208 \end{array}$$

$$\frac{\cancelto{28}{644}}{1}\times\frac{8}{\cancel{23}} = 224.$$

Hence, the quantity of coal received is 224 tons. Ans.

(5) On 1 basket he loses $\frac{1}{100}$ of $\$12.50 = \$.125 = \$.12\frac{1}{2}$; $\$1.87\frac{1}{2} - \$.12\frac{1}{2} = \$1.75$, selling price per basket. Ans.

(6) To reduce the fractions to decimals, annex ciphers to the numerator and divide by the denominator.

$$\begin{array}{r} 4\,)\,\underline{3.00} \\ .75 \\ 8\,)\,\underline{7.000} \\ .875 \\ 16\,)\,\underline{11.0000} \\ .6875 \\ 32\,)\,\underline{17.00000} \\ .53125 \end{array} \qquad \begin{array}{rl} \frac{3}{4} = & .75 \\ \frac{7}{8} = & .875 \\ \frac{11}{16} = & .6875 \\ \frac{17}{32} = & \underline{.53125} \\ \text{Sum} = & 2.84375 \quad \text{Ans.} \end{array}$$

In adding decimals, place the numbers so that the decimal points shall be directly under each other, and add as in whole numbers, placing the decimal point in the sum directly under the decimal points above.

(7) If the diameter of the earth is 7,912 miles and the circumference is 3.1416 times the diameter, the circumference is 7,912 × 3.1416 = 24,856.3392 miles. Ans.

Point off four decimal places in the product as there are four decimal places in the multiplier.

$$\begin{array}{r} 7912 \\ \underline{3.1416} \\ 47472 \\ 7912 \\ 31648 \\ 7912 \\ \underline{23736} \\ 24856.3392 \end{array}$$

(8) $\frac{3}{5}$ of $\frac{2}{3}$ of .0168 = .00672. Ans.

$\frac{3}{5} \times \frac{2}{3} = \frac{2}{5}$. $\frac{2}{5}$ reduced to a decimal equals .4.

The number of decimal places in the product must be five, since there are four in the multiplicand and one in the multiplier (see Art. **15**). Hence, we prefix two ciphers to 672, to make five places in the product.

$$\begin{array}{r} .0168 \\ \underline{.4} \\ .00672 \end{array}$$

(9) .37 of a number + .23 of a number is 33.6.

If .6 of the number is 33.6, the number must be 33.6 ÷ .6 = 56. Ans.

In dividing decimals observe Art. **22**. In this example there is one decimal place in the divisor and one decimal place in the dividend; therefore, there are no decimal places in the quotient, or in other words, the quotient is a whole number.

$$\begin{array}{r} .37 \\ \underline{.23} \\ .60 \end{array} \qquad \begin{array}{r} .6\,)\,\underline{33.6} \\ 56 \end{array}$$

(10) Expressing .8 in a fractional form, $.8 = \frac{\overset{4}{\cancel{8}}}{\underset{5}{\cancel{10}}}$, or $\frac{4}{5}$. Subtracting $\frac{2}{3}$ from $\frac{4}{5}$,

$$\frac{4}{5} - \frac{2}{3} = \frac{12}{15} - \frac{10}{15} = \frac{2}{15}.$$

87.8 is $\frac{2}{15}$ of the number required; hence, $\frac{1}{15}$ of the number is $\frac{1}{2}$ of 87.8, or 43.9, and the number is 15 × 43.9 = 658.5. Ans

(11) $14\frac{5}{8} - 9\frac{1}{2} = 5\frac{1}{8} = 5.125$. Since 5.125 yards of cloth are bought for \$8.20, the price per yard $=$ \$8.20 $\div$ 5.125 $=$ \$1.60. Ans.

```
5.125)8.20000(1.60
      5125
      30750
      30750
```

(12) In 1 mile there are 5,280 feet; therefore, 4,620 feet $= \frac{4620}{5280}$ mile.

Reducing $\frac{4620}{5280}$ to a decimal, 4,620 ft. $=$ .875 mile. Ans.

```
5280)4620.000(.875
     42240
      39600
      36960
       26400
       26400
```

(13) 6 barrels of flour at \$6.375 per barrel cost \$38.25. 28 bushels of potatoes at \$.875 per bushel cost \$24.50. 120 pounds of sugar at \$$.03\frac{1}{8}$ a pound cost \$3.75.

```
  6.375         .875      .03 1/8 = .03125
      6           28      .03125
 38.250         7000         120
               1750       3.75000
              24.500
```

Cost of flour,	\$38.25
Cost of potatoes,	\$24.50
Cost of sugar,	\$ 3.75
Entire cost,	\$66.50 Ans.

(14) If the lot cost \$3,300, and this is .165 of the cost of the house, then the house cost \$3,300 $\div$ .165 $=$ \$20,000. Ans.

```
.165)3300.000
      20000
```

(15) The increase per day is \$2.25 $-$ \$1.875, or \$.375; hence, the increase for 300 days is \$.375 $\times$ 300 $=$ \$112.50. Ans.

```
$2.250        $.375
$1.875          300
$ .375     $112.500
```

(16)

Paid to butcher,	.143
Paid to grocer,	.347
Paid for clothing,	.256
Other expenses,	.154
Total expenses,	.900

Since he spent .9 of his salary, he saved .1 of it. If his butcher receives \$327.47, which is .143 of his entire salary, the salary must be \$327.47 $\div$ .143 $=$ \$2,290.

He therefore saves \$2,290 $\times$ .1 $=$ \$229. Ans.

```
.143)327.470(2290
     286
      414
      286
      1287
      1287
```

(17) On 1 acre the gain was \$112.50 − \$87.50 = \$25. A gain of \$3,670 would require as many acres as 25 is contained in 3,670, or 146.8 acres. Ans.

$$3,670 \div 25 = 36.70 \times 4 = 146.80 \text{ (see Art. } \mathbf{66}).$$

(18) $.\dot{1}\dot{8} = \frac{18}{99} = \frac{2}{11}$ (see Art. **52**).

368.5 bushels is $\frac{2}{11}$ of the crop, $\frac{1}{11}$ is $\frac{1}{2}$ of 368.5 bushels, or 184.25 bushels, and $\frac{11}{11}$, or the entire crop, is 184.25 bushels $\times$ 11 = 2,026.75 bushels. Ans.

(19) If 1 cubic inch weighs .03617 pound, 231 cubic inches will weigh $.03617 \times 231 = 8.355+$ pounds. Ans.

```
   .0 3 6 1 7
        2 3 1
   ----------
      3 6 1 7
  1 0 8 5 1
  7 2 3 4
  -----------
  8.3 5 5 2 7, or 8.355+
```

(20) (*a*) Since $12\frac{1}{2} = 100 \times \frac{1}{8}$, $475 \div 12\frac{1}{2} = 4.75 \times 8 = 38$. Ans.

(*b*) Using the rule in Art. **66**, $\frac{25}{1} \div 16\frac{2}{3} = .25 \times 6 \div 1 = 1.5$, or $1\frac{1}{2}$. Ans.

(21) Reducing $\frac{5}{13}$ to a decimal, $\frac{5}{13} = .\dot{3}8461\dot{5}$. Ans.

```
13 ) 5.0 0 0 0 0 0 ( .3̇ 8 4 6 1 5̇
     3 9
     -----
     1 1 0
     1 0 4
     -------
         6 0
         5 2
         -----
           8 0
           7 8
           -----
             2 0
             1 3
             -----
               7 0
               6 5
               -----
                 5 0
```

It will be noticed that after obtaining the decimal .384615, if we continue dividing, the number 384615 will repeat, as .384615 384615, and the process will never terminate. Now instead of writing such a result, it may be written $.\dot{3}8461\dot{5}$, the dots over the 3 and 5 indicating that the figures 384615 repeat.

(22) By Art. **52**, $.\dot{3}5\dot{1} = \frac{351}{999} = \frac{117}{333} = \frac{39}{111} = \frac{13}{37}$. Ans.

(23) By Art. **53**, $.30208\dot{3} = .30208\frac{3}{9} = .30208\frac{1}{3}$. By Art. **41**, $\frac{.30208\frac{1}{3} \times 24}{24} = \frac{7.25}{24}$, or $\frac{7\frac{1}{4}}{24}$. Ans.

```
  .3 0 2 0 8 1/3
           2 4
  --------------
             8
  1 2 0 8 3 2
  6 0 4 1 6
  ------------
  7.2 5 0 0 0
```

(24) By Art. **41**, $.7292 \times \frac{40}{40} = \frac{29.168}{40}$, say $\frac{29}{40}$. Ans.

```
   .7 2 9 2
        4 0
  ----------
  2 9.1 6 8 0
```

(25)

$$\frac{\overset{37.13}{\cancel{74.26}} \times \overset{\substack{.0357\\ \cancel{.0714}\\ \cancel{.7854}}}{\cancel{3.1416}} \times \overset{6.5}{\cancel{19.5}} \times \overset{6.5}{\cancel{19.5}} \times \overset{35}{\cancel{350}}}{\underset{\substack{\cancel{3300}\\ \cancel{300}\\ 100}}{\cancel{33000}} \times \underset{\substack{\cancel{4}\\ \cancel{2}}}{\cancel{12}} \times \cancel{4}} = \frac{37.13 \times .0357 \times 6.5 \times 6.5 \times 35}{100}$$

$$= \frac{1{,}960.14}{100} = 19.601+. \quad \text{Ans.}$$

Decimals may be canceled just as whole numbers, but care must be taken to properly locate the decimal point. After canceling wherever possible to shorten the work of multiplication, divide the product of the numbers in the numerator by the denominator.

ARITHMETIC.

(1) Multiply .67 by 320, the number of rods in a mile, and the product, 214.4, is the number of rods in .67 of a mile. Multiply now the decimal part, .4, by $5\frac{1}{2}$, the number of yards in a rod, and the product, 2.2, is the number of yards in .4 of a rod. Again multiplying the decimal part, .2, by 3, we reduce the .2 of a yard to .6 of a foot. Multiplying by 12 to reduce .6 ft. to inches, we have 7.2 in. Collecting, the final result is 214 rd. 2 yd. 7.2 in. Ans.

$$\begin{array}{r} .67 \\ 320 \\ \hline 134 \\ 201 \\ \hline 214.40 \text{ rd.} \end{array} \qquad \begin{array}{r} .4 \text{ rd.} \\ 5\frac{1}{2} \\ \hline 2 \\ 20 \\ \hline 2.2 \text{ yd.} \end{array} \qquad \begin{array}{r} .2 \\ 3 \\ \hline .6 \text{ ft.} \\ 12 \\ \hline 7.2 \text{ in.} \end{array}$$

(2) As one place is in *west longitude* and the other in *east longitude*, we must add to find the difference. $23° \ 28' + 5° \ 22' = 28° \ 50'$; $50' = \frac{50}{60}°$, or $\frac{5}{6}°$. If in 1° there are 69.16 mi. (see example 9), in $28\frac{5}{6}°$ there are $69.16 \times 28\frac{5}{6} = 1{,}994.11$ mi. Ans.

$$\begin{array}{r} 23° \ 28' \\ 5° \ 22' \\ \hline 28° \ 50' \end{array} = 28\tfrac{5}{6}° \qquad \begin{array}{r} 69.16 \\ 28\frac{5}{6} \\ \hline 6\,)\,34580 \\ 5763\frac{1}{3} \\ 55328 \\ 13832 \\ \hline 1994.11\frac{1}{3} \end{array}\text{, or say } 1{,}994.11$$

(3) $$28 \times \frac{3}{4} \div \frac{7}{8} = \frac{\overset{4}{\cancel{28}}}{1} \times \frac{3}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{\cancel{7}} = 24.$$

The surface to be lined is $28 \times \frac{3}{4}$, or 21 sq. yd. A piece of lining a yard long has an area of $\frac{7}{8}$ square yard; hence, as many yards will be required as $\frac{7}{8}$ is contained in 21. $21 \div \frac{7}{8} = \frac{21}{1} \times \frac{8}{7} = 24$. Hence 24 yd. are required. Ans.

(4) $3 \times \frac{9}{\cancel{2}} \times \overset{3}{\cancel{6}} = 81.$

$$\begin{array}{r} 1.375 \\ 81 \\ \hline 1375 \\ 11000 \\ \hline 111.375 \end{array}$$

The dimensions of the block being 3 ft. × $4\frac{1}{2}$ ft. × 6 ft., it contains 81 cu. ft. If 1 cu. ft. costs \$1.375, 81 cu. ft. will cost 81 × \$1.375 = \$111.375. Ans.

(5) If the diameter is 12 ft., the circumference is 12 × 3.1416 = 37.6992 ft. 2 ft. 8 in. = $2\frac{2}{3}$ ft., or $\frac{8}{3}$ ft. To give 37.6992 ft. $\frac{8}{3}$ ft. must be repeated as many times as $\frac{8}{3}$ is contained in 37.6992, or

$$37.6992 \div \frac{8}{3} = \frac{\overset{4.7124}{\cancel{37.6992}}}{1} \times \frac{3}{\cancel{8}} = 14.1372 \text{ times. Ans.}$$

(6)

	da.	hr.	min.	sec.
	365	5	48	49.7
				5
8)	1,825	25	240	248.5
	228	6	37	61.0625 (1 min. 1.0625 sec.)
			1	
	228	6	38	1.0625

Multiplying the units of each denomination by 5, we have 1,825 da. 25 hr. 240 min. 248.5 sec. Dividing, 1,825 contains 8 228 times with 1 da. remaining; 1 da. = 24 hr., and, adding to the 25 hr. in the dividend, 24 + 25 = 49 hr., 49 ÷ 8 = 6 hr. and 1 hr. remaining; 1 hr. = 60 min., and 60 + 240 = 300 min.; 300 ÷ 8 = 37 min. and 4 min. remaining. Reducing 4 min. to seconds, 4 × 60 = 240 sec.; 240 + 248.5 = 488.5 sec. 488.5 ÷ 8 = 61.0625 sec. But 61.0625 sec. is equal to 1 min. 1.0625 sec. Adding 1 min. to the 37 min., the result is 38 min. The final result is 228 da. 6 hr. 38 min. 1.0625 sec. Ans.

(7) 5 T. 875 lb. = $5\frac{875}{2240}$ T. = $5\frac{25}{64}$ T. = $\frac{345}{64}$ T.

If 5 T. 875 lb., or $\frac{345}{64}$ T., cost \$9.135, 1 T. will cost as many dollars as $\frac{345}{64}$ is contained in \$9.135, or \$1.69. Ans.

$$9.135 \div \frac{345}{64} = \frac{\overset{1.827}{\cancel{9.135}}}{1} \times \frac{64}{\underset{69}{\cancel{345}}} = \frac{116.928}{69} = 1.69$$

(8) An acre contains 160 sq. rd. 8 in. $= \frac{8}{12}$ ft. $= \frac{\frac{8}{12}}{16\frac{1}{2}}$ rd. $= \frac{4}{99}$ rd. $\frac{4}{99} \times 40 = \frac{160}{99}$; hence, the area of a furrow 8 in. wide and 40 rd. long is $\frac{160}{99}$ sq. rd. $160 \div \frac{160}{99} = 160 \times \frac{99}{160} = 99$; therefore, 99 of these furrows are equivalent to 1 A. Ans.

(9) $20' \quad 30'' = 20.5' = \dfrac{20.5^\circ}{60}$ $= .341\frac{2}{3}^\circ$. $42^\circ\ 20'\ 30'' = 42.341\frac{2}{3}^\circ$.
Since 1° is equal to 69.16 mi., $42.341\frac{2}{3}^\circ$ is equal to $69.16 \times 42.341\frac{2}{3}$ = 2,928.3496+ mi. Ans.

```
            4 2.3 4 1 2/3
              6 9.1 6
            -----------
              4 6 1 0 2/3
          2 5 4 0 4 6
          4 2 3 4 1
      3 8 1 0 6 9
    2 5 4 0 4 6
    -------------------
    2 9 2 8.3 4 9 6 6 2/3, or 2,928.3497−
```

(10) 8 ft. 6 in. = 102 in.
We first find the volume of the cube, which is 1,061,208 cu. in. Dividing this by 231 (see Art. **21**), the number of cubic inches in a gallon, we obtain 4,594− gal. as the contents of the cistern. Ans.

```
            1 0 2
            1 0 2
            -----
            2 0 4
        1 0 2
        ---------
        1 0 4 0 4
            1 0 2
        ---------
        2 0 8 0 8
    1 0 4 0 4
    -------------
2 3 1 ) 1 0 6 1 2 0 8 ( 4 5 9 3.9, say 4,594−
        9 2 4
        -----
        1 3 7 2
        1 1 5 5
        -------
          2 1 7 0
          2 0 7 9
          -------
            9 1 8
            6 9 3
            -------
            2 2 5 0
            2 0 7 9
```

(11) From Jan. 14, 1898, to Jan. 14, 1900, is 2 years.

From the table of Art. **68,** the time from Jan. 14 to July 14 is 181 days. From July 14 to July 23 is 9 days. $181 + 9 = 190$; hence, from Jan. 14, 1898, to July 23, 1900, is 2 yr. 190 da. Ans.

February has only 28 da. in 1900; this being a secular year not divisible by 400 (see Art. **28**).

(12) To reduce $60^\circ\ 40'\ 30''$ to the decimal part of a circle:

Reducing 30″ to a decimal of a minute, $30'' = \frac{30}{60}' = .5'$. $40' + .5' = 40.5'$. Reducing 40.5′ to a decimal of a degree, $40.5' = \dfrac{40.5^\circ}{60} = .675^\circ$. $60^\circ + .675^\circ = 60.675^\circ$. In a circle there are 360°; then, 60.675° is $\frac{60.675}{360}$ or .1685+ of a circle. Ans.

```
6 0 ) 3 0.0        6 0 ) 4 0.5
        .5                .6 7 5

3 6 0 ) 6 0.6 7 5 (.1 6 8 5 +
        3 6 0
        -----
        2 4 6 7
        2 1 6 0
        -------
          3 0 7 5
          2 8 8 0
          -------
            1 9 5 0
            1 8 0 0
            -------
              1 5 0 0
```

(13) $6 \times 6\frac{1}{2} \times 8 = 312$.

```
   1728
    312
   ----
   3456
  1728
 5184
 ------
 539136
```

```
2150.42)539136.000(250.7+
        430084
        -------
        1090520
        1075210
        -------
          1531000
          1505294
          -------
```

Multiplying together the linear dimensions of the bin, its volume is found to be 312 cu. ft. Since in 1 cu. ft. there are 1,728 cu. in., in 312 cu. ft. there are $312 \times 1{,}728 = 539{,}136$ cu. in. In 1 bu. there are 2,150.42 cu. in. (see Art. 24), and in 539,136 cu. in. there are as many bushels as 2,150.42 is contained in 539,136, or 250.7+ bu. Ans.

(14)

```
      8 3/4
      80
     -----
  4)240
     60
    640
    ---
    700
```

```
16)700(43 3/4
   64
   --
    60
    48
    --
    12
    -- = 3/4
    16
```

If they used $8\frac{3}{4}$ oz. per day, in 80 da. they would use $80 \times 8\frac{3}{4} = 700$ oz. In 1 lb. there are 16 oz. (see Table VI) and in 700 oz. there are as many pounds as 16 is contained in 700, or $43\frac{3}{4}$ lb. Ans.

(15) 25 ft. square is an area 25 ft. long and 25 ft. wide, or $(25 \times 25) = 625$ sq. ft. $625 - 75 = 550$ sq. ft. Ans.

(16) Since the volume is the product of the length, width, and thickness, it follows that the width is equal to the volume divided by the product of the length and thickness. 10 in. $= \frac{5}{6}$ ft. $40 \div (36 \times \frac{5}{6}) = 40 \div 30 = \frac{4}{3}$. The width is therefore $\frac{4}{3}$ ft. $= \frac{4}{3} \times 12$ in. $= 16$ in. Ans.

(17)

```
 2150.42
     100
 ---------
 215042.00
```

```
231)215042.000(930.917, or say 930.92−
    2079
    ----
     714
     693
     ----
      2120
      2079
      ----
        410
        231
        ----
        1790
        1617
        ----
         173
```

There are 2,150.42 cu. in. in 1 bu. (see Art. **24**) and in 100 bu. there are $100 \times 2{,}150.42 = 215{,}042$ cu. in. In 1 gal. there are 231 cu. in.; therefore, in 215,042 cu. in. there are as many gallons as 231 is contained in 215,042, or 930.92− gal. Ans.

(18) (*a*) 18.75 ft. $= 18\frac{3}{4}$, or $\frac{75}{4}$ ft.
2 ft. 10 in. $= 2\frac{10}{12}$ ft. $= 2\frac{5}{6}$ ft. $= \frac{17}{6}$ ft.

$$\frac{\overset{25}{\cancel{75}}}{4} \times \frac{17}{\underset{2}{\cancel{6}}} = \frac{425}{8} = 53.125.$$

Hence, the area is 53.125 sq. ft. Ans.

(*b*) $53.125 \times \$.08 = \$4.25$. Ans.

(19) (*a*) 1 pt. $= \frac{1}{2}$ qt. 3 qt. 1 pt. $= 3\frac{1}{2}$ qt. $= \dfrac{3\frac{1}{2}}{4}$ gal. $= \frac{7}{8}$ gal. 18 gal. 3 qt. 1 pt. $= 18\frac{7}{8}$ gal. $18\frac{7}{8}$ gal. is $\dfrac{18\frac{7}{8}}{63} = \frac{151}{504}$ of 63 gal. Ans.

(*b*) $\frac{151}{504} = .2996+$. Ans.

```
504)151.0000(.2996+
     1008
     ----
      5020
      4536
      ----
       4840
       4536
       ----
        3040
        3024
        ----
```

(20) In 1 bu. there are 32 qt. and in 6 bu. there are $6 \times 32 = 192$ qt. If 3 qt. sell for \$.25, for 1 qt. he receives $\frac{1}{3}$ of \$.25, or $\$.08\frac{1}{3}$, and for 192 qt. he receives $192 \times \$.08\frac{1}{3} = \$16$. But this is \$4 more than they cost; therefore, the cost of the 6 bu. was $\$16 - \$4 = \$12$. Therefore, 1 bu. cost $\frac{1}{6}$ of \$12, or \$2. Ans.

(21) 8 hr. 40 min. $= 8\frac{40}{60}$ hr. $= 8\frac{2}{3}$ hr. $8\frac{2}{3}$ hr. a day for 5 da. a week for 40 wk. per year and for 4 yr. is $8\frac{2}{3} \times 5 \times 40 \times 4 = 6{,}933\frac{1}{3}$ hr. Ans.

$$\begin{array}{r} 8\frac{2}{3} \\ 5 \\ \hline 40 \\ 3\frac{1}{3} \\ \hline 43\frac{1}{3} \\ 40 \\ \hline 1720 \\ 13\frac{1}{3} \\ \hline 1733\frac{1}{3} \\ 4 \\ \hline 6933\frac{1}{3} \end{array}$$

(22) Write the numbers as for addition, with units of like denomination under each other. Since 6 d. cannot be subtracted from 3 d., 1 s. (= 12 d.) is borrowed from the column of shillings. $12+3 = 15$ d. Then, 15 d. $-$ 6 d. $=$ 9 d. Since 1 shilling was borrowed from 5 s., only 4 s. remain; and as 9 s. cannot be subtracted from 4 s., £1 (= 20 s.) is borrowed from the £ column, reduced to shillings and added to 4 s., giving 24 s.; then, 24 s. $-$ 9 s. $=$ 15 s. Finally, £20 $-$ £13 $=$ £7; and the final remainder is £7 15 s. 9 d. Ans.

£	s.	d.
21	5	3
13	9	6
£7	15 s.	9 d.

(23) There are 640 A. in 1 sq. mi. and in 16 sq. mi. there are $16 \times 640 = 10{,}240$ A. 10,240 A. is to be divided into 62 farms. Dividing 10,240 A. by 62 gives 165 A., with a remainder of 10 A., which $= 10 \times 160$ or 1,600 sq. rd. $1{,}600 \div 62 = 25$ sq. rd., with a remainder of 50 sq. rd. $= 50 \times 30\frac{1}{4} = 1{,}512\frac{1}{2}$ sq. yd.; $1{,}512\frac{1}{2} \div 62 = 24$ sq. yd., with a remainder of $24\frac{1}{2}$ sq. yd. $= 24\frac{1}{2} \times 9 = 220\frac{1}{2}$ sq. ft. $220\frac{1}{2} \div 62 = 3$ sq. ft. and a remainder of $34\frac{1}{2}$ sq. ft. $= 4{,}968$ sq. in. $4{,}968 \div 62 = 80+$ sq. in. The result is 165 A. 25 sq. rd. 24 sq. yd. 3 sq. ft. 80+ sq. in. Ans.

```
      640 A.
       16
     ----
     3840
     640
     ----
62)10240 A. (165 A.
   62
   ---
   404
   372
   ---
    320
    310
    ---
     10 A.
    160
    ----
 62)1600 sq. rd. (25 sq. rd.
    124
    ---
     360
     310
     ---
      50 sq. rd.
      30¼
      ---
      12½
    1500
    ----
    1512½ sq. yd.
```

```
62)1512½ sq. yd. (24 sq. yd.
   124
   ---
    272
    248
    ---
     24½ sq. yd.
      9
     --
      4½
    216
    ---
 62)220½ sq. ft. (3 sq. ft.
    186
    ---
     34½ sq. ft.
    144
    ---
     72
    136
   136
   34
   ----
62)4968 sq. in. (80+ sq. in.
   496
   ---
     8
```

(24) First reduce 2 bu. 3 pk. 6 qt. to bushels and decimals.

$$6 \text{ qt.} = \tfrac{6}{8} \text{ pk.} = \tfrac{3}{4} \text{ pk.}$$

$$3 \text{ pk. } 6 \text{ qt.} = 3\tfrac{3}{4} \text{ pk.} = \frac{3\frac{3}{4}}{4} \text{ bu.} = \tfrac{15}{16} \text{ bu.}$$

2 bu. 3 pk. 6 qt. $= 2\frac{15}{16}$ bu. $= 2.9375$ bu.

If in 1 bbl. there are 2.9375 bu., in 9 bbl. there are $9 \times 2.9375 = 26.4375$ bu. Ans.

(25) See Table II.

1 rd. = 25 li.	25) 4763254 li.
1 ch. = 4 rd.	4) 190530 rd. + 4 li.
1 mi. = 80 ch.	80) 47632 ch. + 2 rd.
	595 mi. + 32 ch.

Hence, 4,763,254 li. = 595 mi. 32 ch. 2 rd. 4 li. Ans.

ARITHMETIC.

(1) If the tank is 3 m. $\times$ 2 m. $\times$ 3 m., its volume is 18 cu. m. 1 cu. m. = 1,000 cu. dm.; therefore, 18 cu. m. = 18 $\times$ 1,000 cu. dm. = 18,000 cu. dm. (See Table III, Art. **15.**) Since 1 cu. dm. of water weighs 1 Kg. (see Art. **19**), 18,000 cu. dm. weigh 18,000 Kg. Ans.

(2) One metric ton is equal to 1.1023 $\times$ 2,000 = 2,204.6 lb. (see Table V). 1.5 metric tons = 1.5 $\times$ 2,204.6 = 3,306.9 lb. At $2\frac{1}{2}$ cents a pound, 3,306.9 lb. will cost 3,306.9 $\times$ .025 = \$82.6725, or \$82.67+. Ans.

(3) 25.875 Kg. = 258.75 Hg. = 2,587.5 Dg. = 25,875 g. (see Table V). 1 g. = 15.432 gr.; therefore, 25,875 g. = 25,875 $\times$ 15.432 = 399,303 gr. At 10 cents a grain, 399,303 gr. will cost 399,303 $\times$ .10 = \$39,930.30. Ans.

(4) The area of the ceiling is (12 $\times$ 10) sq. m. = 120 sq. m. 1 sq. m. = 1.196 sq. yd. (see Table II), and 120 sq. m. = 120 $\times$ 1.196 = 143.52 sq. yd. At 25 cents a square yard, 143.52 sq. yd. will cost 143.52 $\times$ .25 = \$35.88. Ans.

(5) 50 m. = 500 dm. = 5,000 cm. (see Table I). The row is 5,000 cm. long, and if the diameter of the 5-cent nickel coin is 2 cm., there are 5,000 $\div$ 2 = 2,500 nickel coins in the row. If 1 coin is worth 5 cents, 2,500 are worth 2,500 $\times$ .05 = \$125. Ans.

(6) One liter is equal to .9078 of a dry quart (see Table IV); therefore, 1,000 liters = 1,000 $\times$ .9078 = 907.8 qt. At 12 cents a quart, 907.8 qt. will cost 907.8 $\times$ .12 = \$108.94. Ans.

(7) Since 1 m. = 3.28 ft. (see Table I), 56.8 m. = 56.8 $\times$ 3.28 = 186.304 ft. = $\frac{186.304}{3}$ = 62.1+ yd. Ans.

(8) $\sqrt{32^2+255^2} = \sqrt{32\times 32+255\times 255} = \sqrt{66{,}049} = 257.$ Ans.

$$\begin{array}{r} 32 \\ 32 \\ \hline 64 \\ 96 \\ \hline 1024 \end{array} \qquad \begin{array}{r} 255 \\ 255 \\ \hline 1275 \\ 1275 \\ 510 \\ \hline 65025 \\ 1024 \\ \hline 66049 \end{array}$$

To extract the square root of 66,049, first point off the number into periods of two figures each, beginning at the right; this gives 6′60′49. 2 is the number whose square most nearly equals the first period, since $2^2 = 4$, and is taken as the trial root. Divide the first period by twice the trial root, and to the quotient add one-half the trial root. Thus,

$$\frac{6}{2\times 2}+\frac{2}{2} = 1.5+1 = 2.5.$$

Neglect the decimal point and call this result the new trial root. Repeat the process, using the first and second periods for a trial dividend, and 25 for a trial root. Thus,

$$\frac{6'60}{2\times 25}+\frac{25}{2} = 13.2+12.5 = 25.7.$$

Use three periods and 257 for the trial root. Thus,

$$\frac{6'60'49}{2\times 257}+\frac{257}{2} = 128.5+128.5 = 257.0.$$

In 66,049 there are three periods; hence, there must be three figures in the whole-number part of the root; hence, $\sqrt{66{,}049} = 257$.

(9) Substituting the values of the letters,

$$\frac{5(xyz-abc)}{bc-ax} = \frac{5(10\times 4\times 6-3\times 5\times 8)}{5\times 8-3\times 10} = \frac{5(240-120)}{40-30} = \frac{600}{10} = 60.$$

Ans.

(10)

$$\begin{array}{r} 289 \\ 289 \\ \hline 2601 \\ 2312 \\ 578 \\ \hline 83521 \end{array} \qquad \begin{array}{r} 161 \\ 161 \\ \hline 161 \\ 966 \\ 161 \\ \hline 25921 \end{array} \qquad \begin{array}{r} 83521 \\ 25921 \\ \hline 57600 \end{array}$$

$\sqrt{5'76'00} = 240.$ Ans.

$$\frac{5}{2\times 2}+\frac{2}{2} = 1.25+1 = 2.25, \text{ 1st approximation.}$$

Retaining two figures and neglecting the decimal point gives 23 the third figure being 5) as the second trial root. Use this trial root and the first two periods as a trial dividend. Thus,

$$\frac{5'76}{2\times 23}+\frac{23}{2} = 12.5+11.5 = 24.0, \text{ 2d approximation.}$$

Retaining three figures, the third trial root is 240. Repeat the process, using three periods. Thus,

$$\frac{5'76'00}{2\times 240}+\frac{240}{2} = 120+120 = 240, \text{ true root.}$$

If the square root of the sum is 289, the sum is 289^2, or $289 \times 289 = 83{,}521$. If 83,521 is the sum of the squares of two numbers, one of which is 161^2, or 25,921, then the square of the other number must be $83{,}521 - 25{,}921 = 57{,}600$. Since this is the square of the number, the number itself is $\sqrt{57{,}600} = 240$. Ans.

(11) If the field is 40 rd. square, its area is 1,600 sq. rd. 1 A. = 160 sq. rd.; hence, in 1,600 sq. rd. there are $1{,}600 \div 160 = 10$ A. 1 Ha. = 2.471 A. (Table II); hence, in 10 A. there are $\frac{10}{2.471} = 4.0469+$ Ha. Ans.

(12) $\sqrt[3]{87.54} \div \sqrt{12.1} = 1.2765-$. Ans.

Find the trial root by Art. **55.** Thus, $4^3 = 64$; $5^3 = 125$; $(125 - 64) \div 3 + 64 = 84$. Hence, 5 will be used for the first trial root.

The successive approximations are as follows:

$$\frac{87}{3 \times 5^2} + \frac{2 \times 5}{3} = 1.16 + 3.33 = 4.49.$$

$$\frac{87'540}{3 \times 45^2} + \frac{2 \times 45}{3} = 14.4 + 30 = 44.4.$$

$$\frac{87'540'000}{3 \times 444^2} + \frac{2 \times 444}{3} = 148 + 296 = 444.0.$$

$$\frac{87'540'000'000}{3 \times 4{,}440^2} + \frac{2 \times 4{,}440}{3} = 4{,}440.2.$$

Since there is only one period in the whole-number part of the given number, there is only one figure in the whole-number part of the root; hence, $\sqrt[3]{87.54} = 4.4402$.

In obtaining the square root of 12, the first trial root is 3. The successive approximations are as follows:

$$\frac{12}{2 \times 3} + \frac{3}{2} = 2 + 1.5 = 3.5.$$

$$\frac{12'10}{2 \times 35} + \frac{35}{2} = 17.28 + 17.5 = 34.78.$$

$$\frac{12'10'00}{2 \times 348} + \frac{348}{2} = 173.85 + 174 = 347.85.$$

$$\frac{12'10'00'00}{2 \times 3{,}479} + \frac{3{,}479}{2} = 1{,}739 + 1{,}739.5 = 3{,}478.5.$$

Locating the decimal point, we have $\sqrt{12.1} = 3.4785$. Then, $4.4402 \div 3.4785 = 1.2765-$.

(13) One liter is equal in volume to 1 cu. dm. (see Art. **17**); therefore, 4,913 liters = 4,913 cu. dm. The length of an edge is $\sqrt[3]{4{,}913}$ cu. dm. = 17 dm. 1 dm. = 3.937 in. (see Table I); hence, 17 dm. $= 17 \times 3.937 = 66.929$ in. Ans.

The extraction of the cube root is as follows:

$$\frac{4}{3 \times 2^2} + \frac{2 \times 2}{3} = .33 + 1.33 = 1.66.$$

$$\frac{4'913}{3 \times 17^2} + \frac{2 \times 17}{3} = 5.667 + 11.333 = 17.$$

(14) One Ha. = 2.471 A. (Table II); therefore, 40 A. = 40 ÷ 2.471 = 16.1878 Ha. Reduce 16.1878 Ha. to square meters by moving the decimal point two places to the right for each denomination (see Art. **13**); thus, 16.1878 Ha. = 1,618.78 sq. Dm. = 161,878 sq. m. The area of the square is 161,878 sq. m.; to find the length of a side, extract the square root of 161,878.

The first period is 16 and the first trial root is 4.

$$\frac{16}{2\times 4}+\frac{4}{2} = 2+2 = 4.0.$$

$$\frac{16'18}{2\times 40}+\frac{40}{2} = 20.2+20 = 40.2.$$

$$\frac{16'18'78}{2\times 402}+\frac{402}{2} = 201.34+201 = 402.34+.$$

The length of the side is 402.34+ m. Ans.

(15) To find any power of a mixed number, first reduce it to an improper fraction, and then raise the numerator and the denominator separately to the required power.

$$(43\tfrac{1}{2})^2+(4\tfrac{2}{3})^3 = (\tfrac{87}{2})^2+(\tfrac{14}{3})^3 = \frac{87\times 87}{2\times 2}+\frac{14\times 14\times 14}{3\times 3\times 3} = \frac{7,569}{4}+\frac{2,744}{27}$$

$$= \frac{204,363+10,976}{108} = \frac{215,339}{108} = 1,993.879+. \quad \text{Ans.}$$

(16) $\sqrt[3]{72a^2bc^3} = \sqrt[3]{72\times 2\times 2\times 6\times 3\times 3\times 3} = \sqrt[3]{46,656} = 36.$ Ans.

$$\frac{46}{3\times 4^2}+\frac{2\times 4}{3} = 3.62+. \qquad \frac{46'656}{3\times 36^2}+\frac{36\times 2}{3} = 36.$$

(17) One gallon contains 231 cu. in.; therefore, 100 gal. contains 100×231 = 23,100 cu. in. 1 liter = 61.023 cu. in. (see Table IV); hence, in 23,100 cu. in. there are as many liters as 61.023 is contained in 23,100, or 23,100 ÷ 61.023 = 378.546– liters. Ans.

(18) One gram = 15.432 gr. (see Art. **19**). One kilogram, or kilo = 1,000 g. (see Art. **23**) or 1 Kg. = $1,000\times 15.432$ = 15,432 gr. One gold eagle weighs 258 gr., then 1 dollar weighs $\frac{1}{10}$ of 258 gr. = 25.8 gr. If 1 dollar weighs 25.8 gr. in 15,432 gr., there are as many dollars as 25.8 is contained in 15,432, or $598.14–. Ans.

(19) Contents of the block are $(2.5\times 2.8\times 3.75)$ cu. m. = 26.25 cu. m. One cubic meter = 35.3145 cu. ft. (see Table III); then, 26.25 cu. m. = 26.25×35.3145 = 927.005 cu. ft. If 1 cu. ft. weighs 168.75 lb., 927.005 cu. ft. weigh 927.005×168.75 = 156,432.09+, or say 156,432 lb. Ans.

(20) $\sqrt{7{,}921} \times \sqrt{9{,}604} = 89 \times 98 = 8{,}722$. Ans.

$$\frac{79}{2 \times 9} + \frac{9}{2} = 8.9. \quad \frac{79'21}{2 \times 89} + \frac{89}{2} = 89.$$

Hence, $\sqrt{7{,}921} = 89$.

$$\frac{96}{2 \times 10} + \frac{10}{2} = 9.8. \quad \frac{96'04}{2 \times 98} + \frac{98}{2} = 98.$$

Hence, $\sqrt{9{,}604} = 98$.

(21) Reducing $\frac{7}{8}$ to a decimal, it becomes .875. In pointing off a decimal into periods, begin at the decimal point and point off to the right. The first period is .875, and cipher periods may now be annexed until the root has as many figures as desired. The result will be wholly decimal. The first trial root is 10 (see Art. **60**), and the successive approximations are as follows:

$$\frac{875}{3 \times 10^2} + \frac{2 \times 10}{3} = 9.58.$$

$$\frac{875'000}{3 \times 96^2} + \frac{2 \times 96}{3} = 95.6.$$

$$\frac{875'000'000}{3 \times 956^2} + \frac{2 \times 956}{3} = 956.46.$$

$$\frac{875'000'000'000}{3 \times 9{,}565^2} + \frac{2 \times 9565}{3} = 9{,}564.65.$$

Increasing the fifth figure by 1 and locating the decimal point, we have $\sqrt[3]{\frac{7}{8}} = .95647-$. Ans.

(22) The area of 1 lot is $(25 \times 100) = 2{,}500$ sq. ft. One sq. m. $= 10.7637$ sq. ft. (See Table II.) Since the scale in square measure is 100, the decimal point must be moved two places to the right for each denomination. 1 sq. Dm. = 1,076.37 sq. ft.; hence, 1 sq. Hm. = 107,637 sq. ft. If 1 city lot has an area of 2,500 sq. ft., in 107,637 sq. ft., or 1 Ha., there are $107{,}637 \div 2{,}500 = 43$ lots. Ans.

(23) Since 1 m. = 3.28 ft. (see Table I), 100 m. $= 100 \times 3.28 = 328$ ft., the length of the field. In 1 rd. there are $16\frac{1}{2}$ ft., and in 328 ft. there are $\frac{328}{1} \div \frac{33}{2} = \frac{328}{1} \times \frac{2}{33} = \frac{656}{33}$ rd. The area of the field is 1 A., or 160 sq. rd. Dividing the area by one dimension gives the other dimension:

$$\frac{160}{1} \div \frac{656}{33} = \frac{\overset{10}{\cancel{160}}}{1} \times \frac{33}{\underset{41}{\cancel{656}}} = \frac{330}{41} = 8.0488-.$$

Hence, the width of the field is 8.0488– rd. Ans.

(24) If 1 Kg. costs 30 cents, 50 Kg. will cost $50 \times .30 = \$15.00$. Since 1 Kg. = 2.2046 lb. (see Table V), 50 Kg. $= 50 \times 2.2046 = 110.23$ lb. At 18 cents a pound, the selling price of the fish is $110.23 \times .18 = \$19.84+$. The gain is $\$19.84 - \$15 = \$4.84+$. Ans.

(25) As the longest side of a right triangle is equal to the square root of the sum of the squares of the other two sides, it is equal to

$$\sqrt{52^2+675^2} = \sqrt{52 \times 52 + 675 \times 675} = \sqrt{2{,}704+455{,}625} = \sqrt{45'83'29}.$$

Using 7 as the first trial root, the root is extracted by the following series of approximations:

$$\frac{45}{2 \times 7} + \frac{7}{2} = 6.7.$$

$$\frac{45'83}{2 \times 67} + \frac{67}{2} = 67.7.$$

$$\frac{45'83'29}{2 \times 677} + \frac{677}{2} = 677.$$

Hence, the length of the side is 677 ft. Ans.

ARITHMETIC.

(1) Area of the rectangle = base × altitude. $56\frac{1}{3} = \frac{169}{3}$; $16\frac{1}{3} = \frac{49}{3}$; $\frac{169}{3} \times \frac{49}{3} = \frac{8281}{9}$; therefore, the area of the rectangle is $\frac{8281}{9}$ sq. rd. As the area of the square is the same as the area of the rectangle, its area is also $\frac{8281}{9}$ sq. rd. The length of a side of the square is $\sqrt{\frac{8281}{9}}$ rd. $= \frac{91}{3}$ rd. $= 30\frac{1}{3}$ rd. Ans.

To extract the square root of the fraction, extract the root of the numerator and of the denominator separately; thus, $\sqrt{8,281} = 91$, and $\sqrt{9} = 3$; hence, $\sqrt{\frac{8281}{9}} = \frac{91}{3}$.

(2) Since the perimeter of a field is double the sum of the length and width, or 600 yd., then the sum of the length and width is $600 \div 2 = 300$ yd. Length = width + 80 yd.; then, substituting this expression for the length, we have, width + 80 yd. + width = 300 yd., or $2 \times$ width + 80 yd. = 300 yd. Therefore, twice the width is $300 - 80 = 220$ yd.; the width $= 220 \div 2 = 110$ yd., and the length $= 110 + 80 = 190$ yd. The area of the field is $(190 \times 110) = 20,900$ sq. yd. Since 1 A. = 4,840 sq. yd. (see Table III, § 4), the area of the field is $(20,900 \div 4,840) = 4.32-$ A. Ans.

(3) Using rule in Art. **57**, altitude $= \dfrac{2 \times 2,340}{72} = 65$ in. Having the base, 72 in., and the altitude, 65 in., the hypotenuse is found by the rule, Art. **60**. Hypotenuse $= \sqrt{72^2 + 65^2} = \sqrt{5,184 + 4,225} = \sqrt{9,409} = 97$ in. Ans.

(4) Since the diagonal of a square divides it into two right-angled triangles, the short sides of which are equal, and the diagonal is the hypotenuse, then letting l represent the length of the short side, by Art. **58**, the hypotenuse squared, or $100^2 = l^2 + l^2$; $2l^2 = 10,000$; hence, $l = \sqrt{\dfrac{10,000}{2}} = \sqrt{5,000} = 70.71+$ in. Ans.

(5) The area is 2 A., or $2 \times 160 = 320$ sq. rd. By the rule in Art. **72**, $D = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{320}{.7854}} = \sqrt{407.4357} = 20.185$ rd. Ans.

(6) Since the area is 5 A. = 800 sq. rd., one side of the square is $\sqrt{800} = 28.284$ rd., and the perimeter of the square is $4 \times 28.284 = 113.136$ rd. By the rule in Art. **72**, $D = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{800}{.7854}} = \sqrt{1{,}018.59} = 31.9153$ rd. Finding the circumference by Art. **69**, $C = 3.1416\,D = 3.1416 \times 31.9153 = 100.265$ rd. The difference between the perimeter and circumference is $113.136 - 100.265 = 12.87$ rd. Ans.

(7) The circumference of the wheel, or the number of feet it turns in 1 revolution, is $6 \times 3.1416 = 18.8496$ ft. To travel 90 mi., or $90 \times 5{,}280$ ft. $= 475{,}200$ ft., it would make $475{,}200 \div 18.8496 = 25{,}210.08+$ revolutions. Ans.

(8) Using the rule of Art. **85**, perimeter of base $= 18 \times 3.1416 = 56.5488$ in.; $56.5488 \times 24 = 1{,}357.1712$; hence, the convex surface is 1,357.1712 sq. in. Ans.

(9) Perimeter of hexagon = 8 in. $\times 6 = 48$ in. $12 \times 48 = 576$; the convex surface is, therefore, 576 sq. in. Ans.

(10) Using the rule of Art. **119**, area of upper base $= .7854\,D^2 = .7854 \times 18^2 = 254.4696$ sq. in.; area of lower base $= .7854 \times 20^2 = 314.16$ sq. in.; the square root of the product of the areas of the two bases $= \sqrt{254.4696 \times 314.16} = 282.744$. Adding these three results and multiplying by one-third the altitude, $254.4696 + 314.16 + 282.744 = 851.3736$; 2 ft. = 24 in., $851.3736 \times \frac{24}{3} = 6{,}810.9888$. The volume is, therefore, 6,810.9888 cu. in. Ans.

(11) A 7-foot square contains $7 \times 7 = 49$ sq. ft.; a 10-foot square contains $10 \times 10 = 100$ sq. ft. The area of the third square is $49 + 100 = 149$ sq. ft.; therefore, the length of a side $= \sqrt{149} = 12.207-$ ft. Ans.

(12) Using the rule in Art. **88**, area of base $= .7854 \times 40^2 = 1{,}256.64$ sq. in.; altitude = 12 ft., or 144 in.; therefore, the volume of the cylinder $= 1{,}256.64 \times 144 = 180{,}956.16$ cu. in. According to Art. **21**, § 4, 1 gal. contains 231 cu. in.; hence, the tank holds $\frac{180956.16}{231} = 783.36$ gal. At 8 cents a gallon, the oil is worth $783.36 \times .08 = \$62.6688$, or \$62.67. Ans.

(13) Use rule, Art. **99**. Volume of bin $= 12 \times 8 \times 6 = 576$ cu. ft.; or $576 \times 1{,}728 = 995{,}328$ cu. in. In 1 heaped bushel there are 2,747.71 cu. in.; therefore, in 995,328 cu. in. there are $995{,}328 \div 2{,}747.71 = 362.24-$ bu. Ans.

(14) (*a*) The diameter is the diagonal of the square, and the hypotenuse of a right-angled triangle, the other two sides of which are equal, being the sides of a square. By Art. **58,** the square of the hypotenuse is equal to the sum of the squares of the other two sides; then, 48^2, or 2,304, is equal to double the square of one of the sides of the square. Hence, the side of the square is $\sqrt{\frac{2304}{2}}$, and the area of the square is $\left(\sqrt{\frac{2304}{2}}\right)^2 = \frac{2304}{2} = 1{,}152$ sq. in. Ans.

(*b*) Area of circle $= .7854\,D^2$ (Art. **71**) $= .7854 \times 48^2 = .7854 \times 2{,}304 = 1{,}809.5616$ sq. in. Area of square $= 1{,}152$ sq. in.; then the zinc cut off $= 1{,}809.5616 - 1{,}152 = 657.5616$ sq. in. Ans.

(15) Area of the larger outside circle $= 3.1416\ R^2$ (Art. **71**) $= 3.1416 \times 24^2 = 1{,}809.5616$ sq. ft. Area of the inner circle $= 3.1416\ R^2 = 3.1416 \times 15^2 = 706.86$ sq. ft. Number of square feet of the larger circle outside of the inner circle $= 1{,}809.5616 - 706.86 = 1{,}102.7016$ sq. ft. Ans.

(16) 8 ft. 6 in. = 102 in. Volume or contents $= 102 \times 102 \times 102 = 1{,}061{,}208$ cu. in. According to Art. **21,** § 4, 1 gal. contains 231 cu. in. Hence, the cask can hold $\frac{1{,}061{,}208}{231} = 4{,}593.974$ gal. If the faucet discharges 5 gal. per minute, it will take it $\frac{4{,}593.974}{5} = 918.795$ min., or 15 hr. 18.795 min. Ans.

(17) Since 1 bu. contains 2,150.42 cu. in., 1,000 bu. contain $1{,}000 \times 2{,}150.42 = 2{,}150{,}420$ cu. in. Since the volume or cubical contents is 2,150,420 cu. in., the length of an edge $= \sqrt[3]{2{,}150{,}420} = 129+$ in., or 10.75+ ft. Ans.

(18) See Art. **127.** Mean diameter $= \frac{50 + 56}{2} = 53$ in. Length $= 6$ ft. $= 72$ in. Capacity $= 53^2 \times 72 \times .0034 = 687.64+$ gal. Ans.

(19) First find the diagonal of the floor, which is evidently the hypotenuse of a right-angled triangle whose sides are 14 ft. By Art. **60,** hypotenuse $= \sqrt{14^2 + 14^2} = 19.8$ ft. The required line is the hypotenuse of a right triangle, of which the height of the room, 14 ft., is one short side, and the floor diagonal, 19.8 ft., is the other short side; therefore, the length of the line is $\sqrt{19.8^2 + 14^2} = 24.25-$ ft. Ans.

(20) Since the sphere fits exactly inside a cubical box, its diameter is equal to the side or edge of the cube. Edge $= \sqrt[3]{8} = 2$ ft. = diameter of sphere. By Art. **124,** volume of sphere $= .5236 \times 2^3 = 4.1888$ cu. ft. Ans.

(21) Contents of the car $= 40 \times 6\frac{1}{2} \times 4\frac{1}{2} = 1{,}170$ cu. ft. According to Art. **107**, a ton of Schuylkill coal measures 35 cu. ft. Number of tons in car $= \frac{1170}{35} = 33.4+$ T. Ans.

(22)

Perimeter of first room	$= 12 \times 2 + 14 \times 2 =$	52 ft.
Perimeter of second room	$= 15 \times 2 + 15 \times 2 =$	60 ft.
Perimeter of third room	$= 16 \times 2 + 18 \times 2 =$	68 ft.
Perimeter of fourth room	$= 15 \times 2 + 19 \times 2 =$	68 ft.
Entire perimeter of rooms...............		248 ft.

Area of ceiling of first room	$= 12 \times 14 =$	168 sq. ft.
Area of ceiling of second room	$= 15 \times 15 =$	225 sq. ft.
Area of ceiling of third room	$= 16 \times 18 =$	288 sq. ft.
Area of ceiling of fourth room	$= 15 \times 19 =$	285 sq. ft.
Area of ceilings.....................		966 sq. ft.

Area of walls	$= 248 \times 9\frac{1}{3} =$	$2{,}314\frac{2}{3}$ sq. ft.
Area of ceilings.........	$=$	966 sq. ft.
Total area..........		$3{,}280\frac{2}{3}$ sq. ft.

Area in square yards $= 3{,}280\frac{2}{3} \div 9 = 364.52-$.
Cost $= 364.52 \times \$.0475 = \$17.31+$. Ans.

(23) See rule I, Art. **37**. Perimeter of room $= 2 \times 16 + 2 \times 20\frac{1}{2} = 73$ ft. Width of openings $= 5 \times 3 + 3 \times 3 = 24$ ft. Perimeter, allowing for openings, $= 73 - 24 = 49$ ft. $= 16\frac{1}{3}$ yd. $= 32\frac{2}{3}$ half-yards; hence, 33 strips are required. Assuming that the strips extend the height of the baseboard above the bottom edge of the border, the length of a strip is (since 18 in. $= 1\frac{1}{2}$ ft.) $11 - 1\frac{1}{2} = 9\frac{1}{2}$ ft. The number of strips in a double roll is therefore $16 \times 3 \div 9\frac{1}{2} = 5$, and the number of rolls required is $33 \div 5 = 6\frac{3}{5}$, or say 7 double rolls. Ans.

The number of strips in a single roll of 8 yd., or 24 ft. $= 24 \div 9\frac{1}{2} = 2$ strips, and the number of single rolls required is $33 \div 2 = 16\frac{1}{2}$, or say 17. Ans.

Since the perimeter of the room is 73 ft. $= 24\frac{1}{3}$ yd., the number of double rolls of border required is $24\frac{1}{3} \div 16 = 1.52$, say 2. Ans.

(24) The question does not state which way the strips are to run, but it is economical to have them run lengthwise. Width of room $= 15$ ft. $= 180$ in. Width of carpet $= 27$ in. The number of strips $= 180 \div 27 = 6\frac{2}{3}$. Hence, 7 strips must be used. Allowing 1 ft. for matching, length of strip $= 17 + 1 = 18$ ft. $= 6$ yd. Number of yards required $= 7 \times 6 = 42$. Cost $= 42 \times \$1.25 = \$52.50$. Ans.

(25) Outside length of wall $= 2 \times 145 + 2 \times 75 = 440$ ft.
Net length of wall $= 440 - 4 \times 1 = 436$ ft. (see Art. **92**).
Contents of wall $= 436 \times 30 \times 1 = 13{,}080$ cu. ft.
Deduction for windows $= 7 \times 3\frac{1}{2} \times 1 \times 110 = 2{,}695$ cu. ft.
Deduction for 4 doors $= 8 \times 10 \times 1 \times 4 = 320$ cu. ft.
Deduction for 2 doors $= 6 \times 8 \times 1 \times 2 = 96$ cu. ft.
Net contents of wall $= 13{,}080 - (2{,}695 + 320 + 96) = 9{,}969$ cu. ft.
Number of bricks $= 9{,}969 \times 23\frac{3}{11} = 232{,}005$, or 232.005 M.
Cost of bricks $= 232.005 \times \$5.60 = \$1{,}299.23$. Ans.
Cost of laying $= 232.005 \times \$1.45 = \$336.41$. Ans.

ARITHMETIC.

(1) (*a*) The ratio $\frac{3}{4}:\frac{4}{5}=\frac{\frac{3}{4}}{\frac{4}{5}}=\frac{3}{4}\times\frac{5}{4}=\frac{15}{16}$; $\frac{15}{16}\div 5=\frac{\overset{3}{\cancel{15}}}{16}\times\frac{1}{\cancel{5}}=\frac{3}{16}$. Ans.

(*b*) The ratio $.2:.05=\frac{.2}{.05}=4$; $4\div 4=4\times\frac{1}{4}=1$. Ans.

(*c*) The inverse ratio of 12.5 to 125 is

$$125:12.5=\frac{125}{12.5}=10;\ 10\div\frac{3}{2}=\frac{10}{1}\times\frac{2}{3}=6\tfrac{2}{3}.\quad \text{Ans.}$$

(2) (*a*) The ratio $\frac{4}{3}:\frac{3}{4}=\frac{\frac{4}{3}}{\frac{3}{4}}=\frac{4}{3}\times\frac{4}{3}=\frac{16}{9}$; $\frac{16}{9}\times 3=\frac{48}{9}=5\tfrac{1}{3}$. Ans.

(*b*) The volume of a gallon is 231 cu. in., a cubic foot is 1,728 cu. in. The ratio is 231 : 1,728, or $\frac{231}{1{,}728}$; $\frac{231}{\underset{64}{\cancel{1{,}728}}}\times\frac{\cancel{27}}{1}=\frac{231}{64}=3.6+$. Ans.

(*c*) The inverse ratio of $33\tfrac{1}{3}$ to $187\tfrac{1}{2}=187\tfrac{1}{2}:33\tfrac{1}{3}=\frac{187\frac{1}{2}}{33\frac{1}{3}}=\frac{\frac{375}{2}}{\frac{100}{3}}$

$=\frac{\overset{15}{\cancel{375}}}{2}\times\frac{3}{\underset{4}{\cancel{100}}}=\frac{45}{8}$. $\frac{45}{8}\times 4=\frac{45}{2}=22.5$. Ans.

(3) (*a*) The inverse ratio of $7.2:5=5:7.2=\frac{5}{7.2}=\frac{5}{\frac{72}{10}}=5\times\frac{10}{72}=\frac{50}{72}$. Since multiplying both terms of the fraction by the same number does not change the value of the fraction, $\frac{50}{72}=\frac{50\times 2}{72\times 2}=\frac{100}{144}$. $\sqrt{\frac{100}{144}}=\frac{\sqrt{100}}{\sqrt{144}}$ $=\frac{10}{12}=\frac{5}{6}$. Ans.

(*b*) The inverse ratio of 16 to 54 = 54 : 16 = $\frac{54}{16}$. Multiplying both terms of the fraction by 4, $\frac{54}{16}=\frac{216}{64}$. $\sqrt[3]{\frac{216}{64}}=\frac{\sqrt[3]{216}}{\sqrt[3]{64}}=\frac{6}{4}=1.5$. Ans.

(4) (a) $20 : 23 = 39 : 45$. The products $20 \times 45 = 900$ and $23 \times 39 = 897$ are not equal, and the numbers cannot form a proportion, since in a proportion the product of the means must be equal to the product of the extremes (see Art. **25**).

(b) $20 : x = 39 : 45$; $39x = 900$, $x = 23\frac{1}{13}$. Hence, 23 must be increased by $\frac{1}{13}$. Ans.

(5) 2 lb. 3 oz. = 35 oz.; 5 lb. $1\frac{1}{2}$ oz. = $81\frac{1}{2}$ oz.
The ratio is $35 : 81\frac{1}{2}$; multiplying both terms of the ratio by 2, it becomes $70 : 163$. Ans.

(6) $3 : 8 = \frac{3}{8}$ and $5 : 12 = \frac{5}{12}$, $\frac{3}{8} = \frac{9}{24}$; $\frac{5}{12} = \frac{10}{24}$. $\frac{10}{24} - \frac{9}{24} = \frac{1}{24}$.

Hence, the ratio $5 : 12$ is greater by $\frac{1}{24}$. Ans.

(7) Here the vertical line is used instead of the colon to express the ratio (see Art. **2**). The product of all the numbers included between the vertical lines is equal to the product of all the numbers without them; hence,

$$\begin{array}{c|c|c} \frac{12}{15} & \frac{3}{5} & \\ \frac{2}{3} & \frac{5}{9} = x & 20 \\ 7 & 10 & \end{array}$$

$$\frac{12}{15} \times \frac{2}{3} \times 7 \times 20 = \frac{3}{5} \times \frac{5}{9} \times 10 \times x, \text{ or } x = \frac{\frac{12}{15} \times \frac{2}{3} \times 7 \times 20}{\frac{3}{5} \times \frac{5}{9} \times 10}$$

$$= \frac{\overset{4}{\cancel{12}} \times 2 \times 7 \times \overset{2}{\cancel{20}} \times \cancel{5} \times \overset{3}{\cancel{9}}}{\underset{5}{\cancel{15}} \times \cancel{3} \times \cancel{3} \times \cancel{5} \times \cancel{10}} = \frac{112}{5} = 22.4. \text{ Ans.}$$

(8) Let x = price paid for a grindstone 4 ft. in diameter and 9 in. thick. Then, according to the conditions of the problem, the prices of the two grindstones are to each other as the product of the thickness and the square of the diameter of the smaller stone is to the product of the thickness and the square of the diameter of the larger stone. Hence, $5 \times 2^2 : 9 \times 4^2 = \$5 : x$. $20x = \$720$. $x = \$36$. Ans.

(9) Writing first the direct proportion, we have $17.8 : 67.2 = 10 : x$. Since, however, the volume varies inversely as the pressure, the proportion is inverse and must be written $17.8 : 67.2 = x : 10$. From this,

$$x = \frac{17.8 \times 10}{67.2} = 2.65-.$$

Hence, the volume at the higher pressure is 2.65 cu. ft., nearly. Ans.

(10) If 5 is added to the antecedents of the proportion $2 : 5 = 6 : 15$, the proportion will read $7 : 5 = 11 : 15$, which is incorrect, since the product of the means is less than that of the extremes. Denoting the first consequent by x, $7 : x = 11 : 15$; from which $11x = 105$, and $x = 9\frac{6}{11}$. Hence, $9\frac{6}{11} - 5 = 4\frac{6}{11}$ must be added to the first consequent. Ans.

Denoting the second consequent by x, $7:5 = 11:x = 7x = 55$, $x = 7\frac{6}{7}$. Hence, $15 - 7\frac{6}{7} = 7\frac{1}{7}$ must be subtracted from second consequent. Ans.

(11) According to Art. **50,** the volume of a cylinder varies directly as its length and directly as the square of the diameter. Taking the dimensions for the causes and squaring the diameters,

$$\begin{array}{c|c|l} 3^2 & 5^2 & \\ 5 & 9 = 100 & x, \end{array} \text{ or } \begin{array}{c|c|l} & 5 & \\ \cancel{9} & \cancel{25} & \\ \cancel{5} & \cancel{9} = 100 & x; \end{array}$$

whence, $x = 500$. The second cylinder will therefore hold 500 gal. Ans.

(12) Since the cylinders have equal volumes but different diameters, the diameters are to each other inversely as the square roots of their lengths (see Art. **41**). The direct proportion would be

$$15 : x = \sqrt{49} : \sqrt{36}.$$

Therefore the inverse proportion is

$$15 : x = \sqrt{36} : \sqrt{49}, \text{ or } 15 : x = 6 : 7; \text{ whence, } x = \frac{7 \times 15}{6} = 17.5.$$

The diameter of the second cylinder is, therefore, 17.5 in. Ans.

(13) Since $3:7 = 8:12$ is not a correct proportion, substituting x for the first term, the proportion is $x:7 = 8:12$, from which $x = \frac{56}{12} = 4\frac{2}{3}$. Hence, $4\frac{2}{3} - 3 = 1\frac{2}{3}$ is to be added to the first term. Ans.

Substituting x for the second term, $3:7 = 8:12$, whence $x = \frac{36}{8} = 4\frac{1}{2}$. Hence, $7 - 4\frac{1}{2} = 2\frac{1}{2}$ must be subtracted from the second term to form a correct proportion. Ans.

(14) Since 5 men do something, 5 men is a cause, and since they take 9 da. of 10 hr. each to do the work, 9 da. and 10 hr. are each an element of the first cause. For the same reason 8 men working x da. of 9 hr. each constitute a second cause. The first effect, that which is done, is a wall 100 ft. long, 6 ft. high, and 18 in. thick, and the second effect is a wall 120 ft. long, 8 ft. high, and 20 in. thick. Hence, we write the following proportion and cancel where possible:

$$\begin{array}{c|cc|c} & & \cancel{10} & 20 \\ 5 & \cancel{8} & \cancel{100} & \cancel{120} \\ \cancel{9} & x = & \cancel{6} & \cancel{8} \\ & & 9 & \cancel{2} \\ \cancel{10} & \cancel{9} & \cancel{18} & \cancel{20} \end{array}, \text{ or } x = \frac{20 \times 5}{9} = \frac{100}{9} = 11\tfrac{1}{9}.$$

The required time is $11\frac{1}{9}$ da., and since a day's work consists of 9 hr., $\frac{1}{9}$ da. $= 1$ hr.; hence, $11\frac{1}{9}$ da. $= 11$ da. 1 hr. Ans.

(15) Letting $x =$ the length of the second pendulum, then the proportion is $\sqrt{39}$ in. : $\sqrt{x} = 1:2$. Squaring each term of the proportion, 39 in.: $x = 1:4$, or $x = 156$ in. Ans.

(16) See rule, Art. **56.** Sum of proportional parts $= 2+3+5 = 10$. The three proportions are therefore:

$$2:10 = x:\$4{,}750;$$
$$3:10 = x:\$4{,}750;$$
$$5:10 = x:\$4{,}750.$$

Hence, $\quad$ 1st man's share $= \dfrac{\$4{,}750 \times 2}{10} = \$950$. Ans.

2d man's share $= \dfrac{\$4{,}750 \times 3}{10} = \$1{,}425$. Ans.

3d man's share $= \dfrac{\$4{,}750 \times 5}{10} = \$2{,}375$. Ans.

(17) By inspection $\dfrac{1}{2} \div \dfrac{2}{3} = \dfrac{3}{4}$; and $\dfrac{15}{28} \div \dfrac{5}{7} = \dfrac{3}{4}$. Since the ratios are equal, those numbers will form the proportion $\dfrac{1}{2} : \dfrac{2}{3} = \dfrac{15}{28} : \dfrac{5}{7}$. Reducing to common denominator, $\frac{42}{84} : \frac{56}{84} = \frac{45}{84} : \frac{60}{84}$.

Multiplying each term by 84, $42 : 56 = 45 : 60$. Ans.

Proof.—The ratio of the first couplet $\frac{42}{56} = \frac{3}{4}$, and that of the second is $\frac{45}{60} = \frac{3}{4}$. Hence, the proportion is correct.

(18)
$$\begin{array}{r|l} 4^2 & 5^2 \\ 50 & 120 = 5{,}385.6 \text{ gal.} \\ 33 & 44 \end{array} \Bigg| \begin{array}{r|l} & \cancel{2} \\ \cancel{4} & 5 \\ \cancel{16} & \cancel{10} \\ \cancel{2} & \cancel{40} \\ x, \text{ or } \cancel{50} & \cancel{120} = 5{,}385.6 \text{ gal.} \\ \cancel{33} & \cancel{44} \\ \cancel{3} & \cancel{4} \end{array} \Bigg| \; x;$$

whence $x = 5 \times 5{,}385.6$ gal. $= 26{,}928$ gal. Ans.

(19) Reducing the fractions to the least common denominator and adding, $\dfrac{1}{2} + \dfrac{2}{3} + \dfrac{3}{4} = \dfrac{6+8+9}{12} = \frac{23}{12}$. Multiplying all parts by 12, which does not change their relative value, $6+8+9 = 23$. Applying rule, Art. **56,**

1st part, $6:23 = x:100$, or $23x = 600$; $x = 26\frac{2}{23}$. Ans.
2d part, $8:23 = x:100$, or $23x = 800$; $x = 34\frac{18}{23}$. Ans.
3d part, $9:23 = x:100$, or $23x = 900$; $x = 39\frac{3}{23}$. Ans.

(20) Since the weights vary as the cubes of their lengths, the proportion is $20^3 : 30^3 = 10 : x$, or $8{,}000 : 27{,}000 = 10 : x$.

$$x = \frac{27 \times 10}{8} = 33\tfrac{3}{4} \text{ lb.} \quad \text{Ans.}$$

(21) Here taking \$250, 2 yr. 6 mo., and 8% as the first cause and \$450, 3 yr. 10 mo., and 6% as the second cause, \$50 for the first effect, x for the second effect, and reducing the time to months, we have

$$\left.\begin{array}{r|l} 250 & 450 \\ 30 & 46 \\ 8 & 6 \end{array} = 50 \right| x.$$

Canceling where possible,

$$\left.\begin{array}{r|l} & 3 \\ \cancel{5} & \cancel{15} \\ \cancel{250} & \cancel{450} \\ \cancel{30} & 23 \\ \cancel{8} & \cancel{46} \\ \cancel{4} & \cancel{6} \\ 2 & 3 \end{array} = \cancel{50} \right| x,$$

from which

$$x = \frac{3 \times 23 \times 3}{2} = \frac{207}{2} = 103.5.$$

Hence, the interest is \$103.50. Ans.

(22) Since the distances through which bodies will fall vary as the squares of the times, we have the proportion $144 : x = 3^2 : 5^2$. Squaring, $144 : x = 9 : 25$, or

$$x = \frac{25 \times \overset{16}{\cancel{144}}}{\cancel{9}} = 400 \text{ ft. Ans.}$$

(23) The radii of the spheres are respectively $\frac{12}{2} = 6$ in. and $\frac{30}{2} = 15$ in. Since the weight of spheres of similar material vary as the cubes of their radii, we have the proportion $6^3 : 15^3 = 371.75 : x$. Dividing both terms of the first couplet by 3^3 (see Art. **13**), $2^3 : 5^3 = 371.75 : x$, or $8 : 125 = 371.75 : x$; whence,

$$x = \frac{125 \times 371.75}{8} = 5{,}808.59+ \text{ lb. Ans.}$$

(24) Since the times of oscillation of a pendulum vary directly as the square roots of the length of the pendulum, the proportion is

$$\sqrt{39} : \sqrt{20} = 1 : x.$$

Squaring the terms, $39 : 20 = 1 : x^2$, or $x^2 = \frac{20}{39} = .51282+$; whence, $x = \sqrt{.51282} = .7161$ sec. Ans.

(25) The capacities of the bins are proportional respectively to the products, $2 \times 3 \times 5 = 30$; $3 \times 4 \times 6 = 72$; and $4 \times 5 \times 6 = 120$. Using the rule of Art. **56**, $30 + 72 + 120 = 222$. Hence, the proportions are

$$30 : 222 = x : 3{,}700, \text{ or } x = \frac{3{,}700 \times 30}{222} = 500 \text{ bu. Ans.}$$

$$72 : 222 = x : 3{,}700, \text{ or } x = \frac{3{,}700 \times 72}{222} = 1{,}200 \text{ bu. Ans.}$$

$$120 : 222 = x : 3{,}700, \text{ or } x = \frac{3{,}700 \times 120}{222} = 2{,}000 \text{ bu. Ans.}$$

ARITHMETIC.

(1) See Art. **3.** $\frac{1}{2}\% = \frac{\frac{1}{2}}{100} = \frac{1}{2} \times \frac{1}{100} = \frac{1}{200} = .005.$ Ans.

$\frac{3}{4}\% = \frac{\frac{3}{4}}{100} = \frac{3}{4} \times \frac{1}{100} = \frac{3}{400} = .0075.$ Ans.

$1\frac{3}{8}\% = \frac{1\frac{3}{8}}{100} = \frac{11}{8} \times \frac{1}{100} = \frac{11}{800} = .01375.$ Ans.

$\frac{5}{8}$ of $1\% = \frac{5}{8}$ of $\frac{1}{100} = \frac{5}{800} = .00625.$ Ans.

$\frac{2}{3}$ of $\frac{3}{4}\% = \frac{2}{3}$ of $\frac{3}{400} = \frac{2}{\cancel{3}} \times \frac{\cancel{3}}{\underset{200}{\cancel{400}}} = \frac{1}{200} = .005.$ Ans.

(2) The rate per cent. is $6\frac{1}{4}$; that is, $6\frac{1}{4}\%$ of the entire crop is potatoes. The rate is $.06\frac{1}{4} = .0625$. 1,575 bu. is the difference between the entire crop and the number of bushels of potatoes planted. The base is the number of bushels in the crop. Then, by Art. **30,** base = difference ÷ (1 − rate); whence, base = 1,575 ÷ (1 − .0625) = 1,575 ÷ .9375 = 1,680 bu. Ans.

(3) $37\frac{1}{2}\% + 12\frac{1}{2}\% + 16\frac{2}{3}\% = 66\frac{2}{3}\%$, the total per cent. of the salary expended; hence, $100\% - 66\frac{2}{3}\% = 33\frac{1}{3}\%$ = per cent. saved. Hence, \$90 must be $33\frac{1}{3}\%$ of the month's salary. $33\frac{1}{3}\% = \frac{1}{3}$. Since \$90 is $\frac{1}{3}$ of the month's salary, \$90 × 3 = \$270 must be the month's salary. The salary per year is therefore 12 × \$270 = \$3,240. Ans.

(4) If goods are bought at 20% below list price, they cost 100% − 20% = 80% of list price. If the gain is 30%, then by Art. **40,** selling price = cost × (1 + rate of gain) = 80 × (1 + .30) = 80 × 1.30 = 104% of list price. 104% of list price is 104% − 100% = 4% above list price. Ans.

(5) The selling price of each horse is \$120; the rate of gain on one horse is .25 and the rate of loss on the other is .25. To find the cost of each horse we use the rule of Art. **41.** Cost of 1st horse = selling

price ÷ (1 + rate of gain) = \$120 ÷ 1.25 = \$96. Cost of 2d horse = selling price ÷ (1 − rate of loss) = \$120 ÷ .75 = \$160. Gain on 1st horse, \$120 − \$96 = \$24. Loss on 2d horse, \$160 − \$120 = \$40. Net loss = \$40 − \$24 = \$16. Ans.

(6) From the statement of the question, B's buying price is 120% of A's buying price, and C's buying price is 125% of B's. 1.20 × 1.25 = 1.50; hence, C's buying price is 150% of A's buying price, that is, C paid 50% more for the horse than A originally paid for it. But from the statement, C paid \$60 more than A; hence, \$60 must be 50% of what the horse cost A.

$$\text{Original cost} = \text{base} = \frac{\text{percentage}}{\text{rate}} = \frac{\$60}{.50} = \$120. \quad \text{Ans.}$$

B's buying price = 120% of \$120 = \$120 × 1.20 = \$144. Ans.

C's buying price = original cost + \$60 = \$120 + \$60 = \$180. Ans.

(7) Of 206.95 + 32.06 = 239.01 parts of ore there are 206.95 parts of lead; hence, the lead constitutes $\frac{206.95}{239.01} = .865863+ = 86.5863+\%$ of the ore. Ans. Likewise the sulphur constitutes $\frac{32.06}{239.01} = .134137- = 13.4137-\%$ of the ore. Ans. Evidently in this example, the 239.01 is the base and the 206.95 and 32.06 are percentages; hence, to find the rates, we divide the percentages by the base.

(8) After losing 35% of his money he had 100% − 35% = 65% of it left. Since he lost also 10% of the remainder = .10 × .65 = .065 = 6.5% of his money, he had remaining 65% − 6.5% = $58\frac{1}{2}\%$ of it. \$17,550 is $58\frac{1}{2}\%$ of the original sum of money; hence, \$17,550 is the percentage, $58\frac{1}{2}$ is the rate per cent., and the original sum is the base.

$$\text{Base} = \frac{\text{percentage}}{\text{rate}} = \frac{\$17{,}550}{.58\frac{1}{2}} = \$30{,}000.$$

Therefore, he had originally \$30,000. Ans.

(9) According to the statement of the question, the man earned the second year \$120 for every \$100 earned the first year; and for every \$120 earned the second year, he earned 25% more, or \$120 × (1 + .25) = \$150, the third year. The amounts earned in the three years are proportional, therefore, to the numbers 100, 120, and 150. Using the rule of Art. **56**, § 7, the sum of proportional parts = 100 + 120 + 150 = 370, and the three proportions are:

100 : 370 = earnings first year : \$7,400.
120 : 370 = earnings second year : \$7,400.
150 : 370 = earnings third year : \$7,400.

From these proportions

$$\left.\begin{array}{ll} \text{Earnings first year} & = \dfrac{\$7{,}400 \times 100}{370} = \$2{,}000. \\ \text{Earnings second year} & = \dfrac{\$7{,}400 \times 120}{370} = \$2{,}400. \\ \text{Earnings third year} & = \dfrac{\$7{,}400 \times 150}{370} = \$3{,}000. \end{array}\right\} \text{Ans.}$$

(10) By Art. **41,** the cost = selling price ÷ (1 + rate of gain) = \$140 ÷ (1 + .12) = \$140 ÷ 1.12 = \$125. To gain 28%, the selling price must be cost × (1 + rate of gain) = \$125 × (1 + .28) = \$125 × 1.28 = \$160. Ans.

(11) By Art. **83,** 8 francs = 8 × \$.193 = \$1.544; therefore, 18,600 yd. cost 18,600 × \$1.544 = \$28,718.40. Duty = \$28,718 × .22 = \$6,317.96. Ans. The \$.40 is not considered, as it is less than \$.50 (see Art. **85**).

(12) Commission and guarantee = \$8,407.96 × (.02 + .015) = \$8,407.96 × .035 = \$294.28. Total charges = \$294.28 + \$275 = \$569.28. Amount sent to principal = \$8,407.96 − \$569.28 = \$7,838.68. Ans.

(13) Tax on real estate = \$7,800 × .0125 = \$97.50. Tax on personal property = \$5,640 × .0125 = \$70.50. Poll tax = \$.75 × 3 = \$2.25. Total tax = \$97.50 + \$70.50 + \$2.25 = \$170.25. Ans.

(14) By Art. **49,** 1 − .20 = .80; whence, .80 × .80 = .64; single equivalent discount = 1 − .64 = .36 = 36%. Difference in favor of single discount = 40% − 36% = 4%. Gain by taking single discount = \$987.50 × .04 = \$39.50. Ans.

(15) In selling 3 oranges the boy receives the cost of 5 oranges. Hence, his profit on the 3 oranges is equal to the cost of 5 − 3 = 2 oranges. Since in selling 3 oranges he gains the cost of 2 oranges, his gain is $\frac{2}{3}$ of the cost. $\frac{2}{3} = 66\frac{2}{3}\%$. Ans.

(16) By Art. **72,** amount of insurance = premium ÷ rate of premium = \$267 ÷ .015 = \$17,800. Ans.

(17) By Art. **71,** premium = \$75,375 × .0075 = \$565.31+. Ans.

(18) Total expenses = $2\frac{1}{2}\% + \frac{1}{4}\% = 2\frac{3}{4}\%$ of gross proceeds. The net proceeds, \$6,613, is the difference, $2\frac{3}{4}$ is the rate per cent., and the gross proceeds is the base (see Art. **61**); hence, gross proceeds = net proceeds ÷ (1 − rate) = \$6,613 ÷ $(1 - .02\frac{3}{4})$ = \$6,613 ÷ .9725 = \$6,800. Ans.

(19) See Art. **64.** Deducting drayage and storage, the agent has \$520.45 − (\$18.75 + \$8.50) = \$493.20 to pay for peaches and his commission. Since his commission is computed on the prime cost as a base,

\$493.20 must be the amount; hence, prime cost = \$493.20 ÷ (1 + .0275) = \$493.20 ÷ 1.0275 = \$480. At \$.75 a crate, he bought 480 ÷ .75 = 640 crates. Ans.

(20) By Art. **72,** amount of insurance = premium ÷ rate of premium, or \$372 ÷ .015 = \$24,800. Since \$24,800 is $\frac{4}{5}$, or 80%, of the value of the store and stock, \$24,800 ÷ .80 = \$31,000 is the total value of store and stock. Since the value of the store is to the contents as 9 is to 11, we have by the rule for proportional parts the following proportions:

Value of store : \$31,000 :: 9 : 20; whence, value of

$$\text{store} = \frac{\$31{,}000 \times 9}{20} = \$13{,}950. \quad \text{Ans.}$$

Contents : \$31,000 :: 11 : 20; whence,

$$\text{contents} = \frac{\$31{,}000 \times 11}{20} = \$17{,}050. \quad \text{Ans.}$$

(21) For the first series (1 — .20) (1 — .15) (1 — .05) = .80 × .85 × .95 = .646, and the equivalent single discount is 1.000 — .646 = .354 = 35.4%.

For the second series (1 — .20) (1 — .10) (1 — .10) = .80 × .90 × .90 = .648, and the equivalent single discount is 1.000 — .648 = 35.2%.

The difference is 35.4% — 35.2% = .2%; hence, \$25.48 is .2% of the amount of the bill, without discount. Since \$25.48 is the percentage and .2 is the rate per cent., the base or amount of the bill is \$25.48 ÷ .002 = \$12,740. Ans.

(22) By Art. **71,** premium = \$25,000 × .00875 = \$218.75. Premium for amount reinsured in second company = \$25,000 × .28 × .005 = \$35. Premium for amount reinsured in the third company = \$25,000 × .48 × .0075 = \$90. Amount paid out = \$35 + \$90 = \$125. The company reinsured 28% + 48% = 76% of the risk. On this portion of the risk they received a premium of \$25,000 × .76 × .00$\frac{7}{8}$ = \$166.25; the company therefore gains \$166.25 — \$125 = \$41.25. Ans.

(23) A discount of 20% on $\frac{3}{4}$ of a bill is equivalent to a discount of 20% × $\frac{3}{4}$ = 15% on the whole bill. A discount of 15% on the remainder, or $\frac{1}{4}$ of the bill, is equivalent to a discount of 15% × $\frac{1}{4}$ = 3.75% on the whole. The two discounts are therefore equivalent to a single discount of 15% + 3.75% = 18.75% on the whole bill, which is 2.75% greater than a discount of 16%. From the statement of the question, \$324.50 is 2.75% of the whole bill; hence, the amount of the bill is \$324.50 ÷ .0275 = \$11,800. Ans.

(24) Allowing the first discount series, the dealer then receives (1 — .20) (1 — .10) (1 — .10) = .80 × .90 × .90 = .648 = 64.8% of his list price. With the second series he receives (1 — .25) (1 — .15) (1 — .05)

$= .75 \times .85 \times .95 = .605625 = 60.5625\%$ of the list price. Difference, $64.8\% - 60.5625\% = 4.2375\%$. Since \$33.90 is 4.2375% of the list price, this price is $\$33.90 \div .042375 = \$800$. Ans.

(25) A gain of 20% on 20% of the stock is equivalent to a gain of $.20 \times .20 = .04 = 4\%$ on the whole stock. After selling 20% of the stock there is 80% of it remaining. 30% of this remainder is $.80 \times .30 = .24 = 24\%$ of the original stock. Since this was sold for 1.25 of the cost, the gain was 25%. A gain of 25% on 24% of the stock is equivalent to a gain of $.24 \times .25 = .06 = 6\%$ on the original stock. The remainder of the stock, after selling 20% and 24% of it, is 56% $(100 - 20 - 24)$ of it. By selling this for $\frac{7}{8}$ of its cost, he loses $\frac{1}{8}$, or $12\frac{1}{2}\%$. A loss of $12\frac{1}{2}\%$ on 56% of the stock is equivalent to a loss of $.56 \times .125 = .07 = 7\%$. The total gain is $4\% + 6\% = 10\%$, and the loss is 7%; hence, the net gain is $10\% - 7\% = 3\%$ of the cost. Ans.

ARITHMETIC.

(1) At $3\frac{4}{5}\%$ per year the rate for 5 yr. $= 5 \times 3\frac{4}{5}\% = 19\%$.

$$
\begin{array}{rl}
\$\,5{,}6\,2\,8.4\,0 & \\
.1\,9 & \\
\hline
\$\,1{,}0\,6\,9.3\,9\,6\,0 & = \text{interest for 5 yr.} \\
1\,0\,6.9\,3\,9\,6 & = \text{interest for 6 mo.} = \text{5 yr.} \div 10. \\
1\,7.8\,2\,3\,3 & = \text{interest for 1 mo.} = \text{6 mo.} \div 6. \\
8.9\,1\,1\,6 & = \text{interest for 15 da.} = \text{1 mo.} \div 2. \\
2.9\,7\,0\,5 & = \text{interest for 5 da.} = \text{15 da.} \div 3. \\
5.3\,4\,7\,0 & = \text{interest for 9 da.} = \text{1 mo.} \times .3. \\
\hline
\end{array}
$$

$1,2 1 1.3 8 8 0, or $1,211.39 = interest for 5 yr. 7 mo. 29 da. Ans.

(2)

$$
\begin{array}{rl}
\text{Interest of \$1 at 6\% for 1 yr.} & = \$.0\,6 \\
\text{Interest of \$1 at 6\% for 11 mo.} & = .0\,5\,5 \\
\text{Interest of \$1 at 6\% for 27 da.} & = .0\,0\,4\,5 \\
\hline
\text{Interest of \$1 at 6\% for 1 yr. 11 mo. 27 da.} & = \$.1\,1\,9\,5
\end{array}
$$

Interest at 6% $= \$.1195 \times 5{,}670.80 = \$677.6606$.
Interest at 1% $= \$677.6606 \div 6 = \$112.9434$.
Interest at $4\frac{1}{2}\%$ $= \$112.9434 \times 4\frac{1}{2} = \$508.2453+$, or $508.25. Ans.

(3) By Art. **20**,

$$
\begin{array}{llll}
\text{Interest for 6 0 da.} & & & = \$\,4\,6.8\,9\,5 \\
\text{Interest for 2 0 da.} & = & \frac{1}{3} \text{ of 60 da.} & = \$\,1\,5.6\,3\,1\,7 \\
\text{Interest for 6 da.} & = & \frac{1}{10} \text{ of 60 da.} & = \$\quad 4.6\,8\,9\,5 \\
\text{Interest for 1 da.} & = & \frac{1}{6} \text{ of 6 da.} & = \$\quad .7\,8\,1\,6 \\
\hline
\text{Interest for 8 7 da.} & & & = \$\,6\,7.9\,9\,7\,8, \text{ or \$68.}
\end{array}
$$

Ans.

(4) 3 yr. 8 mo. 15 da. = $\frac{89}{24}$ yr. $5\frac{1}{2}\% - 4\frac{2}{3}\% = \frac{5}{6}\% = .00\frac{5}{6}$. $\$6,600 \times .00\frac{5}{6} \times \frac{89}{24} = \$203.96$. Ans.

(5) By the formula, Art. **37**, $t = \frac{100\,I}{Pr} = \frac{100 \times .2365}{1 \times 6} = \frac{23.65}{6}$ $= 3.94\frac{1}{6}$ yr. $.94\frac{1}{6} \times 12 = 11.3$ mo.; $.3 \times 30 = 9$ da. Therefore, the time is 3 yr. 11 mo. 9 da. Ans.

(6) In Feb., 1896, there were 29 da., it being a leap year; 29 da. $= \frac{29}{366}$ yr. Exact interest $= \$100,000 \times .06 \times \frac{29}{366} = \$475.41$. Feb., 1897, had 28 da.; and 28 da. $= \frac{28}{365}$ yr. The exact interest $= \$100,000 \times .06 \times \frac{28}{365} = \$460.27$. Difference, $\$475.41 - \$460.27 = \$15.14$. Ans.

(7) By Art. **15**, the interest on \$1 for 3 yr. 5 mo. 18 da. at 6% = \$.208. At $4\frac{1}{2}\%$ the interest is $\$.208 \times \frac{4\frac{1}{2}}{6} = \$.208 \times \frac{3}{4} = \$.156$. Since \$.156 is .156 of \$1.00, the interest is .156 of the principal. Ans.

(8) From April 8, 1899, to Oct. 17, 1899 = 192 da., or $\frac{192}{365}$ yr. Exact interest $= \$928.60 \times .07 \times \frac{192}{365} = \$34.19$. Ans.

(9) Since the interest is compounded semiannually, in 3 yr. 7 mo. 20 da. there are seven periods of 6 mo. each, and 1 mo. 20 da. in addition. Since the annual rate is 6%, for 6 mo. the rate is 3%. Referring to the compound-interest table, Art. **54**, the amount of \$1 for 7 yr. at 3% is \$1.229874. The simple interest for 1 mo. 20 da., or 50 da., at 6% is $\$1.229874 \times .008\frac{1}{3} = \$.01024895$. The total amount is $\$1.229874 + \$.01024895 = \$1.24012295$, say \$1.240123. $3,690 \times \$1.240123 = \$4,576.05+$. $\$4,576.05 - \$3,690 = \$886.05$. Ans.

(10) 270 da. $= \frac{270}{360}$ yr. $= \frac{3}{4}$ yr., considering 30 da. as a month. Ordinary interest $= \$3,600 \times .06 \times \frac{3}{4} = \$162$.
According to Art. **23**, the difference is $\frac{1}{73}$ of the ordinary interest; $\$162 \div 73 = \$2.22$. Ans.

(11) 2 yr. 3 mo. 18 da. = 2.3 yr. By Art. **40**,

$$\text{Principal} = \frac{100 \times \$535.20}{100 + 5 \times 2.3} = \frac{\$53,520}{111.5} = \$480. \quad \text{Ans.}$$

(12) 6 mo. 24 da. $= \frac{17}{30}$ yr. By Art. **46**,

$$\text{Present worth} = \frac{100 \times \$1,260}{100 + 5 \times \frac{17}{30}} = \frac{\$126,000}{100 + \frac{17}{6}} = \$126,000 \times \frac{6}{617}$$
$$= \$1,225.28. \quad \text{Ans.}$$

(13) See Art. **52.**

$240	=	prin. 1st yr.
$ 12	=	int. 1st yr. = $240 × .05.
$252	=	prin. 2d yr.
$ 12.60	=	int. 2d yr. = $252 × .05.
$264.60	=	prin. 3d yr.
$ 13.23	=	int. 3d yr. = $264.60 × .05.
$277.83	=	prin. 4th yr.
$ 13.89	=	int. 4th yr. = $277.83 × .05.
$291.72	=	prin. for 5 mo. 20 da.
$ 6.89	=	int. for 5 mo. 20 da. = $291.72 × .05 × $\frac{17}{36}$.
$298.61	=	amt. for 4 yr. 5 mo. 20 da.
$240.00	=	original principal.
$ 58.61	=	comp. int. for 4 yr. 5 mo. 20 da. Ans.

(14) See Art. **87.** At 6%, interest on $1 for

90 da. = $.015
3 da. = $.0005
93 da. = $.0155

Proceeds of $1 for 90 + 3 da. = $1 − $.0155 = $.9845. Face of the note = $1,200 ÷ .9845 = $1,218.89. Ans.

(15) Interest of $1 at 6% for 60 + 3 da., or 63 da. = $.0105. Bank discount = $2,400 × .0105 = $25.20. Ans.

(16)

$6,078$\frac{51}{100}$. *Newark, N. J., Dec. 6, 1898.*
Ninety days after date, for value received, I promise to pay Charles Allen, or order, Six Thousand Seventy-Eight and $\frac{51}{100}$ *Dollars, at the Ninth National Bank.* *John Clark.*

Proceeds of $1 at 5% for 90 + 3 da. = $.9870$\frac{5}{6}$. By Art. **87,** face of the note = $6,000 ÷ .9870$\frac{5}{6}$ = $6,078.51+. Ans.

(17) See Art. **84.**
Maturity, Oct. 2, 1900.
Amount of note at maturity $1,224.80
Term of discount, 31 da.
Discount 6.33
Proceeds .... $1,218.47. Ans.

(18) By Art. **38**,

$$\text{Time} = \frac{\overset{5}{\cancel{100}} \times \overset{23.22}{\cancel{139.32}}}{\underset{43}{\cancel{860}} \times \cancel{6}} = \frac{116.1}{43} = 2.7 \text{ yr.}$$

.7 yr. = .7 × 12 mo. = 8.4 mo.; .4 mo. = .4 of 30 da. = 12 da. Hence, the time is 2 yr. 8 mo. 12 da. Ans.

(19) By Art. **38**,

$$\text{Time} = \frac{\overset{20}{\cancel{100}} \times \overset{.4}{\cancel{58.40}}}{\underset{146}{\cancel{2,920}} \times \cancel{5}} = .4 \text{ yr.}$$

.4 yr. = .4 of 365 da. = 146 da. 146 da. after April 1, 1900, is Aug. 25, 1900. Ans.

(20)

Principal	$12,000.00
Exact interest from May 10, 1894, to Jan. 9, 1895 (244 da.)	481.32
Amount	$12,481.32
Sum of first and second payments ($200 + $960)	1,160.00
New principal	$11,321.32
Exact interest from Jan. 9 to Mar. 1 (51 da.)	94.91
Amount	$11,416.23
Third payment	150.00
New principal	$11,266.23
Exact interest from Mar. 1 to May 1 (61 da.)	112.97
Amount	$11,379.20
Fourth payment	500.00
New principal	$10,879.20
Interest from May 1 to Sept. 1 (123 da.)	219.97
Amount due Sept. 1, 1895, by U. S. rule	$11,099.17

By Art. **62**.

Principal	$12,000.00
Interest of $12,000 from May 10, 1894, to Sept. 1, 1895 (1 yr. 114 da.)	944.88
Amount	$12,944.88

Amount of $200 from Sept. 1, 1894, to Sept. 1, 1895 (1 yr.)	$212.00
Amount of $960 from Jan. 9 to Sept. 1 (235 da.)	997.08
Amount of $150 from Mar. 1 to Sept. 1 (184 da.)	154.54
Amount of $500 from May 1 to Sept. 1 (123 da.)	510.11
Sum of payments with interest	$1,873.73

Amount due at time of settlement (12,944.88 − 1,873.73) = $11,071.15. The amount due by the U. S. rule is $11,099.17; the difference is $11,099.17 − $11,071.15 = $28.02. Ans.

(21) By Art. **34,**

$$\text{Rate} = \frac{100 \times 148.31}{1{,}280 \times 2\frac{29}{60}} = \frac{\overset{\overset{5}{10}}{\cancel{100}} \times 148.31 \times \overset{15}{\cancel{60}}}{\underset{\underset{\underset{16}{\cancel{32}}}{\cancel{128}}}{\cancel{1{,}280}} \times 149} = 4\tfrac{2}{3}\%. \quad \text{Ans.}$$

(22) By Art. **31,**

$$\text{Principal} = \frac{100 \times 66}{6 \times \frac{33}{360}} = \frac{6{,}600}{\frac{11}{20}} = 6{,}600 \times \tfrac{20}{11} = \$12{,}000.$$

This is really the amount, or the face of the note and the interest. Then, by Art. **40,**

$$\text{Principal or face} = \frac{100 \times 12{,}000}{100 + 8 \times \frac{33}{360}} = \frac{1{,}200{,}000}{\frac{1511}{15}} = \$11{,}912.64. \quad \text{Ans.}$$

(23) See Art. **29.**

Interest for 1 yr. = $8,000 × .06....................	$ 4 8 0.0 0
Interest for 5 yr. 7 mo. 24 da. ($\frac{339}{60}$ yr.) = $480 × $\frac{339}{60}$..	$ 2,7 1 2 0 0
Interest of $480 for $4\frac{39}{60} + 3\frac{39}{60} + 2\frac{39}{60} + 1\frac{39}{60} + \frac{39}{60}$ = interest for $13\frac{1}{4}$ yr. = $480 × .06 × $13\frac{1}{4}$..............	3 8 1.6 0
	$ 3,0 9 3.6 0
	8,0 0 0.0 0
Amount due = $8,000 + $3,093.60	$ 1 1,0 9 3.6 0
	Ans.

(24) The difference in the rate is $5\frac{1}{2}\% - 4\frac{1}{3}\% = 1\frac{1}{6}\%$. 1 yr. 9 mo. 18 da. = $\frac{9}{5}$ yr.

By Art. **31,**

$$\text{Principal} = \frac{100 \times 42}{1\frac{1}{6} \times \frac{9}{5}} = \frac{4{,}200}{\frac{21}{10}} = \frac{\overset{200}{\cancel{4{,}200}}}{1} \times \frac{10}{\cancel{21}} = \$2{,}000. \quad \text{Ans.}$$

(25) Since $\frac{3}{4}$ of A's money equals $\frac{2}{3}$ of B's, then the interest on $\frac{2}{3}$ of B's = $\frac{1320}{2}$ = \$660, and the interest on $\frac{3}{4}$ of A's = $\frac{1320}{2}$ = \$660. By Art. **31,** principal, or $\frac{3}{4}$ of A's money $= \frac{100 \times 660}{5\frac{1}{2} \times 4} = \frac{66{,}000}{22} = 3{,}000$;

$$3{,}000 = \frac{3}{4};\ \frac{4}{4} = \frac{4}{\cancel{3}} \times \overset{1{,}000}{\cancel{3{,}000}} = \$4{,}000, \text{ A's money. Ans.}$$

$$\text{Since } \tfrac{2}{3} \text{ of B's money} = \$3{,}000,\ \tfrac{3}{3} = \frac{3}{\cancel{2}} \times \frac{\overset{1{,}500}{\cancel{3{,}000}}}{1} = \$4{,}500, \text{ B's money.}$$

Ans.

ARITHMETIC.

(1) Yearly dividends $= 2 \times 3\frac{1}{2}\% = 7\%$. At \$100 a share, the yearly dividend on 1 share would be $\$100 \times .07 = \$7$, and to yield \$5,600 it would take $5{,}600 \div 7 = 800$ shares. Cost $= (108\frac{1}{2} + \frac{1}{8}) \times 800 = \$86{,}900$. Ans.

(2) (*a*) At $\frac{1}{4}\%$ commission, the broker receives on 1 share $.0025 \times \$100 = \$.25$, and to receive \$50 he bought $50 \div .25 = 200$ shares. Ans.

(*b*) Cost $= \$110\frac{1}{2} \times 200 = \$22{,}100$. Ans.

(3) Apply rule, Art. **14** $(114\frac{1}{8} + \frac{1}{8}) \times 125 = \$14{,}281.25$. Ans.

(4) Yearly dividend $= 2\% \times 4 = 8\%$. By Art. **16**, number of shares $= 4{,}800 \div 8 = 600$. Cost $= (112\frac{3}{8} + \frac{1}{8}) \times 600 = \$67{,}500$. Ans.

(5) Per cent. received on investment in first case is $10 \div 110 = 9\frac{1}{11}\%$; in the second case, $8 \div 90 = 8\frac{8}{9}\%$. Therefore, the former is better by $9\frac{1}{11}\% - 8\frac{8}{9}\% = \frac{20}{99}\%$. Ans.

(6) Use rule, Art. **27.**
$\$5{,}600 \times .01125 = \$63$. $\$5{,}600 + \$63 = \$5{,}663$. Ans.

(7) Apply rule given in Art. **30.** Interest on \$8,000 at 6% for 60 + 3 da. $= \$8{,}000 \times .0105 = \$84$. Proceeds of $\$8{,}000 = \$8{,}000 - \$84 = \$7{,}916$. Premium $= \$8{,}000 \times .015 = \$120$. Therefore, the cost is $\$7{,}916 + \$120 = \$8{,}036$. Ans.

(8) Interest of \$6,800 for 93 da. at $5\% = \$6{,}800 \times .0129\frac{1}{6} = \$87.83+$. Proceeds of $\$6{,}800 = \$6{,}800 - \$87.83+ = \$6{,}712.17-$. The discount $= \$6{,}800 \times .0125 = \$85$. Cost of the draft $= \$6{,}712.17 - \$85 = \$6{,}627.17-$. Ans.

(9) Use rule, Art. **27.** $\$2{,}800 \times .0075 = \$21$; $\$2{,}800 + \$21 = \$2{,}821$. Ans.

(10) Use rule, Art. **33.** Interest of \$1 for 63 da. at 6% = .0105. Proceeds of \$1 = \$1 − \$.0105 = \$.9895. Discount on \$1 = \$.015. Proceeds − discount = \$.9895 − \$.015 = \$.9745. Face of draft = \$12,000 ÷ .9745 = \$12,314.01. Ans.

(11) Apply rule, Art. **40.** £400 15s. = £400¾. \$4.855 × 400.75 = \$1,945.64. Ans.

(12) 1 franc = $\frac{\$1}{5.22}$; $\frac{\$1}{5.22} \times 20{,}000 = \frac{20{,}000}{5.22}$ = \$3,831.42. Ans.

(13)

$$\begin{array}{r|l} 2 & \\ \cancel{6} & \\ \cancel{30} \text{ bu. wheat} & x \text{ bu. oats} \\ 5 \text{ bu. oats} & \cancel{3} \text{ bu. corn} \\ 7 \text{ bu. corn} & \cancel{4} \text{ bu. rye} \\ \cancel{8} \text{ bu. rye} & \cancel{5} \text{ bu. wheat} \\ 2 & \end{array}$$

$$x = 2 \times 5 \times 7 \times 2 = 140 \text{ bu. of oats. Ans.}$$

As in example in Art. **44,** place the unknown quantity x on the right-hand side of the vertical line of division, and the quantity to which x is equivalent on the left-hand side. Arrange the other quantities so that a quantity of the same kind appears on each side of the line. Thus, 30 bu. of wheat are equivalent to a certain number of bushels of oats; hence, we place 30 bu. of wheat on the left and x bu. of oats on the right. Now, since 3 bu. of corn are worth 5 bu. of oats, and we already have oats on the right-hand side, we place the 5 bu. of oats on the left-hand side, and the 3 bu. of corn on the right-hand side. We then place 7 bu. of corn on the left and 4 bu. of rye on the right, and also 8 bu. of rye on the left and 5 bu. of wheat on the right. We have then wheat, oats, corn, and rye on each side of the line. Canceling and dividing the product of the numbers on the left by the product of those on the right, x = 140 bu. of oats.

(14)

$$\begin{array}{l|l} \text{A earns in 1 da.} & \text{E earns in } x \text{ da.} \\ \text{E " " 9 "} & \text{D " " 5 "} \\ \text{D " " 11 "} & \text{C " " 6 "} \\ \text{C " " 7 "} & \text{B " " 3 "} \\ \text{B " " 5 "} & \text{A " " 4 "} \end{array}$$

$$x = \frac{\overset{3}{\cancel{1}} \times \cancel{9} \times 11 \times 7 \times \cancel{5}}{\cancel{5} \times \cancel{6} \times \cancel{3} \times \underset{2}{4}} = \frac{77}{8} = 9.625 \text{ da.}$$

It is first necessary to find how many days of E's work are equivalent to 1 da. of A's work; we find this to be 9⅝, or 9.625, da. In other words, A earns 9.625 times as much as E. Hence, while E is earning \$100, A earns 9.625 × \$100 = \$962.50. Ans.

(15)

A goes 50 mi.	D goes x mi.		100	$2x$
D " $8\frac{1}{2}$ "	C " 9 "		17	18
C " $9\frac{1}{2}$ "	B " 10 "	or,	19	20
B " $11\frac{1}{2}$ "	A " 12 "		23	24

We multiply all the numbers by 2 to get rid of the fractions. Therefore,

$$x = \frac{\overset{5}{\cancel{100}} \times 17 \times 19 \times 23}{2 \times 18 \times \cancel{20} \times 24} = \frac{5 \times 17 \times 19 \times 23}{2 \times 18 \times 24} = 42.99+ \text{ mi.} \quad \text{Ans.}$$

(16)

$$\begin{array}{rcrl} \$800 \text{ for } 30 \text{ da.} & = & \$24000 & \text{for 1 da.} \\ \$500 \text{ for } 20 \text{ da.} & = & 10000 & \text{for 1 da.} \\ \$400 \text{ for } 60 \text{ da.} & = & \underline{24000} & \text{for 1 da.} \\ & & \$58000 & \text{for 1 da.} \end{array}$$

\$58,000 at interest for 1 da. equals \$1,000 at interest for $\frac{58000}{1000}$, or 58 da. Ans.

(17)

$$\begin{array}{lcrl} \$1200 \text{ for } 30 \text{ da.} & = & \$\ 36000 & \text{for 1 da.} \\ \$3600 \text{ for } 60 \text{ da.} & = & 216000 & \text{for 1 da.} \\ \underline{\$1200} \text{ for } 90 \text{ da.} & = & \underline{108000} & \text{for 1 da.} \\ \$6000 & &)\$360000 & \\ & & \overline{\quad 60} & \text{da.} \end{array}$$

The whole debt may be equitably paid 60 da. after the obligation is incurred. Ans.

(18)

$$\begin{array}{rcr} \$\ \ 800 \times 0 & = & 0 \\ \$\ \ 900 \times 39 & = & \$\ 35100 \\ \$1000 \times 59 & = & 59000 \\ \underline{\$1200} \times 91 & = & \underline{109200} \\ \$3900 & &)\$203300 \\ & & \overline{52+} \text{ da.} \end{array}$$

52 da. after Sept. 1, or Oct. 23. Ans.

(19) The use of $\frac{1}{3}$ for 20 da. equals the use of the whole bill for $6\frac{2}{3}$ da.; of $\frac{1}{4}$ for 30 da. equals the use of all for $7\frac{1}{2}$ da.; of $\frac{1}{6}$ for 60 da. equals the use of all for 10 da., and the use of the remainder, or $\frac{1}{4}$, for 90 da., equals the use of all the bill for $22\frac{1}{2}$ da. Hence, the sum of all the credits equals the use of the whole bill for $46\frac{2}{3}$, or 47, da. Ans.

$$\begin{array}{lcr} \frac{1}{3} \times 20 & = & 6\frac{2}{3} \\ \frac{1}{4} \times 30 & = & 7\frac{1}{2} \\ \frac{1}{6} \times 60 & = & 10 \\ \underline{\frac{1}{4}} \times 90 & = & \underline{22\frac{1}{2}} \\ \frac{12}{12} & & 46\frac{2}{3} \end{array}$$

(20) Applying rule, Art. **55**, using June 1 as the focal date, and adding three days of grace to the time the draft has to run, since no state is mentioned in the example.

June 18,	$1200 \times 17 =$	20400	July 1,	$1000 \times 30 =$	30000
Sept. 8,	$2000 \times 99 =$	198000	Aug. 27,	$500 \times 87 =$	43500
Sept. 19,	$3000 \times 110 =$	330000	Sept. 30,	$800 \times 121 =$	96800
Dec. 24,	$2400 \times 206 =$	494400	Oct. 10,	$6000 \times 131 =$	786000
Dec. 31,	$5000 \times 213 =$	1065000	Nov. 20,	$900 \times 172 =$	154800
	13600	2107800		9200	1111100
	9200	1111100			
	4400	) 996700			

226.5+, or 227 da.

227 days after June 1, 1899, is Jan. 14, 1900. Ans.

(21) Use rule, Art. **72.**

$$\begin{array}{rcr}
\$.25 \times 12 & = & \$\ \ 3.00 \\
\$.35 \times 12 & = & 4.20 \\
\$.45 \times 48 & = & 21.60 \\
72 &) & \$28.80 \\
 & & \$.40
\end{array}\quad \text{Ans.}$$

(22) Use rule, Art. **72.**

$$\begin{array}{rcr}
\$1080 \times 24 & = & \$25920 \\
\$2160 \times 36 & = & 77760 \\
\$3240 \times 48 & = & 155520 \\
108 &) & \$259200 \\
\text{Average cost of a lot} & = & \$2400 \\
\text{Average gain on 1 lot} & = & 100 \\
\text{Selling price,} & & \$2500
\end{array}\quad \text{Ans.}$$

(23) Use rule, Art. **78.**

$$40\left\{\begin{array}{l|c|c|l}
29 & 7 & & 7 \times 29 = 203 \\
37 & & 1 & 1 \times 37 = \ \ 37 \\
41 & & 3 & 3 \times 41 = 123 \\
47 & 11 & & 11 \times 47 = 517 \\
 & & & \qquad\quad\ \ 880
\end{array}\right.\quad \text{Ans.}$$

(24) Use rule, Art. **78.**

$$\$5\left\{\begin{array}{l|c|c|c|c}
\$2\frac{1}{2} & 2\frac{1}{2} & & & 2\frac{1}{2} \\
3\frac{1}{2} & & 2\frac{1}{2} & & 2\frac{1}{2} \\
4\frac{1}{2} & & & 2\frac{1}{2} & 2\frac{1}{2} \\
7\frac{1}{2} & 2\frac{1}{2} & 1\frac{1}{2} & \frac{1}{2} & 4\frac{1}{2}
\end{array}\right\} \times 2 = \left|\begin{array}{c} 5 \\ 5 \\ 5 \\ 9 \end{array}\right| \quad \text{Ans.}$$

(25) If 1,280 bu. cost \$960, 1 bu. cost \$960 ÷ 1,280 = \$.75.

$$75\left\{\begin{array}{r|c|c|rcr} 100 & 35 & & 35\times 16 & = & 560 \\ 80 & & 15 & 15\times 16 & = & 240 \\ 60 & & 5 & 5\times 16 & = & 80 \\ 40 & 25 & & \underline{25}\times 16 & = & \underline{400} \\ & & & 80 & & 1280 \end{array}\right\}\text{Ans.}$$

1,280 ÷ 80 = 16.

$$75\left\{\begin{array}{r|c|c|rcr} 100 & 15 & & 15\times 16 & = & 240 \\ 80 & & 35 & 35\times 16 & = & 560 \\ 60 & 25 & & 25\times 16 & = & 400 \\ 40 & & 5 & \underline{5}\times 16 & = & \underline{80} \\ & & & 80 & & 1280 \end{array}\right\}\text{Ans.}$$

The sum of the proportions 35, 15, 5, and 25 = 80 bu. Also 15, 35, 25, and 5 = 80 bu. The miller bought 1,280 bu., or 1,280 ÷ 80 = 16 times as many. Hence, the proportions must be multiplied by 16.

INDEX

NOTE:—All items in this index refer first to the section and then to the page of the section. Thus, "Fractions 2 1" means that fractions will be found on page 1 of section 2.

HOW TO OPEN A NEW BOOK.......

From "Modern Bookbinding Practically Considered,"
By Wm. Matthews.

HOLD the book with its back on a smooth or covered table; let the front board down, then the other, holding the leaves in one hand while you open a few leaves at the back, then a few at the front, and so go on, alternately opening back and front, **gently** pressing open the sections till you reach the center of the Volume. Do this two or three times, and you will obtain the best results. Open the Volume violently or carelessly in any one place, and you will likely break the back, and cause a start in the leaves.

Never force the back. If it does not yield to gently opening, rely upon it the back is too tightly or strongly lined.

A connoisseur, many years ago, an excellent customer of mine, who thought he knew perfectly how to handle books, came into my office when I had an expensive binding just brought from the bindery, ready to be sent home; he, before my eyes, took hold of the Volume, and, **tightly holding** the leaves in each hand, instead of allowing them **free play,** violently opened it in the center and exclaimed, "How beautifully your bindings open!" I almost fainted. He had broken the back of the Volume, and it had to be rebound.

INTERNATIONAL TEXTBOOK COMPANY,
Scranton, Pa.

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